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BUSINESS MATHEMATICS

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Business Mathematics Note 1 Unit 1: Introduction: Scope, Data Collection and Classification CONTENTS Objectives Introduction 1.1 Meaning and Definitions of Statistics 1.1.1 Statistics as Data 1.1.2 Statistics as a Science 1.2 Importance of statistics 1.3 Scope of Statistics 1.3 .1 Statistics and Industry 1.3.2 Statistics and Commerce 1.3.3 Statistics and Agriculture 1.3.4 Statistics and Economics 1.3.5 Statistics and Education 1.3.6 Statistics and Planning 1.3.7 Statistics and Medicine: 1.4 Variable and Attribute 1.4.1 Notation for Variables and Attributes 1.5 Primary Data and Secondary Data 1.5.1 Distinction between Primary and secondary Data : 1.6 Population and Sample 1.7 Complete Enumeration and Sample Survey 1.7.1 Sample survey 1.8 Statistical Enquiry 1.9 Classification 1.9.1 Objectives of Classification 1.9.2 Requisites of a Good Classification 1.9.3 Types of Classification 1.10 Tabulation 1.10.1 Objectives of Tabulation 1.10.2 Difference between Classification and Tabulation 1.10.3 Main Parts of a Table Contd....

Unit 1: Introduction: Scope, Data Collection and Classification Note 2 1.10.4 Rules for Tabulation 1.10.5 Type of Tables 1.11 Manual and Mechanical Methods of Tabulation 1.12 Summary 1.13 Keywords 1.14 Review Questions 1.15 Further Readings Objectives After studying this unit, you will be able to: 1. Define Statistics and discuss its scope and importance 2. Discuss the meaning of variable and attribute 3. Describe Primary Data and Secondary Data, Population and Sample, Complete Enumeration and Sample Survey, Statistical Enquiry 4. Focus on Classification of data 5. Explain Tabulation, manual and mechanical tabulation Introduction The collected data are a complex and unorganised mass of figures which is very difficult to analyse and interpret. Therefore, it becomes necessary to organise this so that it becomes easier to grasp its broad features. Further, in order to apply the tools of analysis and interpretation, it is essential that the data are arranged in a definite form. This task is accomplished by the process of classification and tabulation. In this unit, we will define Statistics and discuss its scope and importance. Further we will discuss the meaning of variable and attribute. We will also focus on Primary Data and Secondary Data, Population and Sample, Complete Enumeration and Sample Survey, Statistical Enquiry. Finally we will move to classification of data, tabulation and mechanical tabulation. 1.1 Meaning and Definitions of Statistics The meaning of the word 'Statistics' is implied by the pattern of development of the subject. Since the subject originated with the collection of data and then, in later years, the techniques of analysis and interpretation were developed, the word 'statistics' has been used in both the plural and the singular sense. Statistics, in plural sense, means a set of numerical figures or data. In the singular sense, it represents a method of study and therefore, refers to statistical principles and methods developed for analysis and interpretation of data. Statistics has been defined in different ways by different authors. These definitions can be broadly classified into two categories. In the first category are those definitions which lay emphasis on statistics as data whereas the definitions in second category emphasize statistics as a science.

Business Mathematics Note 3 1.1.1 Statistics as Data Statistics used in the plural sense implies a set of numerical figures collected with reference to a certain problem under investigation. It may be noted here that any set of numerical figures cannot be regarded as statistics. There are certain characteristics which must be satisfied by a given set of numerical figures in order that they may be termed as statistics. Before giving these characteristics it will be advantageous to go through the definitions of statistics in the plural sense, given by noted scholars. 1. "Statistics are numerical facts in any department of enquiry placed in relation to each other." —A.L. Bowley The main features of the above definition are: (i) Statistics (or Data) implies numerical facts. (ii) Numerical facts or figures are related to some enquiry or investigation. (iii) Numerical facts should be capable of being arranged in relation to each other. On the basis of the above features we can say that data are those numerical facts which have been expressed as a set of numerical figures related to each other and to some area of enquiry or research. We may, however, note here that all the characteristics of data are not covered by the above definition. 2. "By statistics we mean quantitative data affected to a marked extent by multiplicity of causes." —Yule & Kendall This definition covers two aspects, i.e., the data are quantitative and affected by a large number of causes. 3. "Statistics are classified facts respecting the conditions of the people in a state - especially those facts which can be stated in numbers or in tables of numbers or in any other tabular or classified arrangement." —Webster 4. "A collection of noteworthy facts concerning state, both historical and descriptive." —Achenwall Definitions 3 and 4, given above, are not comprehensive because these confine the scope of statistics only to facts and figures related to the conditions of the people in a state. However, as we know that data are now collected on almost all the aspects of human and natural activities, it cannot be regarded as a state-craft only. 5. "Statistics are measurements, enumerations or estimates of natural or social phenomena, systematically arranged, so as to exhibit their interrelations." —L.R. Connor This definition also covers only some but not all characteristics of data. 6. "By statistics we mean aggregate of facts affected to a marked extent by a multiplicity of causes, numerically expressed, enumerated or estimated according

Unit 1: Introduction: Scope, Data Collection and Classification Note 4 to a reasonable standard of accuracy, collected in a systematic manner for a predetermined purpose and placed in relation to each other." —H. Secrist This definition can be taken as a comprehensive definition of statistics since most of the characteristics of statistics are covered by it.` Caution! When statistics are being obtained by measurement of units, it is necessary to maintain a reasonable degree or standard of accuracy in measurements. Example: Would you regard the following information as statistics? Explain by giving reasons. (i) The height of a person is 160 cms. (ii) The height of Ram is 165 cms and of Shyam is 155 cms. (iii) Ram is taller than Shyam. (iv) Ram is taller than Shyam by 10 cms. (v) The height of Ram is 165 cms and weight of Shyam is 55 kgs. Solution. Each of the above statement should be examined with reference to the following conditions : (a) Whether information is presented as aggregate of numerical figures (b) Whether numerical figures are homogeneous or comparable (c) Whether numerical figures are affected by a multiplicity of factors On examination of the given information in the light of these conditions we find that only the information given by statement (ii) can be regarded as statistics. It should be noted that condition (c) will be satisfied, almost invariably. In order to illustrate the circumstances in which this condition is not satisfied, we assume that a relation between quantity demanded and price of a commodity is given by the mathematical equation q = 100 ? 10p and the quantity demanded at various prices, using this equation, is shown in the following table, p 1 2 3 4 5 6 7 8 9 10 q 90 80 70 60 50 40 30 20 10 0 The above information cannot be regarded as statistics because here quantity demanded is affected by only one factor, i.e., price and not by a multiplicity of factors. Contrary to this, the figures of quantity demanded obtained from a market at these very prices are to be regarded as statistics. 1.1.2 Statistics as a Science The use of the word 'STATISTICS' in singular form refers to a science which provides methods of collection, analysis and interpretation of statistical data. Thus, statistics as a science is defined on the basis of its functions and different scholars have defined it

Business Mathematics Note 5 in a different way. In order to know about various aspects of statistics, we now state some of these definitions. 1. "Statistics is the science of counting." — A.L. Bowley 2. "Statistics may rightly be called the science of averages." — A.L. Bowley 3. "Statistics is the science of measurement of social organism regarded as a whole in all its manifestations." — A.L. Bowley 1. "Statistics is the science of estimates and probabilities." — Boddington All of the above definitions are incomplete in one sense or the other because each considers only one aspect of statistics. Task: Go to website http://psy1.clarion.edu/mm/General/Methods/Methods.html and differentiate between experimental and non experimental methods. According to the first definition, statistics is the science of counting. However, we know that if the population or group under investigation is large, we do not count but obtain estimates. The second definition viz. statistics is the science of averages, covers only one aspect, i.e., measures of average but, besides this, there are other measures used to describe a given set of data. The third definition limits the scope of statistics to social sciences only. Bowley himself realised this limitation and admitted that scope of statistics is not confined to this area only. The fourth definition considers yet another aspect of statistics. Although, use of estimates and probabilities have become very popular in modern statistics but there are other techniques, as well, which are also very important. Task: Discuss in group statistics in singular and plural form The following definitions cover some more but not all aspects of statistics. 5. "The science of statistics is the method of judging collective, natural or social phenomena from the results obtained by the analysis or enumeration or collection of estimates." — W.I. King 6. "Statistics or statistical method may be defined as collection, presentation, analysis and interpretation of numerical data." — Croxton and Cowden This is a simple and comprehensive definition of statistics which implies that statistics is a scientific method.

Unit 1: Introduction: Scope, Data Collection and Classification Note 6 7. "Statistics is a science which deals with collection, classification and tabulation of numerical facts as the basis for the explanation, description and comparison of phenomena." — Lovitt 8. "Statistics is the science which deals with the methods of collecting, classifying, presenting, comparing and interpreting numerical data collected to throw some light on any sphere of enquiry." — Seligman The definitions given by Lovitt and Seligman are similar to the definition of Croxton and Cowden except that they regard statistics as a science while Croxton and Cowden has termed it as a scientific method. With the development of the subject of statistics, the definitions of statistics given above have also become outdated. In the last few decades the discipline of drawing conclusions and making decisions under uncertainty has grown which is proving to be very helpful to decision-makers, particularly in the field of business. Although, various definitions have been given which include this aspect of statistics also, we shall now give a definition of statistics, given by Spiegel, to reflect this new dimension of statistics. 9. "Statistics is concerned with scientific method for collecting, organising, summarising, presenting and analysing data as well as drawing valid conclusions and making reasonable decisions on the basis of such analysis." Notes Statistics, in singular sense, is a science which consists of various statistical methods that can be used for collection, classification, presentation and analysis of data relating to social, political, natural, economical, business or any other phenomena. The results of the analysis can be used further to draw valid conclusions and to make reasonable decisions in the face of uncertainty. Self Assessment Fill in the blanks: 1. Statistics used in the .... sense implies a set of numerical figures collected with reference to a certain problem under investigation 2. In the .... sense, it represents a method of study and therefore, refers to statistical principles and methods developed for analysis and interpretation of data. 3. Statistics as a science is defined on the basis of its .... and different scholars have defined it in a different way. 4. Statistics is the science of .... 5. Statistics is a science which deals with collection, classification and tabulation of numerical facts as the basis for the explanation, description and comparison of ....

Business Mathematics Note 7 1.2 Importance of statistics It is perhaps difficult to imagine a field of knowledge which can do without statistics. To begin with, the State started the use of statistics and now it is being used by almost every branch of knowledge such as physics, chemistry, biology, sociology, geography, economics, business, etc. The use of statistics provides precision to various ideas and can also suggest possible ways of tackling a problem relating to any of the above subjects. The importance of statistics has been summarised by A.L. Bowley as, "A knowledge of statistics is like a knowledge of foreign language or of algebra. It may prove of use at any time under any circumstances." We shall discuss briefly, the importance of statistics in the following major areas: (a) Importance to the State (b) Importance in economics (c) Importance in national income accounting (d) Importance in planning (e) Importance in business (a) We know that the subject of statistics originated for helping the ancient rulers in the assessment of their military and economic strength. Gradually its scope was enlarged to tackle other problems relating to political activities of the State. In modern era, the role of State has increased and various governments of the world also take care of the welfare of its people. Therefore, these governments require much greater information in the form of numerical figures for the fulfillment of welfare objectives in addition to the efficient running of their administration. In a democratic form of government, various political groups are also guided by the statistical analysis regarding their popularity in the masses. Thus, it can be said that it is impossible to think about the functioning of modern state in the absence of statistics. (b) Importance in economics: Statistics is an indispensable tool for a proper understanding of various economic problems. It also provides important guidelines for the formulation of various economic policies. Almost every economic problem is capable of being expressed in the form of numerical figures, e.g., the output of agriculture or of industry, volume of exports and imports, prices of commodities, income of the people, distribution of land holding, etc. In each case, the data are affected by a multiplicity of factors. Further, it can be shown that the other conditions prescribed for statistical data are also satisfied. Thus, we can say that the study of various economic problems is essentially the one of a statistical nature. Inductive method of generalisation, popularly used in economics, is also based on statistical principles. Did u Know? Various famous laws in economics such as, the law of diminishing marginal utility, the law of diminishing marginal returns, the theory of revealed preference, etc., are

Unit 1: Introduction: Scope, Data Collection and Classification Note 8 based on generalisations from observation of economic behaviour of a large number of individuals. Statistical methods are also useful in estimating a mathematical relation between various economic variables. For example, the data on prices and corresponding quantities demanded of a commodity can be used to estimate the mathematical form of the demand relationship between two variables. Further, the validity of a generalisation or relation between variables can also be tested by using statistical techniques. Statistical analysis of a given data can also be used for the precise understanding of an economic problem. For example, to study the problem of inequalities of income in a society, we can classify the relevant data and, if necessary, compute certain measures to bring the problem into focus. Using statistics, suitable policy measures can also be adopted for tackling this problem. Similarly, statistical methods can also be used to understand and to suggest a suitable solution for problems in other areas such as industry, agricultural, human resource development, international trade, etc. Realising the importance of statistics in economics, a separate branch of economics, known as econometrics has been developed during the recent years. Did u know? The techniques of econometrics are based upon the principles of economics, statistics and mathematics. (c) Importance in national income accounting: The system of keeping the accounts of income and expenditure of a country is known as national income accounting. These accounts contain information on various macro-economic variables like national income, expenditure, production, savings, investments, volume of exports and imports, etc. The national income accounts of a country are very useful in having an idea about the broad features of its economy or of a particular region. The preparation of these accounts require data, regarding various variables, at the macro-level. Since such information is very difficult, if not impossible, to obtain, is often estimated by using techniques and principles of statistics. (d) Importance in planning: Planning is indispensable for achieving faster rate of growth through the best use of a nation's resources. It also requires a good deal of statistical data on various aspects of the economy. One of the aims of planning could be to achieve a specified rate of growth of the economy. Using statistical techniques, it is possible to assess the amounts of various resources available in the economy and accordingly determine whether the specified rate of growth is sustainable or not. The statistical analysis of data regarding an economy may reveal certain areas which might require special attention, e.g., a situation of growing unemployment or a situation of rising prices during past few years. Statistical techniques and principles can also guide the Government in adopting suitable policy measures to rectify such situations. In addition to this, these techniques can be used to assess various policies of the Government in the past. Thus, it is rather impossible to think of a situation where planning and evaluation of various policies can be done without the use of statistical techniques. In view of this it is sometimes said that, "Planning without statistics is a ship without rudder

Business Mathematics Note 9 and compass". Hence statistics is an important tool for the quantification of various planning policies. (e) Importance in business: With the increase in size of business of a firm and with the uncertainties of business because of cut throat competition, the need for statistical information and statistical analysis of various business situations has increased tremendously. Prior to this, when the size of business used to be smaller without much complexities, a single person, usually owner or manager of the firm, used to take all decisions regarding its business. For example, he used to decide, from where the necessary raw materials and other factors of production were to be obtained, how much of output will be produced, where it will be sold, etc. This type of decision making was usually based on experience and expectations of this single man and as such had no scientific basis. Note Statistical methods are explicit in nature and provide clearly defined measure of error. On the other hand, normative techniques based on the judgment and rule of thumb, although help in effective decision-making but fail to specify estimate of error. The modern era is an era of mass production in which size and number of firms have increased enormously. The increase in the number of firms has resulted into cut throat competition among various firms and, consequently, the uncertainties in business have become greater than before. Under such circumstances, it has become almost impossible for a single man to take decisions regarding various aspects of a rather complex business. It is precisely this point from where the role of statistics started in business. Now a days no business, large or small, public or private, can prosper without the help of statistics. Statistics provides necessary techniques to a businessman for the formulation of various policies with regard to his business. Did u Know? The process of collection and analysis of data becomes necessary right from the stage of launching a particular business. Case let: Basic Statistics and Fraud Detection Pam Mantone is a Forensic Certified Public Accountant and a Certified Fraud Examiner and has earned the Certified in Financial Forensics designation from the American Institute of Certified Public Accountants (AICPA). She practices in the areas of audit and attestation with emphasis on forensic accounting, fraud examinations, financial institutions, non-profit organizations, and governments. Pam has performed forensic and fraud auditing services for organizations, including the gathering of forensic evidence and testifying to findings. She also provides consulting services regarding implementation of fraud prevention and fraud detection internal control systems. Pam was recently designated as a Certified Information Technology Professional. Pam has supervised audits for city governments and their component units, including local airports, industrial boards, school boards, utility systems, and both city and county school systems, while attaining the GFOA Certificate of

Unit 1: Introduction: Scope, Data Collection and Classification Note 10 Achievement for Excellence in Financial Report for those entities. She also supervised the successful implementation of GASB 34 requirements. Her experience includes conducting and supervising audits of local banks, credit unions, local non-profit organizations and HUD audits. She manages and performs external and internal audits of financial institutions, including publicly-traded entities. She also manages and performs audits of internal control systems for various types of business entities. Pam is active in the high school liaison program sponsored by the Tennessee Society of Certified Public Accountants (TSCPA) in which she promotes accounting as a possible career choice to high school students. She also promotes accounting careers to college students seeking accounting degrees. Pam is a member of the AICPA, the TSCPA, and the Forensic Certified Public Accountant Society (FCPA). She is a Past Training Director for the Chattanooga Chapter of the Association of Certified Fraud Examiners (ACFE). She currently serves as the Treasurer of the Chattanooga Chapter of the TSCPA and serves on the association's Forensic and Valuation Services Committee. She also serves on the ACFE Advisory Council and is a CFF Champion for the state of Tennessee. She performs presentations to various organizations on a variety of topics including fraud and forensic techniques, internal control design and weaknesses and auditing techniques. http://thectiblog.com/2013/02/basic-statistics-and-fraud-detection-a-case-study/ Self Assessment State whether the following statements are true or false: 6. The use of statistics provides precision to various ideas and can also suggest possible ways of tackling a problem relating to any specified subjects. 7. It is almost possible to think about the functioning of modern state in the absence of statistics. Almost every economic problem is capable of being expressed in the form of numerical figures, 8. Study of various economic problems is essentially the one of a statistical nature. 9. Statistical analysis of a given data can also be used for the precise understanding of an economic problem. 10. The increase in the number of firms has resulted into cut throat competition among various firms and, consequently, the uncertainties in business have become smaller than before 1.3 Scope of Statistics Statistics is not a mere device for collecting numerical data, but as a means of developing sound techniques for their handling, analyzing and drawing valid inferences from them. Statistics is applied in every sphere of human activity – social as well as physical – like Biology, Commerce, Education, Planning, Business Management, Information Technology, etc. It is almost impossible to find a single department of human activity where statistics cannot be applied.

Business Mathematics Note 11 1.3 .1 Statistics and Industry Statistics is widely used in many industries. In industries, control charts are widely used to maintain a certain quality level. In production engineering, to find whether the product is conforming to specifications or not, statistical tools, namely inspection plans, control charts, etc., are of extreme importance. In inspection plans we have to resort to some kind of sampling – a very important aspect of Statistics. 1.3.2 Statistics and Commerce Statistics are lifeblood of successful commerce. Any businessman cannot afford to either by under stocking or having overstock of his goods. In the beginning he estimates the demand for his goods and then takes steps to adjust with his output or purchases. Thus statistics is indispensable in business and commerce. As so many multinational companies have invaded into our Indian economy, the size and volume of business is increasing. On one side the stiff competition is increasing whereas on the other side the tastes are changing and new fashions are emerging. In this connection, market survey plays an important role to exhibit the present conditions and to forecast the likely changes in future. 1.3.3 Statistics and Agriculture Analysis of variance (ANOVA) is one of the statistical tools developed by Professor R.A. Fisher, plays a prominent role in agriculture experiments. In tests of significance based on small samples, it can be shown that statistics is adequate to test the significant difference between two sample means. In analysis of variance, we are concerned with the testing of equality of several population means. For example, five fertilizers are applied to five plots each of wheat and the yield of wheat on each of the plots are given. In such a situation, we are interested in finding out whether the effect of these fertilisers on the yield is significantly different or not. In other words, whether the samples are drawn from the same normal population or not. The answer to this problem is provided by the technique of ANOVA and it is used to test the homogeneity of several population means. 1.3.4 Statistics and Economics Statistical methods are useful in measuring numerical changes in complex groups and interpreting collective phenomenon. Nowadays the uses of statistics are abundantly made in any economic study. Both in economic theory and practice, statistical methods play an important role. Alfred Marshall said, " Statistics are the straw only which I like every other economist have to make the bricks". It may also be noted that statistical data and techniques of statistical tools are immensely useful in solving many economic problems such as wages, prices, production, distribution of income and wealth and so on. Did u know? Statistical tools like Index numbers, time series Analysis, Estimation theory, Testing Statistical Hypothesis are extensively used in business mathematics and economics

Unit 1: Introduction: Scope, Data Collection and Classification Note 12 1.3.5 Statistics and Education Statistics is widely used in education. Research has become a common feature in all branches of activities. Statistics is necessary for the formulation of policies to start new course, consideration of facilities available for new courses etc. There are many people engaged in research work to test the past knowledge and evolve new knowledge. These are possible only through statistics. 1.3.6 Statistics and Planning Statistics is indispensable in planning. In the modern world, which can be termed as the "world of planning", almost all the organisations in the government are seeking the help of planning for efficient working, for the formulation of policy decisions and execution of the same. In order to achieve the above goals, the statistical data relating to production, consumption, demand, supply, prices, investments, income expenditure etc and various advanced statistical techniques for processing, analysing and interpreting such complex data are of importance. In India statistics play an important role in planning, commissioning both at the central and state government levels. 1.3.7 Statistics and Medicine: In Medical sciences, statistical tools are widely used. In order to test the efficiency of a new drug or medicine, t - test is used or to compare the efficiency of two drugs or two medicines. More and more applications of statistics are at present used in clinical investigation.1.3.8 Statistics and Modern applications Recent developments in the fields of computer technology and information technology have enabled statistics to integrate their models and thus make statistics a part of decision making procedures of many organisations. There are so many software packages available for solving design of experiments, forecasting simulation problems etc. Notes SYSTAT, a software package offers mere scientific and technical graphing options than any other desktop statistics package. SYSTAT supports all types of scientific and technical research in various diversified fields as follows 1. Archeology: Evolution of skull dimensions 2. Epidemiology: Tuberculosis 3. Statistics: Theoretical distributions 4. Manufacturing: Quality improvement 5. Medical research: Clinical investigations. 6. Geology: Estimation of Uranium reserves from ground water In conclusion, some of the stages of business where statistical analysis has become necessary are briefly discussed below: (i) Decisions regarding business, its location and size: Before starting a business it is necessary to know whether it will be worth while to undertake this. This involves

Business Mathematics Note 13 a detailed analysis of its costs and benefits which can be done by using techniques and principles of statistics. Furthermore, statistics can also provide certain guidelines which may prove to be helpful in deciding the possible location and size of the proposed business. (ii) Planning of production: After a business is launched, the businessman has to plan its production so that he is able to meet the demand of its product and incurs minimum losses on account of over or under production. For this he has to estimate the pattern of demand of the product by conducting various market surveys. Based upon these surveys, he might also forecast the demand of the product at various points of time in future. In addition to this, the businessman has to conduct market surveys of various resources that will be used in the production of the given output. This may help him in the organisation of production with minimum costs. (iii) Inventory control: Sometimes, depending upon the fluctuations in demand and supply conditions, it may not be possible to keep production in pace with demand of the product. Caution! There may be a situation of no demand resulting in over production and consequently the firm might have to discontinue production for some time. On the other hand, there may be a sudden rise in the demand of the product so that the firm is able to meet only a part of the total demand. Under such situations the firm may decide to have an inventory of the product for the smooth running of its business. The optimum limits of inventory, i.e., the minimum and maximum amount of stock to be kept, can be decided by the statistical analysis of the fluctuations in demand and supply of the product. (iv) Quality control: Statistical techniques can also be used to control the quality of the product manufactured by a firm. This consists of the preparation of control charts by means of the specification of an average quality. A control chart shows two limits, the lower control limit and the upper control limit for variation in the quality of the product. The samples of output, being produced, are taken at regular intervals and their quality is measured. If the quality falls outside the control limits, steps are taken to rectify the manufacturing process. (v) Accounts writing and auditing: Every business firm keeps accounts of its revenue and expenditure. All activities of a firm, whether big or small, are reflected by these accounts. Whenever certain decisions are to be taken or it is desired to assess the performance of the firm or of its particular section or sections, these accounts are required to be summarised in a statistical way. This may consist of the calculation of typical measures like average production per unit of labour, average production per hour, average rate of return on investment, etc. Statistical methods may also be helpful in generalising relationships between two or more of such variables. Further, while auditing the accounts of a big business, it may not be possible to examine each and every transaction. Statistics provides sampling techniques to audit the accounts of a business firm. This can save a lot of time and money.

Unit 1: Introduction: Scope, Data Collection and Classification Note 14 (vi) Banks and Insurance companies: Banks use statistical techniques to take decisions regarding the average amount of cash needed each day to meet the requirements of day to day transactions. Furthermore, various policies of investment and sanction of loans are also based on the analysis provided by statistics. Did u Know? The business of insurance is based on the studies of life expectancy in various age groups. Depending upon these studies, mortality tables are constructed and accordingly the rates of premium to be charged by an insurance company are decided. All this involves the use of statistical principles and methods. Notes: Statistics as a Science or an Art We know that science is a body of systematized knowledge. How this knowledge is to be used for solving a problem is the work of an art? In addition to this, art also helps in achieving certain objectives and to identify merits and demerits of methods that could be used. Since statistics possesses all these characteristics, it may be reasonable to say that it is also an art. Thus, we conclude that since statistical methods are systematic and have general applications, therefore, statistics is a science. Further since the successful application of these methods depends, to a considerable degree, on the skill and experience of a statistician, therefore, statistics is an art also. Self Assessment Fill in the blanks: 11. Statistics is applied in every sphere of .... activity 12. ....said, " Statistics are the straw only which I like every other economist have to make the bricks". 13. .... are lifeblood of successful commerce 1.4 Variable and Attribute To understand the meaning of variable and attribute consider following example: There are 4 persons and their heights in inches are 55, 56, 72 and 74. Here height is a characteristic and the figures 55, 56, 72 and 74 are the values of a variable. These figures are the result of measurements. You know that the measurements generate the continuous variable. Thus the variable on heights is a continuous variable. Suppose we select 4 bulbs from a certain lot and inspect them. The lot contains good as well as defective bulbs. The sample may contain 0, 1, 2, 3, 4 defective bulbs. The values 0, 1, 2, 3 and 4 are the values of a discrete variable. Out of 4 persons whose heights are given above, 2 are tall with heights 72 and 74 inches and 2 are short with heights 55 and 56 inches. When we use the words, tall and short, any variable is not under consideration. We do not make any measurements. We only see who is tall and who is short. Here level of height tall or short is not a variable, it is called an attribute. Out of 4 bulbs 2 are good and 2 are defective. Here also any variable is not under consideration. We only count the defective bulbs and good bulbs. We examine whether the quality of being defective is present in a bulb or not. The status of the bulb is an attribute with two outcomes good and defective. Thus, attribute is a quality and the data is collected to see how many objects possess the quality of being defective and how many elements do not possess this quality. Other famous examples
Business Mathematics Note 15 of the attributes are level of education, level of smoking, level of social work, level of income, religion and color etc. The data on the attribute is the result of recording the presence and absence of a certain quality (attribute) in the individuals. Did u Know? The data on the variables are called the quantitative data whereas the data on the attributes are called qualitative data or count data. As the data on the variables is collected for the purpose of analysis of data and for inference about the population parameters, similarly the data on the attribute or attributes is collected for the purpose of analysis of data and for testing of hypotheses about the attributes. 1.4.1 Notation for Variables and Attributes For a single variable we use the symbol X and if there are two variables, we use the symbols X and Y for them. When there is a single attribute like height, the word, 'tall' may be denoted by A and 'short' may be denoted by . If the tall and the short persons are divided into intelligent and 'non-intelligent' persons, then 'intelligent' may be denoted by B and may be used for the opposite attribute 'non-intelligent'. It may be noted that the word attribute is used for the main group like intelligence and the sub- groups 'intelligent' and 'non-intelligent' are also called attributes. Self Assessment State whether the following statements are true or false: 14. Attribute is a quality and the data is collected to see how many objects possess the quality of being defective and how many elements do not possess this quality. 15. Famous examples of the variable are level of education, level of smoking, level of social work, level of income, religion and color etc. 16. The data on the attribute is the result of recording the presence and absence of a certain quality (attribute) in the individuals. 1.5 Primary Data and Secondary Data Data means information. Statistical data are usually of two types : (i) Primary, (ii) Secondary Data collected expressly for a specific purpose are called 'Primary data'. They are collected for the first time, for a specific purpose. Examples: Data collected by a particular person or organisation from the primary source for his own use. Collection of data about the population by censuses and surveys, etc Monthly Abstract of Statictics Monthly Statistical Digest International Labour Bulletin (monthly). Data collected and published by one organisation and subsequently used by other organisations are called 'Secondary data'. The data used in an investigation, which have been originally collected by some one else. The various sources of collection for

Unit 1: Introduction: Scope, Data Collection and Classification Note 16 secondary data are: newspapers and periodicals; publications of trade associations; research papers published by university departments, U.G.C. or research bureaus; official publications of central, state and the local and foreign governments, etc. For example, Data relating to national income collected by government are primary data, but the same data will be secondary while those will be used by a different concern. Data collected by one department are known as primary data. And if, any other private concern use these related data for any other purpose, then the data will be known as secondary data to them. Data are primary to the collector, but secondary to the user. The collection expenses of primary data are more than secondary data. Secondary data should be used with care. The various methods of collection of primary data are: (i) Direct personal investigation (interview/observation); (ii) Indirect oral investigation; (iii) Data from local agents and correspondents; (iv) Mailed questionnaires; (v) Questionnaires to be filled in by enumerators; (vi) Results of experiments, etc. Data collected in this manner are called 'raw data'. These are generally voluminous and have to be arranged properly before use. 1.5.1 Distinction between Primary and secondary Data : (i) Primary data are those data which are collected for the first time and thus original in character. Secondary data are those data that have already been collected earlier by some other persons. (ii) Primary data are in the form of raw materials to which statistical methods are applied for them purpose of analysis. On the other hand, secondary data are in form of finished products as they have been already statistically applied. (iii) Primary data are collected directly from the people to which enquiry is related. Secondary data are collected from published materials. (iv) Observed closely the difference is one of degree only. Data are primary to an institution collecting it, while they secondary for all others. Thus data which are primary in the hands of one, are secondary in the hands of other Did u Know? The difference between primary and secondary data is only in terms of degree.
Business Mathematics Note 17 Self Assessment Fill in the blanks 17. .... may be collected either from a primary or from a secondary source. 18. Data from a .... source are collected, for the first time, keeping in view the objective of investigation. 19. .... data are available from certain publications or reports. 20. The choice of a particular method depends, apart from objective, scope and nature of investigation, on the ...., 21. The data, collected and used by some other person or agency for an investigation in the past, when used for the investigation of a current problem, is known as .... data. 1.6 Population and Sample A population is the totality of the units under the field of investigation. These units are also called the items or objects or individuals or sampling units, which may be animates or inanimates. For example, if we want to study the marks obtained by students of B.Com. of a University, the population will consist of all the B.Com. students of that University. Further, if we wish to determine the average yield of wheat per acre in a particular year, the population will be all those acres of land which were under wheat crop in that year. Thus we can say that, "A population consist of all the individuals or objects in a well-defined group about which information is needed to answer a question". A sample is a subset or fraction of a population. When only some representative items of a population are selected, the data collected from these units constitute a sample from the population. 1.7 Complete Enumeration and Sample Survey If detail information regarding every individual person or items of a given universe is collected, then the enquiry will be complete enumeration. Another common name of complete enumeration is census. If it is required to compute the average height or weight of all the employees working under the Government region by the complete enumeration, then the heights or weights of all such employees are to be counted. (No one should be excluded). Since this methods requires time, expenditure, strength of working person, etc., application of the method is less. But for the interest of accurate observation of a particular individual item of the universe or if universe is small, them this method may be applied. In case of cesus of any country, detail enquiries of age, education, religion, occupation, income etc. of every individual (man of woman) are collected. In our counry census is made after every ten years. In certain cases complete enumeration is impossible. For export purpose it is not possible to test the quality.of every grain of rice or wheat in a bag.

Unit 1: Introduction: Scope, Data Collection and Classification Note 18 1.7.1 Sample survey Sample survey is the technique used to study about a population with the help of a sample. Population is the totality all objects about which the study is proposed. Sample is only a portion of this population, which is selected using certain statistical principles called sampling designs. Once the sample decided information will be collected from this sample, which process is called sample survey. Sample surveys involve the selection and study of a sample of items from a population. A sample is just a set of members chosen from a population, but not the whole population. A survey of a whole population is called a census. A sample from a population may not give accurate results but it helps in decision making. Therefore, a sample survey is a study that obtains data from a subset of a population, in order to estimate population attributes. Examples of sample surveys: 1. Phoning the fifth person on every page of the local phonebook and asking them how long they have lived in the area. (Systematic Sample) 2. Dropping a quad. in five different places on a field and counting the number of wild flowers inside the quad. (Cluster Sample) 3. Selecting sub-populations in proportion to their incidence in the overall population. For instance, a researcher may have reason to select a sample consisting 30% females and 70% males in a population with those same gender proportions. (Stratified Sample) 4. Selecting several cities in a country, several neighbourhoods in those cities and several streets in those neighbourhoods to recruit participants for a survey (Multi-stage sample) The term random sample is used for a sample in which every item in the population is equally likely to be selected. Task Discuss in group some of the application areas where sample survey is needed Self Assessment State whether the following statements are true or false: 22. In case of cesus of any country, detail enquiries of age, education, religion, occupation, income etc. of every individual (man of woman) are collected. 23. In India census is made after every five years. 24. For export purpose it is not possible to test the quality.of every grain of rice or wheat in a bag. 25. Sample survey is the technique used to study about a population with the help of a sample

Business Mathematics Note 19 1.8 Statistical Enquiry Statistical enquiry is a statement containing data where the object or subject is unidentifiable. There is no way to determine who and what the data applies to in these types of enquiries. A Statistical Enquiry is with a Predefined purpose and dealing with collection of data in a systematic manner. Data collected from such enquiry are called statistical data. It may be Descriptive Field Surveys or Analytical Experimental study under controlled conditions. Steps in conducting statistics inquiries include: 1. Collection of data 2. Organization 3. Processing 4. Presentation 5. Analysis of data 6. Interpretation of data 7. Inferences/ modeling 1.9 Classification The collected data are a complex and unorganized mass of figures which is very difficult to analyze and interpret. Therefore, it becomes necessary to organize this so that it becomes easier to grasp its broad features. Further, in order to apply the tools of analysis and interpretation, it is essential that the data are arranged in a definite form. This task is accomplished by the process of classification. Classification is the process of arranging the available data into various homogeneous classes and subclasses according to some common characteristics or objective of investigation. In the words of L.R. Connor, "Classification is the process of arranging things (either actually or notionally) in the groups or classes according to the unity of attributes that may subsist amongst a diversity of individuals.” The chief characteristics of any classification are: 1. The collected data are arranged into homogeneous groups. 2. The basis of classification is the similarity of characteristics or features inherent in the collected data. 3. Classification of data signifies unity in diversity. 4. Classification of data may be actual or notional. 5. Classification of data may be according to certain measurable or non-measurable characteristics or according to some combination of both. 1.9.1 Objectives of Classification The main objectives of any classification are: 1. To present a mass of data in a condensed form.

Unit 1: Introduction: Scope, Data Collection and Classification Note 20 2. To highlight the points of similarity and dissimilarity. 3. To bring out the relationship between variables. 4. To highlight the effect of one variable by eliminating the effect of others. 5. To facilitate comparison. 6. To prepare data for tabulation and analysis. 1.9.2 Requisites of a Good Classification A good classification must possess the following features: 1. Unambiguous: The classification should not lead to any ambiguity or confusion. 2. Exhaustive: A classification is said to be exhaustive if there is no item that cannot be allotted a class. 3. Mutually Exclusive: Different classes are said to be mutually exclusive if they are non-overlapping. When a classification is mutually exclusive, each item of the data can be placed only in one of the classes. 4. Flexibility: A good classification should be capable of being adjusted according to the changed situations and conditions. 5. Stability: The principle of classification, once decided, should remain same throughout the analysis, otherwise it will not be possible to get meaningful results. In the absence of stability, the results of the same type of investigation at different time periods may not be comparable. 6. Suitability: The classification should be suitable to the objective(s) of investigation. 7. Homogeneity: A classification is said to be homogeneous if similar items are placed in a class. 8. Revealing: A classification is said to be revealing if it brings out essential features of the collected data. This can be done by selecting a suitable number of classes. Making few classes means over summarisation while large number classes fail to reveal any pattern of behaviour of the variable. 1.9.3 Types of Classification The nature of classification depends upon the purpose and objective of investigation. The following are some very common types of classification: 1. Geographical (or spatial) classification 2. Chronological classification 3. Conditional classification 4. Qualitative classification 5. Quantitative classification 1. Geographical (or spatial) classification: When the data are classified according to geographical location or region, it is called a geographical classification.

Business Mathematics Note 21 For the purpose of immediate location or comparison of the data, it is necessary that it should be presented either in alphabetical or in ascending (or descending) order of the figures. 2. Chronological classification: When the data are classified on the basis of its time of occurrence, it is called a chronological classification. Various time series such as ; National Income figures (annual), annual output of wheat, monthly expenditure of a household, daily consumption of milk, etc., are some examples of chronological classification. 3. Conditional classification: When the data are classified according to certain conditions, other than geographical or chronological, it is called a conditional classification. 4. Qualitative classification or classification according to attributes: When the characteristics of the data are non-measurable, it is called a qualitative data. The examples of non-measurable characteristics are sex of a person, marital status, colour, honesty, intelligence, etc. These characteristics are also known as attributes. When qualitative data are given, various items can be classified into two or more groups according to a characteristic. If the data are classified only into two categories according to the presence or absence of an attribute, the classification is termed as dichotomous or twofold classification. On the other hand, if the data are classified into more than two categories according to an attribute, it is called a manifold classification. For example, classification of various students of a college according to the colour of their eyes like black, brown, grey, blue, etc. The conditional classification, given above, is also an example of a manifold classification. If the classification is done according to a single attribute, it is known as a one- way classification. On the other hand, the classification done according to two or more attributes is known as a two-way or multiway classification respectively. A two way classification of this type is shown as: Figure 1.1: Two-way Classification The example of a three-way classification, where population is dichotomized according to each attribute; sex, honesty and smoking habit, is given below: Figure 1.2: Three-way Classification

Unit 1: Introduction: Scope, Data Collection and Classification Note 22 We note that there will be eight subgroups of individuals like (male, honest, smokers), (male, honest, nonsmokers), etc. In the classification, given above, the population is dichotomised with respect to each of the three attributes. There may be situations where classification with respect to one attribute is dichotomous while it is manifold with respect to the other. 5. Quantitative classification or classification according to variables: In case of quantitative data, the characteristic is measurable in terms of numbers and is termed as variable, e.g., weight, height, income, the number of children in a family, the number of crime cases in a city, life of an electric bulb of a company, etc. A variable can take a different value corresponding to a different item of the population or universe. Variables can be of two types (a) Discrete and (b) Continuous. (a) Discrete Variable: A discrete variable can assume only some specific values in a given interval. For example, the number of children in a family, the number of rooms on each floor of a multistoried building, etc. (b) Continuous Variable: A continuous variable can assume any value in a given interval. For example, monthly income of a worker can take any value, say, between $ 1,000 to 2,500. The income of a worker can be $ 1,500.25, etc. Similarly, the life of an electric bulb is a continuous variable that can take any value from 0 to? ?. It must be pointed out here that, in practice, data collected on a continuous variable also look like the data of a discrete variable. This is due to the fact that measurements, done even with the finest degree of accuracy, can only be expressed in a discrete form. For example, height measured even with accuracy upto three places after decimal gives discrete values like 167.645 cms, 167.646 cms, etc. Similarly age, income, time, etc., are continuous variables but their actual measurements are expressed in terms of discrete numbers. In the classification according to variables, the data are classified by the values of the variables for each item. As in the case of attributes, the classification on the basis of a single variable is termed as a one-way classification. Similarly, there can be a two-way and multi-way classification of the data. For example, if the students of a class are classified on the basis of their marks in statistics, we get a one-way classification. However, if these students are simultaneously classified on the basis of marks in statistics and marks in economics, it becomes a two-way classification. It should be noted here that in a two-way classification, it is possible to have simultaneous classification according to an attribute and a variable. For example, the classification of students of a class on the basis of their marks in statistics and the sex of the person.

Business Mathematics Note 23 Self Assessment Fill in the blanks 26. Classification is the process of arranging the available data into various ....classes and subclasses according to some common characteristics or objective of investigation. 27. .... is the process of arranging things (either actually or notionally) in the groups or classes according to the unity of attributes that may subsist amongst a diversity of individuals. 28. The nature of classification depends upon the purpose and objective of .... 29. When the data are classified according to geographical location or region, it is called a .... classification. 30. .... methods are sometimes used in place of .... computations to save time and labour, State whether the following statements are true or false: 31. When the data are classified according to geographical location or region, it is called a geographical classification. 32. When the data are classified on the basis of its time of occurrence, it is called a chronological classification. 33. When the data are classified according to certain conditions, other than geographical or chronological, it is called a unconditional classification. 34. When the characteristics of the data are non-measurable, it is called a qualitative data. 35. In a one-way classification, it is possible to have simultaneous classification according to an attribute and a variable. 1.10 Tabulation Tabulation is a systematic presentation of numerical data in rows and columns. Tabulation of classified data make it more intelligible and fit for statistical analysis. According to Tuttle, "A statistical table is the logical listing of related quantitative data in vertical columns and horizontal rows of numbers, with sufficient explanatory and qualifying words, phrases and statements in the form of titles, headings and footnotes to make clear the full meaning of the data and their origin." The classified data presented in tabular form helps to bring out their essential features. 1.10.1 Objectives of Tabulation The main objectives of tabulation are: (i) To simplify complex data. (ii) To highlight chief characteristics of the data. (iii) To clarify objective of investigation. (iv) To present data in a minimum space.

Unit 1: Introduction: Scope, Data Collection and Classification Note 24 (v) To detect errors and omissions in the data. (vi) To facilitate comparison of data. (vii) To facilitate reference. (viii) To identify trend and tendencies of the given data. (ix) To facilitate statistical analysis. 1.10.2 Difference between Classification and Tabulation The basic points of difference between classification and tabulation, inspite of the fact that these are closely related, are as given below: (i) Classification of data is basis for tabulation because first the data are classified and then tabulated. (ii) Classification is a process of statistical analysis while tabulation is a process of presentation. (iii) By classification the data are divided into various groups and subgroups on the basis of their similarities and dissimilarities while tabulation is a process of arranging the classified data in rows and columns with suitable heads and subheads. 1.10.3 Main Parts of a Table The main parts of a table are as given below: (i) Table Number: This number is helpful in the identification of a table. This is often indicated at the top of the table. (ii) Title: Each table should have a title to indicate the scope, nature of contents of the table in an unambiguous and concise form. (iii) Captions and stubs: A table is made up of rows and columns. Headings or subheadings used to designate columns are called captions while those used to designate rows are called stubs. A caption or a stub should be self explanatory. Caution! A provision of totals of each row or column should always be made in every table by providing an additional column or row respectively. (iv) Main Body of the Table: This is the most important part of the table as it contains numerical information. The size and shape of the main body should be planned in view of the nature of figures and the objective of investigation. The arrangement of numerical data in main body is done from top to bottom in columns and from left to right in rows. (v) Ruling and Spacing: Proper ruling and spacing is very important in the construction of a table. Vertical lines are drawn to separate various columns with the exception of sides of a table. Horizontal lines are normally not drawn in the body of a table, however, the totals are always separated from the main body by horizontal lines. Further, the horizontal lines are drawn at the top and the bottom of a table. Business Mathematics Note 25 Spacing of various horizontal and vertical lines should be done depending on the available space. Major and minor items should be given space according to their relative importance. (vi) Head-note: A head-note is often given below the title of a table to indicate the units of measurement of the data. This is often enclosed in brackets. (vii) Foot note: Abbreviations, if any, used in the table or some other explanatory notes are given just below the last horizontal line in the form of footnotes. (viii) Source-Note: This note is often required when secondary data are being tabulated. This note indicates the source from where the information has been obtained. Source note is also given as a footnote. The main parts of a table can also be understood by looking at its broad structure given below: Structure of a table Table No: .... Title: .... Foot Note: Source: 1.10.4 Rules for Tabulation General Rules (i) The table should be simple and compact which is not overloaded with details. (ii) Tabulation should be in accordance with the objective of investigation. (iii) The unit of measurements must always be indicated in the table. (iv) The captions and stubs must be arranged in a systematic manner so that it is easy to grasp the table. (v) A table should be complete and self explanatory. (vi) As far as possible the interpretative figures like totals, ratios and percentages must also be provided in a table. (vii) The entries in a table should be accurate. (viii) Table should be attractive to draw the attention of readers. Unit 1: Introduction: Scope, Data Collection and Classification Note 26 Self Assessment Fill in the Blanks: 36. .... is a systematic presentation of numerical data in rows and columns. 37. According to.........................., "A statistical table is the logical listing of related quantitative data in vertical columns and horizontal rows of numbers, with sufficient explanatory and qualifying words, phrases and statements in the form of titles, headings and footnotes to make clear the full meaning of the data and their origin." 38. Classification is a process of statistical .... while tabulation is a process of .... 39. Headings or subheadings used to designate columns are called .... while those used to designate rows are called .... 40. A ....is often given below the title of a table to indicate the units of measurement of the data. 41. Horizontal lines are normally .... in the body of a table. 42 The totals are always separated from the main body by .... lines. 43. Tabulation should be in accordance with the objective of .... 44. A table should be complete and .... 45. ....indicates the source from where the information has been obtained 1.10.5 Type of Tables Statistical tables can be classified into various categories depending upon the basis of their classification. Broadly speaking, the basis of classification can be any of the following: (i) Purpose of investigation (ii) Nature of presented figures (iii) Construction Different types of tables, thus, obtained are shown in the following chart. Figure 1.3: Basic of Classification

Business Mathematics Note 27 1. Classification on the basis of purpose of investigation: These tables are of two types viz. (i) General purpose table and (ii) Special purpose table. (i) General purpose table: A general purpose table is also called as a reference table. This table facilitates easy reference to the collected data. In the words of Croxton and Cowden, "The primary and usually the sole purpose of a reference table is to present the data in such a manner that the individual items may be readily found by a reader." A general purpose table is formed without any specific objective, but can be used for a number of specific purposes. Such a table usually contains a large mass of data and are generally given in the appendix of a report. (ii) Special purpose table: A special purpose table is also called a text table or a summary table or an analytical table. Such a table presents data relating to a specific problem. According to H. Secrist, " These tables are those in which are recorded, not the detailed data which have been analysed, but rather the results of analysis." Such tables are usually of smaller size than the size of reference tables and are generally found to highlight relationship between various characteristics or to facilitate their comparisons. 2. Classification on the basis of the nature of presented figures: Tables, when classified on the basis of the nature of presented figures can be (i) Primary table (ii) Derivative table. (i) Primary Table: Primary table is also known as original table and it contains data in the form in which it were originally collected. (ii) Derivative Table: A table which presents figures like totals, averages, percentages, ratios, coefficients, etc., derived from original data. A table of time series data is an original table but a table of trend values computed from the time series data is known as a derivative table. 3. Classification on the basis of construction: Tables when classified on the basis of construction can be (i) Simple table, (ii) Complex table or (iii) Cross-classified table. (i) Simple Table: In this table the data are presented according to one characteristic only. This is the simplest form of a table and is also known as table of first order. The following blank table, for showing the number of workers in each shift of a company, is an example of a simple table. Table No. .... Shifts No. of Workers Total I II III (ii) Complex Table: A complex table is used to present data according to two or more characteristics. Such a table can be two-way, three-way or multi-way, etc.
Unit 1: Introduction: Scope, Data Collection and Classification Note 28 (a) Two-way table: Such a table presents data that is classified according to two characteristics. In such a table the columns of a table are further divided into sub-columns. The example of such a table is given below. Table No. .... Distribution of workers of a factory according to shifts and sex Shifts No. of Workers Males Females Total Total I II III (b) Three-way table: When three characteristics of data are shown simultaneously, we get a three-way table as shown below. Table No. .... Distribution of workers of a factory according to shifts, sex and training No. of Workers Shifts I II III Males Skilled Skilled Unskilled Unskilled Total No. of Workers Total Total Females (c) Multi-way table: If each shift is further classified into three departments, say, manufacturing, packing and transportation, we shall get a four-way table, etc. (iii) The Cross-Classified Table: Tables that classify entries in both directions, i.e., row-wise and column-wise, are called cross-classified tables. The two ways of classification are such that each category of one classification can occur with any category of the other. The cross-classified tables can also be constructed for more than two characteristics also. A cross-classification can also be used for analytical purpose, e.g., it is possible to make certain comparisons while keeping the effect of other factors as constant. Example Draw a blank table to show the population of a city according to age, sex and unemployment in various years. Solution. Table No. .... Population of a city according to age, sex and unemployment in various years
Business Mathematics Note 29 Note: The table can be extended for the years 2013, 2014,...etc. Example In a sample study about coffee habit in two towns; the following information were received: Town A: Females were 40%; total coffee drinkers were 45%; and male non-coffee drinkers were 20%. Town B: Males were 55%; male non-coffee drinkers were 30%; and female coffee drinkers were 15%. Represent the above data in a tabular form. Solution. Table No. ...... Distribution of population, according to sex and coffee habit, in two towns Note: The figures are in percentage. Example Prepare a blank table for showing the percentage of votes polled by various political parties in India according to states, during 1996 general elections. Solution. Table No. ...... Percentage distribution of votes polled by political parties according to States in India during the particular year for general elections

Unit 1: Introduction: Scope, Data Collection and Classification Note 30 Self Assessment State whether the following statements are true or false: 46. A general purpose table is also called as a preference table 47. A general purpose table is formed without any specific objective, but can be used for a number of specific purposes. 48. A special purpose table is also called a text table or a summary table or an analytical table. 49. According to H. Secrist, “These tables are those in which are recorded, not the detailed data which have been analysed, but rather the results of analysis.” 50. Primary table is also known as original table. 51. A table which presents figures like totals, averages, percentages, ratios, coefficients, etc., derived from original data are called derivative table. 52. Simple form of a table is also known as table of first order. 53. A complex table is used to present data according to two or more characteristics. Such a table can be two-way, three-way or multi-way, etc. 1.11 Manual and Mechanical Methods of Tabulation Tabulation of the collected data can be done in two ways: (i) By Manual Method, and (ii) By Mechanical Method. (i) Manual Method: When field of investigation is not too large and the number of characteristics are few, the work of tabulation can be done by hand. (ii) Mechanical Method: This method is used when the data are very large. The use of machines save considerable amount of labour and time. With the development of high speed computers, the work of tabulation and analysis of data can be done very quickly and with greater accuracy. Self Assessment Multiple Choice Questions 54. When field of investigation is not too large and the number of characteristics are few then we use .... method of investigation. (a) Manual (b) Semi manual (c) Mechanical (d) Automatic 55. .... method is used when the data are very large. (a) Manual (b) Semi manual (c) Mechanical (d) Automatic Case Study: Tabulation A survey of 370 students from the Commerce Faculty and 130 students from the Science Faculty revealed that 180 students were studying for only C.A. examinations, 140 for only Costing examinations and 80 for both C.A. and Costing

Business Mathematics Note 31 examinations. The rest had offered part-time Management courses. Of those studying Costing only, 13 were girls and 90 boys belonged to the Commerce Faculty. Out of 80 students studying for both C.A. and Costing, 72 were from the Commerce Faculty amongst which 70 were boys. Amongst those who offered part- time Management courses, 50 boys were from the Science Faculty and 30 boys and 10 girls from the Commerce faculty. In all there were 110 boys in the Science Faculty. Question Present the above information in a tabular form. Find the number of students from the Science Faculty studying for part-time Management courses. 1.12 Summary ? Classification is the process of arranging the available data into various homogeneous classes and subclasses according to some common characteristics or objective of investigation. ? In the words of L.R. Connor, "Classification is the process of arranging things (either actually or notionally) in the groups or classes according to the unity of attributes that may subsist amongst a diversity of individuals. ? The nature of classification depends upon the purpose and objective of investigation. The following are some very common types of classification: ? Geographical (or spatial) classification ? Chronological classification ? Conditional classification ? Qualitative classification ? Quantitative classification ? When the data are classified according to geographical location or region, it is called a geographical classification. ? When the data are classified on the basis of its time of occurrence, it is called a chronological classification. Various time series such as ; National Income figures (annual), annual output of wheat, monthly expenditure of a household, daily consumption of milk, etc., are some examples of chronological classification. ? When the data are classified according to certain conditions, other than geographical or chronological, it is called a conditional classification ? When the characteristics of the data are non-measurable, it is called a qualitative data. The examples of non-measurable characteristics are sex of a person, marital status, colour, honesty, intelligence, etc. These characteristics are also known as attributes. ? In case of quantitative data, the characteristic is measurable in terms of numbers and is termed as variable, e.g., weight, height, income, the number of children in a family, the number of crime cases in a city, life of an electric bulb of a company, etc.

Unit 1: Introduction: Scope, Data Collection and Classification Note 32 ? A variable can take a different value corresponding to a different item of the population or universe. ? Tabulation is a systematic presentation of numerical data in rows and columns. ? Tabulation of classified data make it more intelligible and fit for statistical analysis. ? According to Tuttle, "A statistical table is the logical listing of related quantitative data in vertical columns and horizontal rows of numbers, with sufficient explanatory and qualifying words, phrases and statements in the form of titles, headings and footnotes to make clear the full meaning of the data and their origin." ? The classified data presented in tabular form helps to bring out their essential features. 1.13 Keywords Statistics: Statistics is a science which deals with collection, classification and tabulation of numerical facts as the basis for the explanation, description and comparison of phenomena. sample. The techniques of forecasting are also included in inductive statistics. Inferential Statistics: It includes all those methods which are used to test certain hypotheses regarding characteristics of a population. Modern era : The modern era is an era of mass production in which size and number of firms have increased enormously. Statistics in plural sense: Statistics used in the plural sense implies a set of numerical figures collected with reference to a certain problem under investigation. Science : Science is a body of systematised knowledge developed by generalisations of relations based on the study of cause and effect. Classification: Classification is the process of arranging things (either actually or notionally) in the groups or classes according to the unity of attributes that may subsist amongst a diversity of individuals. Dichotomous classification: When a characteristic is an attribute, the data can be classified into two classes according to this attribute, known as dichotomous classification. Complex Table: A complex table is used to present data according to two or more characteristics. Such a table can be two-way, three-way or multi-way, etc. Cross-Classified Table: Tables that classify entries in both directions, i.e., row-wise and column-wise, are called cross-classified tables Derivative Table: A table which presents figures like totals, averages, percentages, ratios, coefficients, etc., derived from original data. Foot note: Abbreviations, if any, used in the table or some other explanatory notes are given just below the last horizontal line in the form of footnotes. General purpose table: A general purpose table is also called as a reference table. This table facilitates easy reference to the collected data.

Business Mathematics Note 33 Manual Method: When field of investigation is not too large and the number of characteristics are few, the work of tabulation can be done by hand. Mechanical Method: This method is used when the data are very large. The use of machines save considerable amount of labour and time Primary Table: Primary table is also known as original table and it contains data in the form in which it were originally collected. Simple Table: In this table the data are presented according to one characteristic only. This is the simplest form of a table and is also known as table of first order. Source-Note: This note is often required when secondary data are being tabulated. This note indicates the source from where the information has been obtained. Source note is also given as a footnote. Special purpose table: A special purpose table is also called a text table or a summary table or an analytical table. Such a table presents data relating to a specific problem. Statistical table: A statistical table is the logical listing of related quantitative data in vertical columns and horizontal rows of numbers, with sufficient explanatory and qualifying words, phrases and statements in the form of titles, headings and footnotes to make clear the full meaning of the data and their origin. Tabulation: Tabulation is a systematic presentation of numerical data in rows and columns 1.14 Review Questions 1. What do you understand by secondary data? State their chief sources and point out dangers involved in their use. What precaution must be taken while using such data for further investigation? 2. What do you mean by Classification and Tabulation? Explain their importance in statistical studies. 3. What are the different factors that should be kept in mind while classifying data? 4. Distinguish between classification and tabulation. Discuss the purpose and methods of classification. 5. What are objects of classification of data? Discuss different methods of classification. 6. Construct a frequency distribution of the marks obtained by 50 students in economics as given below: 42, 53, 65, 63, 61, 47, 58, 60, 64, 45, 55, 57, 82, 42, 39, 51, 65, 55, 33, 70, 50, 52, 53, 45, 45, 25, 36, 59, 63, 39, 65, 30, 45, 35, 49, 15, 54, 48, 64, 26, 75, 20, 42, 40, 41, 55, 52, 46, 35, 18. (Take the first class interval as 10 - 20) 7. The following figures give the ages, in years, of newly married husbands and their wives. Represent the data by an appropriate frequency distribution.

Unit 1: Introduction: Scope, Data Collection and Classification Note 34 8. Define the term tabulation. 9. What is the difference between tabulation and classification? 10. What is the need for tabulation? 11. What are the various parts of table? 12. What is the difference between primary table and derivative table? 13. What is the difference between footnote and source note? 14. What is the difference between simple and complex table? 15. What is the difference between manual and mechanical method of tabulation? 16. Tabulate the following information: In a trip organized by a college, there were 80 persons each of whom paid $15.50 on an average. There were 60 students, each of whom paid $16. Members of the teaching staff were charged at a higher rate. The number of servants was 6, all males and they were not charged anything. The number of ladies was 20% of the total of which one was a lady staff member. 17. There were 850 union and 300 non union workers in a factory in 2009. Of these, 250 were females out of which 100 were non union workers. The number of union workers increased by 50 in 2010 out of which 40 were males. Of the 350 non union workers, 125 were females. In 2011, there were 1,000 workers in all and out of 400 non union workers there were only 100 females. There were only 400 male workers in the union. Present the above information in a tabular form. 18. A super market divided into five main sections; grocery, vegetables, medicines, textiles and novelties, recorded the following sales in 2009, 2010 and 2011: 19. In 2011, though total sales remained the same as in 1986, grocery fell by $22,000, vegetables by $32,000, medicines by $10,000 and novelties by $12,000. Tabulate the above data. 20. A survey of 370 students from the Commerce Faculty and 130 students from the Science Faculty revealed that 180 students were studying for only C.A. examinations, 140 for only Costing examinations and 80 for both C.A. and Costing examinations. The rest had offered part-time Management courses. Of those studying Costing only, 13 were girls and 90 boys belonged to the Commerce Faculty. Out of 80 students studying for both C.A. and Costing, 72 were from the Commerce Faculty amongst which 70 were boys. Amongst those who offered part-time Management courses, 50 boys were from the Science Faculty and 30 boys and 10 girls from the Commerce faculty. In all there were 110 boys in the Science Faculty.

Business Mathematics Note 35 Present the above information in a tabular form. Find the number of students from the Science Faculty studying for part-time Management courses. Answers: Self Assessment 1. plural 2. singular 3. functions 4. counting 5. phenomena 6. True 7. False 8. True 9. True 10. False 11. Human 12. Alfred Marshall 13. Statistics 14. True 15. False 16. True 17. Data 18. Primary 19. Secondary 20. Availability of resources 21. Secondary 22. True 23. False 24. True 25. True 26. Homogeneous 27. Classification 28. Investigation 29. Geographical 30. Graphic, mathematical 31. True 32. True 33. False 34. True 35. False 36. Tabulation 37. Tuttle 38. analysis, presentation 39. captions, stubs 40. head-note 41. not drawn 42. horizontal 43. investigation. 44. self explanatory 45. Source note 46. False 47. True 48. True 49. True 50. True 51. True 52. True 53. True 54. Manual 55. Mechanical

Unit 1: Introduction: Scope, Data Collection and Classification Note 36 1.15 Further Readings Books Kenneth Rosen, Discrete Mathematics and Its Applications, 7/e, 2012, McGraw-Hill. Bathul, Shahnaz, Mathematical Foundations of Computer Science, PHI Learning. Lambourne, Robert, Basic Mathematics for the Physical Sciences,2008, John Wiley & Sons. Bari, Ruth A.; Frank Harary. Graphs and Combinatorics, Springer. F. Ernest Jerome, Connect for Jerome, Business Mathematics in Canada, 7e, Canadian Edition. S Rajagopalan and R Sattanathan, Business Mathematics, 2 edition, 2009, Tata McGraw Hill Education. Garrett H.E. (1956), Elementary Statistics, Longmans, Green & Co., New York. Guilford J.P. (1965), Fundamental Statistics in Psychology and Education, Mc Graw Hill Book Company, New York. Hannagan T.J. (1982), Mastering Statistics, The Macmillan Press Ltd., Surrey. Lindgren B.W (1975), Basic Ideas of Statistics, Macmillan Publishing Co. Inc., New York. Selvaraj R., Loganathan C., Quantitative Methods in Management. Sharma J.K., Business Statistics, Pearson Education Asia Walker H.M. and J. Lev, (1965), Elementary Statistical Methods, Oxford & IBH Publishing Co., Calcutta. Wine R.L. (1976), Beginning Statistics, Winthrop Publishers Inc., Massachusetts. Online links en.wikipedia.org/wiki/Statistics www.thefreedictionary.com/statistics www.investopedia.com/terms/s/statistics.asp - www.usa.gov/Topics/Reference-Shelf/Data.shtml www.cdc.gov/datastatistics www.thefreedictionary.com/Statistical+data

Business Mathematics Note 37 Unit 2: Permutation and Combinations CONTENTS Objectives Introduction 2.1 Fundamentals Rule of Counting 2.1.1 Factorial Notation 2.2 Permutation 2.3 Combination 2.3.1 Ordered Partitions 2.4 Result/Application of Permutation and Combination 2.4.1 The Telephone Numbering System 2.4.2 Application of the Principles of Permutation and Combination 2.4.3 Technical Interpretations of the Mathematical Results 2.4 Summary 2.5 Keywords 2.6 Review Questions 2.7 Further Readings Objectives After studying this unit, you will be able to: 1. Define the term permutation 2. State fundamentals rule of counting 3. Understand results on Permutation 4. Explain combination with suitable examples Introduction The word permutation means arrangement of things, here arrangement is used in the sense, if the order of things is considered. On the other hand combination means selection of things, here selection is used, when the order of things has no importance. Considering an example, let we have to form a numbers, consisting of three digits using the digits 1,2,3,4, To form this number the digits have to be arranged. Different numbers will get formed depending upon the order in which we arrange the digits. This is an example of Permutation. Now, let we have to make a team of 11 players out of 20 players, This is an example of combination, because the order of players in the team will not result in a change in the

Note Unit 2: Permutation and Combinations 38 team. No matter in which order we list out the players the team will remain the same! For a different team to be formed at least one player will have to be changed. In this unit, we will discuss the term permutation. We will also focus on fundamentals rule of counting. Further, we will focus on different results on Permutation and combinations. 2.1 Fundamentals Rule of Counting Counting techniques or combinatorial methods are often helpful in the enumeration of total number of outcomes of a random experiment and the number of cases favourable to the occurrence of an event. 2.1.1 Factorial Notation This product of first 'n' natural numbers is denoted by !n or n . This is read as 'factorial n'. ! 1 2 3 .... (1) 1! 1 2! 1 2 2 3! 1 2 3 6 4! 1 2 3 4 24 n n n ? Did u Know? 0! is defined to be equal to 1. n! can be written as (1)!or (1)(2)! n n n n n ? ? ? etc. For example, 10! 10 9! 10 9 8! etc. ? ? ? ? ? 10! 10 9 8 7 6 5 4 3 2 1 3628800 ? ? ? ? ? ? ? ? ? ? ? 6! 6 5 4 3 2 1 720 ? ? ? ? ? ? ? Fundamental rule of Counting If the first operation can be performed in any one of the m ways and then a second operation can be performed in any one of the n ways, then both can be performed together in any one of the m n ways. This rule can be generalized. If first operation can be performed in any one of the n 1 ways, second operation in any one of the n 2 ways, ... k th operation in any one of the n k ways, then together these can be performed in any one of the n 1 × n 2 × ...... × n k ways.

Business Mathematics Note 39 Example: Suppose there are cities A, B, C and there are three different ways of travelling from A to B and two different ways to travelling from B to C. In how many different ways one can travel from A to C via B? There are 3 × 2 different ways of travelling from A to C via B. Self Assessment Fill in the blanks: 1. .... methods are often helpful in the enumeration of total number of outcomes of a random experiment and the number of cases favourable to the occurrence of an event. 2. If first operation can be performed in any one of the n1 ways, second operation in any one of the n2 ways, ... kth operation in any one of the nk ways, then together these can be performed in any one of the .... ways. 2.2 Permutation The different arrangements which can be made out of a given set of things, by taking some or all of them at a time are called permutations. Thus, a permutation is an arrangement of a given set of objects in a definite order. Thus composition and order both are important in a permutation. Permutations play an important role in the theory of probability which is being used in many areas. The problems on permutations are sheer amusement. Theorem:

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The number of permutations of n things taken r at a time is given by (1)(2).....(1) n r p n n n n

r ? ? ? ? ? Proof: Let 1 2 3 , , ,......., n a a a a be n different things.

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The number of permutations of n things taken r at a time is the same as the number of ways of

filling up r blanks in a row. Figure 2.1: Permutations The first blank space can be filled in n ways. After filling up this space in one of these ways, the second space can be filled in (1) n ? ways. After filling up the second space in one of these ways, the third space can be filled in (2) n ? ways etc., the th r space

Note Unit 2: Permutation and Combinations 40 can be filled in (1) 1 n r n r ? ? ? ? ? ways. By fundamental principle, the number of ways of filling up the r boxes in succession is equal to (1)(2).....(1)

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n n n n r ? ? ? ? (1)(2).....(1) n r p n n n n r ? ? ? ? ? ? In factorial notation, (1)(2).....(1) n r p n n n n r ? ? ? ? ? can be written as (1)(2).....(1)( )! ( )! n r n n n n r n r p n r ? ? ? ? ? ? ? ? ! ( )! n r n p n r ? ? ? Did u Know? Particular Cases If 0 ! ! 0,then 1 (0)! ! n n n r p n n ? ? ? ? ? If ! ! ,then ! ( )! 0! n n n n r n p n n n ? ? ? ? ? If 1 ! (1)! 1,then (1)! (1)! n n n n r p n n

n ? ? ? ? ? ? ?
Let us discuss different cases of permutations: 1. Permutations of n objects: The total number of permutations of n distinct objects is n!. Using symbols, we can write = n!, (where n denotes the permutations of n objects, all taken together). Let us assume there are n persons to be seated on n chairs. The first chair can be occupied by any one of the n persons and hence, there are n ways in which it can be occupied. Similarly, the second chair can be occupied in n – 1 ways and so on. Using the fundamental principle of counting, the total number of ways in which n chairs can be occupied by n persons or the

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permutations of n objects taking all at a time is given by = n(n – 1)(n – 2) ...... 3.2.1 = n! 2. Permutations of n objects taking r at a time:

In terms of the example, considered above, now we have n persons to be seated on r chairs, where

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r ≤ n. Thus, = n(n – 1)(n – 2) ...... [n – (r – 1)] = n(n – 1)(n – 2) ...... (n – r + 1). On multiplication and division of the R.H.S. by (n – r)!, we get ( )( ) ( )( ) ( ) n r n n 1 n 2 .... n r 1 n r ! n! P n r ! (n r)! - - - + - = = - - 3.

Permutations of n objects taking r at a time
when any object may be repeated any number of times: Here, each of the r places can be filled in n ways. Therefore, total number of permutations is n r .
Business Mathematics Note 41 Notes Permutations with restrictions: If out of n objects n 1 are alike of one kind, n 2 are alike of another kind, ... n k are alike, the number of permutations are 1 2 k n! n !n ! .... n ! Since permutation of n i objects, which are alike, is only one (i = 1, 2, ... k). Therefore, n! is to be divided by n 1 !, n 2 ! .... n k !, to get the required permutations. Example:

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The number of permutations of 4 things taken 3 at a time is 4 3 4! 4! 4 3 2 1 24 (4 3)! p ? ? ? ? ? ? ? ? ? . The number of permutations of 4 things taken all at a time is 4 4 4! 4! 4 3 2 1 24 0! p ? ? ? ? ? ? ? Permutation of things when some of them are alike The number of permutations of n things when p things are alike of one kind, q things are alike of second kind, r things are alike of

third kind etc., is given by the formula ! ! ! !..... n p q r . Circular Permutations Suppose that there are three persons A, B and C, to be seated on the three chairs 1, 2 and 3, in a circular order. Then, the following three arrangements are identical: Figure 2.2: Circular Permutations Similarly, if n objects are seated in a circle, there will be n identical arrangements of the above type. Thus, in order to obtain distinct permutation of n objects in circular order we divide n n P by n, where n n P denotes number of permutations in a row. Hence, the number of permutations in a circular order Caution! When things are arranged in a row, we find two ends in each arrangement, while when the things are arranged in circle, there is no such end. Here, instead of arranging the things along a line, we arrange the things along a circle, which is called Circular Permutation.

Note Unit 2: Permutation and Combinations 42 To find the number of circular permutations of n things taken all at a time. Let us take 4 n ? Let the four things be A, B, C, D. Let us arrange these along a circle. Let the number of circular permutations of these four things be x. Let us consider one of these circular permutations. Figure 2.3: Circular Permutations By shifting A to B, B to C, C to D, D to A. Figure 2.4: Circular Permutations We get the above arrangements. We observe that these four circular arrangements are not different as the relative position of each with the others is the same. But the corresponding linear permutations are A B C D, D A B C, C D A B , B C D A which are all different. Therefore, one circular permutation gives 4 linear permutations. ? x circular permutations gives 4x linear permutations which are equal to 4!. 4 4! 4! 3! (4 1)! 4 x x ? ? ? ? ? ? ? Now if we take n things, the number of circular permutations (1)! n? ? Note: If the n things are different say persons, then the number of circular permutations (1)! n ? ? because anticlockwise and clockwise arrangements are different. But if the n things are alike say beads to form a necklace, then the anticlockwise and clockwise arrangements are same. Therefore, the number of circular permutations will be half of (1)! (1)! i.e., 2 n n ? ?

Business Mathematics Note 43 Example: Find 10 7 6 4 3 2 , , p p p Solution: 10 4 7 3 6 2 10 9 8 7 5040 7 6 5 210 6 5 30 p p p ? ? ? ? ? ? ? ? ? ? ? ? Example: If 720 n n p ? , find n. Solution: 720 n n p ? i.e., ! 6 5 4 3 2 1 n ? ? ? ? ? ? i.e., ! 6! n ? 6 n ? ? Example: How many three digit numbers can be formed using the digits 2, 3, 4, 5, 6, repetitions being allowed? Solution: The first digit can be chosen in 5 ways. The second digit can be chosen in 5 ways. The third digit can be chosen in 5 ways. Therefore, the three digits can be arranged in 5 5 5 ? ? ways. Therefore, the number of three-digit numbers is 5 5 5 125 ? ? ? . Example: Find the number of three digit numbers using 2, 3, 4, 5, 6, repetitions being not allowed. Solution: The first digit can be selected in 5 ways. After fixing the first digit in one of these ways, the second digit can be fixed in 4 ways. After fixing the second digit in one of these ways, the third digit can be fixed in 3 ways. Therefore, the number of three digit numbers 5 4 3 60 ? ? ? ? . Example: How many three digit numbers that can be formed which are less than 400 using the digits 2, 3, 4, 5, 6, repetitions being not allowed. Solution: The first digit can be either 2 or 3. Therefore, the first digit can be chosen in 2 ways. After choosing the first digit in one of these 2 ways, the second digit can be chosen in 4 ways and the third digit can be chosen in 3 ways. Therefore, the number of three digit numbers less than 400 2 4 3 24 ? ? ? ? Example: Find the number of three digit even numbers that can be formed using 2, 3, 4, 5, 6, repetitions being allowed.

Note Unit 2: Permutation and Combinations 44 Solution: The first digit can be chosen in 5 ways, the second digit can be chosen in 5 ways and the last digit can be chosen in 3 ways. ? The number of three digit even numbers is 5 5 3 75 ? ? ? . Example: If 4 2 12

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n n p p ? find n. Solution: 4 2 12 n n p p ? (1)(2)(3) 12 (1)(2)(3) n n n n n n n n ? ? ? ? ? ? ? 2 2 5 6 12 0 5 6 0 (6)(1) 0 6 0, 1 0 6, 1 n n n n n n n n n n ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? But n cannot be –1 6 n? ? Example: Find the number of permutations of the letters of the

word 'LOOK'. Solution: The word 'LOOK' has four letters in which there are two Os. Therefore, the number of permutations 4! 4 3 2 1 12 2! 2 1 ? ? ? ? ? ? ? Example: Prove that, (2)! 2 [1 3 5 ...... (2 1)](!)

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n n n n ? ? ? ? ? ? Solution: (2)! (2)(2 1)(2 2)(2 3)(2 4)......4 3 2 1 n n n n n n ? ? ? ? ? ? ? ? (2)! (2)(2 2)(2 4)......4 2 1 3....(2 1) 2 2(1) 2(2).....2 2 2 1 3...(2 1) (2 2 2....2) (1)(2)...1 1 3...(2 1) 2 (!)[1 3 5...(2 1)] n n n n n n n n n n n n n n n n n ?

Example:

In how many ways can 7 Mathematics books, 4 Physics books and 5 Chemistry books be arranged in a shelf so that: (i) Physics books are together, (ii) Chemistry books are together and Mathematics books are to be together, (iii) no two Mathematics books are together, and (iv) books of the same subjects are together. Solution: (i) 4 Physics books are to be together. Hence, they can be considered as 1 unit. Therefore, total number of books =13.

Business Mathematics Note 45 These can be arranged in 13! ways and 4 Physics can be arranged among themselves in 4! ways. Therefore, required number of ways 13! 4! ? ? (ii) 5 Chemistry books are to be together. Hence, they can be considered as 1 unit and they can be arranged among themselves in 5! ways. 7 mathematics books are to be together. Hence, they can be considered as 1 unit and they can be arranged among themselves in 7! ways. The total number of books now can be taken as 6 and they can be arranged in 6! ways. Therefore, the number of ways of arranging the books 6! 5! 7! ? ? ? (iii) No two Mathematics books are to be together. Let us arrange the remaining books. 4 Physics and 5 Chemistry books i.e., totally 9 books can be arranged in 9! ways. Consider one of these arrangements. 1 2 3 4 1 2 3 4 5 p p p p c c c c c ? ? ? ? ? ? ? ? ? ? Therefore, for Mathematics books no two of which are to be together, the number of places is 10 as marked by ? . ? 7 Mathematics books can be arranged in 10 7 p ways. ? the required number of ways of arranging the books is 10 7 9!. p ? (iv) 7 Mathematical books are considered as 1 unit and they can be arranged among themselves in 7! ways. 4 Physics books are considered as 1 unit and they can be arranged among themselves in 4! ways. 5 Chemistry books are considered as 1 unit and they can be arranged among themselves in 5! ways. ? the required number of ways of arranging the books is 3! 7! 4! 5!. ? ? ? Example: How many four digit numbers that can be formed using 0, 1, 3, 5 which are divisible by 5 (without repetition)? Solution: Since the numbers are to be divisible by 5, the last digit has to be 0 or 5. If it is 0, the first digit can be chosen in 3 ways, the second digit can be chosen in 2 ways and the third digit can be chosen in 1 way. Therefore, with 0 as the last digit, the number of four digit numbers is 3 2 1 6 ? ? ? . If the digit is 5, the first digit can be chosen in 2 ways, the second digit can be chosen in 2 ways and third digit can be chosen in 1 way. ? with 5 as the last digit, the number of four digit numbers is 2 2 1 4 ? ? ? ways. ??The total number of four digit numbers which are divisible by 5 is 6 4 10. ? ?
Note Unit 2: Permutation and Combinations 46 Example What is the total number of ways of simultaneous throwing of (i) 3 coins, (ii) 2 dice and (iii) 2 coins and a die ? Solution 1. Each coin can be thrown in any one of the two ways, i.e, a head or a tail, therefore, the number of ways of simultaneous throwing of 3 coins = 2 3 = 8. 2. Similarly, the total number of ways of simultaneous throwing of two dice is equal to 6 2 = 36 and 3. The total number of ways of simultaneous throwing of 2 coins and a die is equal to 2 2 6 = 24. Example In how many ways the first, second and third prize can be given to 10 competitors? Solution: There are 10 ways of giving first prize, nine ways of giving second prize and eight ways of giving third prize. Therefore, total no. ways is 1098 = 720. Alternative Method: Here n = 10 and r = 3, ( ) 10 3 10! P 720 10 3 ! \ = = - Example 1. There are 5 doors in a room. In how many ways can three persons enter the room using different doors? 2. A lady is asked to rank 5 types of washing powders according to her preference. Calculate the total number of possible rankings. 3. In how many ways 6 passengers can be seated on 15 available seats? 4. If there are six different trains available for journey between Delhi to Kanpur, calculate the number of ways in which a person can complete his return journey by using a different train in each direction. 5. In how many ways President, Vice-President, Secretary and Treasurer of an association can be nominated at random out of 130 members? Solution 1. The first person can use any of the 5 doors and hence can enter the room in 5 ways. Similarly, the second person can enter in 4 ways and third person can enter in 3 ways. Thus, the total number of ways is 5 3 5! P 60 2! = = . 2. Total number of rankings are 5 5 5! P 120 0! = = . (Note that 0! = 1)
Business Mathematics Note 47 3. Total number of ways of seating 6 passengers on 15 seats are: 15 6 15! P 9! = = 36,03,600. 4. Total number of ways of performing return journey, using different train in each direction are 65 = 30, which is also equal to . 5. Total number of ways of nominating for the 4 post of association are: 130 4 130! P 27,26,13,120 126! = = . Example Three prizes are awarded each for getting more than 80% marks, 98% attendance and good behaviour in the college. In how many ways the prizes can be awarded if 15 students of the college are eligible for the three prizes? Solution Note that all the three prizes can be awarded to the same student. The prize for getting more than 80% marks can be awarded in 15 ways, prize for 90% attendance can be awarded in 15 ways and prize for good behaviour can also be awarded in 15 ways. Thus, the total number of ways is n r = 15 3 = 3,375.

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Example 1. In how many ways can the letters of the word EDUCATION be arranged?

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In how many ways can the letters of the word STATISTICS be arranged?

In how many ways can 20 students be allotted to 4 tutorial groups of 4, 5, 5 and 6 students respectively? In how many ways 10 members of a committee can be seated at a round table if (i) they can sit anywhere (ii) president and secretary must not sit next to each other? Solution 1. The given word EDUCATION has 9 letters. Therefore, number of permutations of 9 letters is 9! = 3,62,880. 2. The word STATISTICS has 10 letters in which there are 3S' s , 3T' s , 2I' s , 1A and 1C. Thus, the required number of permutations 10! 3!3!2!1!1! = 50,400. 3. Required number of permutations 20! 4!5!5!6! = 9,77,72,87,522 4. (i) Number of permutations when they can sit anywhere = (10 –1)! = 9! = 3,62,880.
Note Unit 2: Permutation and Combinations 48 (ii) We first find the number of permutations when president and secretary must sit together. For this we consider president and secretary as one person. Thus, the number of permutations of 9 persons at round table = 8! = 40,320. ? The number of permutations when president and secretary must not sit together = 3,62,880 – 40,320 = 3,22,560. Example 1.

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In how many ways 4 men and 3 women can be seated in a row such that

women occupy the even places? 2.

39%	MATCHING BLOCK 12/248	SA	Objective Mathematics -II (BMGE- 02)- FINAL.pdf (D162012805)

In how many ways 4 men and 4 women can be seated such that men and women occupy alternative places? Solution 1. 4 men can be seated in 4! ways and 3 women can be seated in 3! ways.

Since each arrangement of men is associated with each arrangement of women, therefore, the required number of permutations = 4! 3! = 144. 2. There are two ways in which 4 men and 4 women can be seated MWMWMWMWMW or WMWMWMWMWM The required number of permutations = 2 .4! 4! = 1,152 Example There are 3 different books of economics, 4 different books of commerce and 5 different books of statistics. In how many ways these can be arranged on a shelf when 1. all the books are arranged at random, 2. books of each subject are arranged together, 3. books of only statistics are arranged together, and 4. books of statistics and books of other subjects are arranged together? Solution 1. The required number of permutations = 12! 2. The economics books can be arranged in 3! ways, commerce books in 4! ways and statistics book in 5! ways. Further, the three groups can be arranged in 3! ways. ? The required number of permutations = 3! 4! 5! 3! =1,03,680. 3. Consider 5 books of statistics as one book. Then 8 books can be arranged in 8! ways and 5 books of statistics can be arranged among themselves in 5! ways. ? The required number of permutations = 8! 5! = 48,38,400. 4. There are two groups which can be arranged in 2! ways. The books of other subjects can be arranged in 7! ways and books of statistics can be arranged in 5! ways. Thus, the required number of ways = 2! 7! 5! = 12,09,600.
Business Mathematics Note 49 Example Find the number of ways in which a group of 6 men and 4 women be seated at a round table. Solution: Total number of people to be seated is 10. Therefore, the number of ways of these 10 people to be seated at a round table is (10 1)! 9! ? ? Example In how many ways can 12 beads of the same colour and size be strung together to form a necklace. Solution: Since the beads are of the same colour and size, the anticlockwise and clockwise arrangements are same. Therefore, the number of permutations 1 (1)! 2 n ? ? 1 (12 1)! 2 11! 2 ? ? ? Example

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In how many ways 7 boys and 7 girls be seated at a round table so that no two girls

shall sit next to each other. Figure 2.5: Circular Permutations Solution: First arrange 7 boys at a round table. This can be done in (7 1)! 6! ? ? ways. After arranging these 6 boys in any one of these 6! ways, the 7 girls can be arranged in 7! ways. Therefore the number of ways that 7 boys and 7 girls be seated at a round table so that no two girls shall sit together is 6! x7!. Task A father, mother, 2 boys, and 3 girls are asked to line up for a photograph. Discuss in group of 7, considering each identity to determine the number of ways they can line up if 1. there are no restrictions 2. the parents stand together 3. the parents do not stand together 4. all the females stand together
Note Unit 2: Permutation and Combinations 50 Self Assessment State whether the following statements are true or false: 3. A permutation is an arrangement of a given set of objects in any order. 4. Composition and order both are important in a permutation. 5. Permutations play an important role in the theory of probability which is being used in many areas 6. The problems on permutations are sheer amusement. 2.3 Combination A combination is a selection of things. When no attention is given to the order of arrangement of the selected objects, we get a combination. We know that

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the number of permutations of n objects taking r at a time is n

r P . Since r objects can be arranged in r! ways, therefore, there are r! permutations corresponding to one combination. Thus,

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the number of combinations of n objects taking r at a time, denoted by n r C , can be obtained by dividing by r!, i.e., . ( ) n n r

r P n! C r! r! n r ! = = - Caution! If the order do matter then we have a permutation. Otherwise, it is a combination. Definition: The number of ways of selecting 'r' things out of 'n' things is called
the number of combinations of n things taken r at a time and is denoted by .

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n r C Theorem 1 : ! n n r r P C r ? Theorem 2 : n n r n r C C ? ? Theorem 3 : 1 1 n n n r r r C C C ? ? ? ? Particular Cases 0 ! ! 1 (0)! 0! ! n n n C n n ? ? ? ? 1 ! ! (1)! (1)! 1! (1)! (1)! n n n n n C n n n n ? ? ? ? ? ? ? ? ? ! ! ! ! 1 ( )! ! 0! ! 0! ! ! n n n n n n C n n n n n n ? ? ? ? ? ? Business Mathematics Note 51 Note: (a) Since n n r n r

C C - = , therefore, is also equal to the combinations of n objects taking (n – r) at a time. The total

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number of combinations of n distinct objects taking 1, 2, ... n respectively, at a time is n n n n 1 2

n C C ...... C 2 1 + + + = - . 2.3.1 Ordered Partitions 1. Ordered Partitions (distinguishable objects) (a) The total number of ways of putting n distinct objects into r compartments which are marked as 1, 2, ... r is equal to r n . Since first object can be put in any of the r compartments in r ways, second can be put in any of the r compartments in r ways and so on. (b) The number of ways in which n objects can be put into r compartments such that the first compartment contains n 1 objects, second contains n 2 objects and so on the rth compartment contains n r objects, where n 1 + n 2 + ... + n r = n, is given by n! n 1 !n 2 ! ...... n r ! . To illustrate this, let r = 3. Then n 1 objects in the first compartment can be put in ways. Out of the remaining n ? n 1 objects, n 2 objects can be put in the second compartment in ways. Finally the remaining n ? n 1 ? n 2 = n 3 objects can be put in the third compartment in one way. Thus, the required number of ways is

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n C n 1 ´ n - n 1 C n 2 = n! n 1 !n 2 !n 3 ! . 2.

Ordered Partitions (identical objects) (a) The total number of ways of putting n identical objects into r compartments marked as 1, 2, ... r, is n+r- 1 C r- 1 , where each compartment may have none or any number of objects. We can think of n objects being placed in a row and partitioned by the (r ? 1) vertical lines into r compartments. This is equivalent to permutations of (n + r ? 1) objects out of which n are of one type and (r ? 1) of another type. The required number of permutations are n +r - 1 a f ! n! r - 1 a f ! , which is equal to n+r- 1 a f C n or n+r- 1 a f C r- 1 a f . (b) The total number of ways of putting n identical objects into r compartments is or , where each compartment must have at least one object. Note Unit 2: Permutation and Combinations 52 In order that each compartment must have at least one object, we first put one object in each of the r compartments. Then the remaining (n ? r) objects can be placed as in (a) above. (c) The formula, given in (b) above, can be generalised. If each compartment is supposed to have at least k objects, the total number of ways is, n- kr b g + r- 1 a f C r- 1 a f where k = 0, 1, 2, ... etc. such that k > n r . Example Find the values of 7 3 C and 10 2 C . Solution: 7 3 7! 7 6 5 4! 7 6 5 35 (7 3)! 3! 4! 3 2 6 C ? ? ? ? ? ? ? ? ? ? ? 10 2 10! 10 9 8! 45 (8)! 2! 8! 2 1 C ? ? ? ? ? ? ? Example Find n if 10 4 n

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n C C ? Solution: 10 4 n n C C ? 10 4 ( ) 4 10 14 n n n n n r n r C C C C n ? ? ? ? ? ? ? ? ? ? ? Example 3: From a group of 20 people,

how many selections of 12 people can be made so as to exclude 5 particular persons. Solution: Since 5 particular persons are to be excluded in each selection, only 15 people remain out of which 12 are to be selected. ? Required number of selections 15 12 15 14 13 3 2 1 5 7 13 455 C ? ? ? ? ? ? ? ? ? ? Example From 7 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done to include atleast one lady? Solution: The committee may be formed in the following ways. (i) 1 lady and 4 gentlemen and this can be done in 4 7 1 4 7 6 5 4 140 3 2 1 C C ? ? ? ? ? ? ? ? ? ways. (ii) 2 ladies and 3 gentlemen and this can be done in 4 7 2 3 4 7 6 5 210 2 1 3 2 1 C C ? ? ? ? ? ? ? ? ? ? 3 ways
Business Mathematics Note 53 (iii) 3 ladies and 2 gentlemen and this can be done in 4 7 3 2 7 6 4 84 2 1 C C ? ? ? ? ? ? ways (iv) 4 ladies and 1 gentleman and this can be done in 4 7 4 1 1 7 7 C C ? ? ? ? ways Therefore, total number of 5 member committee with atleast one lady 140 210 84 7 441 ? ? ? ? ? . Example In how many ways two balls can be selected from 8 balls? Solution balls can be selected from 8 balls in 8 2 8! C 28 2!6! = = ways. Example In how many ways a group of 12 persons can be divided into two groups of 7 and 5 persons respectively? Solution 2. Since , therefore, the number of groups of 7 persons out of 12 is also equal to the number of groups of 5 persons out of 12. Hence, the required number of groups is 12 7 12! C 792 7!5! = = . Alternative Method. We may regard 7 persons of one type and remaining 5 persons of another type. The required number of groups are equal to the number of permutations of 12 persons where 7 are alike of one type and 5 are alike of another type. Example A committee of 8 teachers is to be formed out of 6 science, 8 arts teachers and a physical instructor. In how many ways the committee can be formed if 1. Any teacher can be included in the committee. 2. There should be 3 science and 4 arts teachers on the committee such that (i) any science teacher and any arts teacher can be included, (ii) one particular science teacher must be on the committee, (iii) three particular arts teachers must not be on the committee? Solution 3. (a) 8 teachers can be selected out of 15 in 15 8 15! C 6,435 8!7! = = ways. (b) (i) 3 science teachers can be selected out of 6 teachers in ways and 4 arts teachers can be selected out of 8 in ways and the physical instructor can be
Note Unit 2: Permutation and Combinations 54 selected in 8 4 C way. Therefore, the required number of ways = 6 8 1 3 4 1 C C C ´ ´ = 20 × 70 × 1 = 1400. (ii) 2 additional science teachers can be selected in ways. The number of selections of other teachers is same as in (i) above. Thus, the required number of ways = 5 8 1 2 4 1 C C C ´ ´ = 10 × 70 × 1 = 700. (iii) 3 science teachers can be selected in ways and 4 arts teachers out of remaining 5 arts teachers can be selected in ways. The required number of ways = 6 5 3 4 C C ´ = 20 × 5 = 100. Example In how many ways can a person choose one or more of the goods: T.V., Refrigerator, Washing machine, Radiogram? Solution: 1. good

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can be chosen in 4 1 4 C ? ways. 2. goods can be chosen in 4 2 4 3 6 2 1 C ? ? ? ? ways. 3. goods can be chosen in 4 4 3 1 4 C C ? ? ways. 4.

goods can be chosen in 4 4 1 C ? way. ? ?? the total number of ways of choosing the goods is 4 6 4 1 15 ? ? ? ? . Example In an examination paper, there are two parts each containing 4 questions. A candidate is required to attempt 5 questions but not more than 3 questions from any part.

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In how many ways can 5 questions be selected? Solution: 5 questions can be selected in the following ways: (i) 2 and 3: This can be done in 4 4 2 3 C C ? ways 4 3 4 24 2 1 ? ? ? ? ? (ii) 3 and 2: This can be done in 4 4 3 2 C C ? ways 4 3 4 24 2 1 ? ? ? ? ? ?? 5

questions can be selected in 24 24 48 ? ? ways. Example There are 12 points in a plane of which 4 are collinear. Find the number of (i) straight lines, (ii) triangles which can be formed from these points.

Business Mathematics Note 55 Solution: (i) Two points are required for a line. If no three of the given 12 points are collinear then the number of straight lines that can be formed is 12 2 C . But 4 points are given to be collinear. From these 4 points, we get only one straight line. But if these 4 points were non- collinear, we would get 4 2 C number of straight lines. ? Number of straight lines 12 4 2 2 1 C C ? ? ? 12 11 4 3 1 2 1 2 1 66 1 6 61 ? ? ? ? ? ? ? ? ? ? ? ? (ii) Three non-collinear points are required to form a triangle. If none of the 12 points are not collinear, then the number of triangles that can be formed is 12 3 .C Since 4 of these points are collinear, we would not get 4 3 C triangles from these 4 points. ? number of triangles 12 4 3 3 C C ? ? 12 11 10 4 3 2 1 ? ? ? ? ? ? ? 2 11 10 4 220 4 ? ? ? ? ? ? ? 216 ? Example Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. In how many different ways we can place the balls so that no box remains empty? (IIT-1981) Solution: The five balls of different colours can be placed in three boxes in the following ways. Case (i) I Box II Box III Box (a) 1 ball 1 ball 3 balls (b) 1 ball 3 balls 1 ball (c) 3 balls 1 ball 1 ball 3 balls for one box can be selected in 5 3 C ways. 1 ball for second box can be selected in 2 1 C ways the last ball can be placed in the III box in 3 ways. ? number of ways 5 2 3 1 3 10 2 3 60 C C ? ? ? ? ? ? ?

Note Unit 2: Permutation and Combinations 56 Case (ii) I Box II Box III Box (a) 1 ball 2 balls 2 balls (b) 2 balls 1 ball 2 balls (c) 3 balls 3 balls 1 ball 2 balls can be selected for one box in 5 2 C ways. 2 balls can be selected for second box from the remaining 3 balls in 3 2 C ways. The last ball can be placed in the third box in 3 ways. ?? number of ways 5 3 2 2 3 10 3 3 90 C C ? ? ? ? ? ? ? ? ?? total number of ways 60 90 150 ? ? ? Example: (a) In how many ways two balls can be selected from 8 balls? (b) In how many ways a group of 12 persons can be divided into two groups of 7 and 5 persons respectively? (c) A committee of 8 teachers is to be formed out of 6 science, 8 arts teachers and a physical instructor. In how many ways the committee can be formed if 1. Any teacher can be included in the committee. 2. There should be 3 science and 4 arts teachers on the committee such that (i) any science teacher and any arts teacher can be included, (ii) one particular science teacher must be on the committee, (iii) three particular arts teachers must not be on the committee? Solution (a) 2 balls can be selected from 8 balls in 8 C 2 = 8! 2!6! =28 ways. (b) Since n C r = n C n- r , therefore, the number of groups of 7 persons out of 12 is also equal to the number of groups of 5 persons out of 12. Hence, the required number of groups is 12 C 7 = 12! 7!5! =792 . Alternative Method. We may regard 7 persons of one type and remaining 5 persons of another type. The required number of groups are equal to the number of permutations of 12 persons where 7 are alike of one type and 5 are alike of another type. (c) 1. 8 teachers can be selected out of 15 in 15 C 8 = 15! 8!7! =6,435 ways. 2. (i) 3 science teachers can be selected out of 6 teachers in ways and 4 arts teachers can be selected out of 8 in ways and the physical instructor

Business Mathematics Note 57 can be selected in way. Therefore, the required number of ways = 6 8 1 3 4 1 20 70 1 1400 C C C ? ? ? ? ? ? ? . (ii) 2 additional science teachers can be selected in ways. The number of selections of other teachers is same as in (i) above. Thus, the required number of ways . 5 8 1 2 4 1 10 70 1 700 C C C ? ? ? ? ? ? ? (iii) 3 science teachers can be selected in ways and 4 arts teachers out of remaining 5 arts teachers can be selected in ways. ? ? The required number of ways 6 5 3 4 20 5 100 C C ? ? ? ? ? ? . Example 4 couples occupy eight seats in a row at random. What is the probability that all the ladies are sitting next to each other? Solution Eight persons can be seated in a row in 8! ways. We can treat 4 ladies as one person. Then, five persons can be seated in a row in 5! ways. Further, 4 ladies can be seated among themselves in 4! ways. ?? The required probability = 5!4! 8! = 1 14 . Example 12 persons are seated at random (i) in a row, (ii) in a ring. Find the probabilities that three particular persons are sitting together. Solution (i) The required probability = 10!3! 12! = 1 22 . (ii) The required probability ? ? 9!3! 11! 3 55 . Example 5 red and 2 black balls, each of different sizes, are randomly laid down in a row. Find the probability that (i) the two end balls are black, (ii) there are three red balls between two black balls and (iii) the two black balls are placed side by side. Solution. The seven balls can be placed in a row in 7! ways.

Note Unit 2: Permutation and Combinations 58 (i) The black can be placed at the ends in 2! ways and, in-between them, 5 red balls can be placed in 5! ways. ? The required probability = 2!5! 7! = 1 21 . (ii) We can treat BRRRB as one ball. Therefore, this ball along with the remaining two balls can be arranged in 3! ways. The sequence BRRRB can be arranged in 2! 3! ways and the three red balls of the sequence can be obtained from 5 balls in ways. ? The required probability = 3!2!3! 7! ´ 5 C 3 = 1 7 . (iii) The 2 black balls can be treated as one and, therefore, this ball along with 5 red balls can be arranged in 6! ways. Further, 2 black ball can be arranged in 2! ways. ? The required probability = 6!2! 7! = 2 7 . Example Each of the two players, A and B, get 26 cards at random. Find the probability that each player has an equal number of red and black cards. Solution Each player can get 26 cards at random in 52 C 26 ways. In order that a player gets an equal number of red and black cards, he should have 13 cards of each colour, note that there are 26 red cards and 26 black cards in a pack of playing cards. This can be done in 26 26 13 13 C C ? ways. Hence, the required probability 26 26 13 13 52 26 C C C ? ? . Example 8 distinguishable marbles are distributed at random into 3 boxes marked as 1, 2 and 3. Find the probability that they contain 3, 4 and 1 marbles respectively. Solution Since the first, second .... 8th marble, each, can go to any of the three boxes in 3 ways, the total number of ways of putting 8 distinguishable marbles into three boxes is 3 8 . The number of ways of putting the marbles, so that the first box contains 3 marbles, second contains 4 and the third contains 1, are 8! 3!4!1! ?? The required probability 8 8 1 280 341 6561 3 ? ? ? ! ! ! ! .

Business Mathematics Note 59 Self Assessment Fill in the blanks: 7. A .... is a selection of things. 8. When no attention is given to the order of arrangement of the selected objects, we get a .... 9. r objects can be arranged in ....ways, 10. The number of combinations of n objects taking r at a time, denoted by , can be obtained by dividing by r!, i.e., ... 2.4 Result/Application of Permutation and Combination The principles/theories of permutation and combination in mathematics applied in the allocation of telephone numbers from country code to actual telephone/fax numbers for home or businesses. The results obtained are compared to the present world population. The principle of permutation, in mathematics, is about arranging a group of numbers or objects in a specific order while that of combination is about arranging a group of numbers or objects in no specific order. Every person living on earth, in developed, underdeveloped and undeveloped countries deserves to have a telephone because of its necessity. One would then ask: Is it possible to assign telephone numbers to every person living on earth? The answer to this question is "Yes." Let 's explain it how? 2.4.1 The Telephone Numbering System The allocation of telephone numbers primarily starts with the International Telecommunications Union (ITU). The ITU determines and assigns telephone country codes to all countries in the world. Since this task is regulated and controlled, the allocation of those numbers must be in specific order. Usually, these numbers range from 1 to 3 digits. Next to country codes are city codes. City codes are usually assigned by the Ministry of Telecommunications of each country. Usually, these range from 1 to 3 digits as well. Next, in the series of numbers, is usually the telephone numbers themselves; a set of numbers that can go up to 7 digits. Hence, generally a telephone number system appears as follows. 2.4.2 Application of the Principles of Permutation and Combination (a) The Allocation Of Country Telephone Codes The allocation of country telephone code must be in specific order. As already mentioned, the ITU controls this function. The emphasis on this arrangement/allocation is in the word "order." For this reason, this implies the principle of permutation in mathematics. The allocation can be efficiently carried out

Note Unit 2: Permutation and Combinations 60 by posing the following mathematical question: In how many ways can 3 digits be arranged from a group of 10, in particular orders? Answer to this question is provided by the principle of permutation in mathematics. It should be understood that one is free to choose the 3 digits from the decimal numbering system (0...........9), in particular orders. Hence, the answer to that question is 10 P 3 , which is: This means that there are 720 ways to arrange 3 digits from a group of 10 digits, in particular orders. What does this imply in allocation of telephone codes for countries in the world? There are 273 countries in the world today. Using the result from equation (2), this means that all the 273 countries in the world could easily be assigned telephone codes. Comparing the number of countries in the world to the result obtained from equation (2), it can easily be seen that this freedom of choice is more than twice possible and available. That takes care of the first segment in equation (1). (b) The Allocation of City/Area Codes Telephone city/area codes of all countries in the world are usually assigned by the country's Ministry of Telecommunications. Usually, these digits are 3, selected from the decimal number system, 0....9. Grouping of these 3 digits must also be in a particular order. What this implies is that the grouping could be efficiently implemented using the principle of permutation in mathematics, similar to that of allocation of country codes. That means arranging digits in groups of 3 selected from the decimal digit system, (0....9), in orders. This is represented mathematically as 10P3. What this implies is that 720 combinations could be obtained for 3 digits arrangements from 10 digits (0....9). In a country like the United States of America (USA) which has 50 states, this principle would allow up to 720 combinations for area codes. (c) The Allocation of Phone Numbers The principle for efficient allocation of phone numbers is technically different from those of country and city/area codes. This usually constitutes 7 digits and are the actual telephone numbers which provide telecommunications services for homes and businesses. This principle is equally applicable to all types of communication lines. To have a wider latitude of numbers, the 7 digits could be broken into 2 segments, of 3 and 4 digits, as follows: X X X X X X X --- (4) The arrangement here follows no particular order. Hence the principle of "combination" in mathematics applies. By this application the number of arrangements for the first 3 digits, combined from a group of 10 digits (0.....9), are obtained from the following expression.

Business Mathematics Note 61 From the result in equation (5), these would give 120 combinations of 3 digits, chosen from 10 digits (0….9). The last 4 digits would similarly be arranged using the same principle of "Combination." This would be as follows: There would hence be 210 arrangements of 4 digits, chosen from 10 digits, 0…..9. 2.4.3 Technical Interpretations of the Mathematical Results The mathematical results obtained could technically and practically, be implemented as illustrated in the following chart. Figure 2.6: An Illustration of Telephone Allocation Numbers from Country Codes to Actual Telephone Numbers The explanation of the illustration in figure 2.6 is that each number allocation at point A will have 720 possible combinations at point B. Each allocation at point B will have 720 possible combinations at point C. Each allocation at point C will have 120 possible combinations at point D, while each allocation at point D would finally have 210 possible combinations. These allocations and possible combinations of numbers would give 720 x 720 x 120 x 210 = 13,063,680,000--------------------------------------- (7) This very principle of allocation of communication lines covers all types of telephones (including cellular telephones), fax and beeper lines all over the world. Conclusion This study, with the accompanying result, had been made possible by the unique application of the principles/theories of permutation and combination in Mathematics. It would prove an efficient way for the International Telecommunications Union (ITU), Ministries of Telecommunications and telephone companies, in different countries around the world to adopt, in the process of assigning telephone and fax numbers for homes and businesses. The same principle can be applied for economic and social planning of countries and communities

Note Unit 2: Permutation and Combinations 62 around the world. It would particularly be useful in the allocation of social security and national identification numbers and in income identification/classification schemes. Case Study: Randomized Response Scheme The principal of a large high school would like to determine the proportion of students in her school who used drugs during the past week. Because the results of directly asking each student" Have you used drugs during the past week?" would be unreliable, the principal might use the following randomized response scheme. Each student rolls a fair die once, and only he or she knows the outcome. If a 1 or 2 is rolled, the student must answer the sensitive question truthfully. However, if a 3, 4, 5, or 6 is rolled, the student must answer the question with the opposite of the true answer. In this way, the principal would not know whether a yes response means the student used drugs and answered truth- fully or the student did not use drugs and answered untruthfully. If 60% of the students in the school respond yes , what proportion of the students actually did use drugs during the past week? (Note: Split-ting the outcomes of the roll of the die as described represents only one possibility. The outcomes could have been split differently but not in such a way that the probability of answering truthfully is .5. In this case, it would not be possible to estimate the proportion of all students who actually used drugs over the past week.) Questions: 1. What is Randomized Response Scheme? 2. What do you infer from the case? Source: http://www.wiley.com/college/sc/mann/4 Self Assessment State whether the following statements are true or false: 11. The principles/theories of permutation and combination in mathematics applied in the allocation of telephone numbers from country code to actual telephone/fax numbers for home or businesses. 12. The principle of permutation, in mathematics, is about arranging a group of numbers or objects in no specific order. 13. The principle of combination, in mathematics, is about arranging a group of numbers or objects in specific order. 14. The allocation of telephone numbers primarily starts with the International Telecommunications Union (ITU). 15. The ITU determines and assigns telephone country codes to all countries in the world. 16. Country codes range from 1 to 5 digits. 17. Next to country codes are city codes. City.

Business Mathematics Note 63 18. City codes are usually assigned by the Ministry of Telecommunications of each country. 19. City codes range from 1 to 3 digits. 20. Generally a telephone number system appears as: 2.4 Summary ? If the first operation can be performed in any one of the m ways and then a second operation can be performed together in any one of the m × n ways. ? A permutation is an arrangement of a given number of objects in a definite order. ? The total

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number of permutations of n distinct objects is n!, i.e., n r P = n! ? Permutation of n objects taking r at a time: ! ( )!

n r n P n r ? ? ? If we take n things, the number of circular permutations = (n –1)! ? If n things are alike, then the number of circular permutations will be (1)! 2 n ? . ? When no attention is given to the order of arrangement of the selected objects, we get a combination. ? ! ! !( )!

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n p n n r c r r r n r ? ? ? ? n c n-r n c r ? ? The total number of combinations of n distinct objects at a time = 2

n – 1. 2.5 Keywords Circular Permutations: Instead of arranging the things along a line, we arrange the things along a circle, which is called circular permutation. Permutation: A permutation is an arrangement of a given number of objects in a definite order. Permutation Notation: ! (,) ( )! n r n p n r p npr n r ? ? ? ? Combination: The number of ways of selecting 'r' things out of 'n' things is called the number of combinations of 'n' things taken r at a time.
Note Unit 2: Permutation and Combinations 64 Combination Notation: ! (,) !( )! n r n r n n c n r c c r r n r ? ? ? ? ? ? ? ? ? ? ?
2.6 Review Questions 1. In how many ways three models (gold, silver and bronze) can be distributed to 5 persons? 2.

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In how many ways can the letters of the word MATHEMATICS be arranged? 3.

In how many ways can 25 students of management be allotted to 5 classrooms of 5,6,4,5 and 5 students respectively? 4. Find the number of permutations of the letters of the word 'Book'. 5. Find the number of ways in which a group of 5 girls and 6 boys can be seated at a round table. 6. Find the value of n if (i) 3 5 2 n n p p ? (ii) 4 2 12 n n p p ? (iii) 1 1 3 3 12 5 n n p p ? ? ? (iv) 4 2 42 n n p p ? (v) 2 3 5 n n p p? (vi) 720 n n p ? (vii) 40320 n n p ? 7. Find in how many ways can: (i) 5 persons sit in a row (ii) 10 persons be seated in a row 5 at a time (iii) 7 boys be seated in a row 4 at a time (iv) 5 boys and 4 girls be seated in a row if boys and girls are to sit separately. (v) 3 gents and 7 ladies be seated in a row if no two gents are together. 8. In how many ways can the colours of the rainbow be arranged so that the red and the blue colours are always together? 9.

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In how many ways can 3 boys and 5 girls be arranged in a row,

so that
all the 3 boys are together? 10. There are 5 red and 4 black balls of different sizes. Find in how many ways they may

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be arranged in a row if balls of the same colour are to be together. 11.

How many number of ways of arranging 3 books on Hindi, 4 books on English, and 5 books on Sanskrit if books on Hindi are never together.
Business Mathematics Note 65 12. How many numbers between 200 and 600 can be formed by using 1, 2, 3, 4, 5, 6 without repetition. How many of them are even? 13. How many numbers lying between 100 and 1000 can be formed from the digits 1, 2, 3, 4, 5 each digit not occurring more than once in the number? 14. How many numbers between 3000 and 7000 can be formed using 0, 1, 2, 3, 5, 8, 9 without repeating the digits? 15. How many even numbers greater than 300 can be formed with the digits 1, 2, 3, 4, 5 with repetition not allowed? 16. How many odd numbers of five digits can be formed with the digits 0, 2, 3, 4, 7 when no digit is repeated? 17. How many three-digit numbers formed out of 2, 3, 4, 5, 6 (without repetition) are greater than 357? 18. Find the number of arrangements that can be made out of the letters of the following words: (a) SUBJECT (b) COLLEGE (c) PROBLEMS (d) SOLUTIONS (e) MATHEMATICS (f) ENGINEERING (g) PRINCIPAL 19. In how many ways 5 boys can form a ring? 20. In how many ways can 4 commerce and 4 science students be arranged alternatively in a round table? 21. In how many ways can 7 students and 6 teachers sit round a table so that no two teachers sit together? 22. In how many ways 5 different beads be strung on a necklace? 23. In how many ways can 4 men and 2 ladies be arranged at a round table if the two ladies (1) sit together, (2) are separated? 24. Show that 2 2 [1 3 5 .....(2 1)] n n n n ? ? ? ? ? ? 25. Show that 2 r ? if 6 360 r p ? . 26. Show that 5 n ? if 2 2 2 2 50 . n n p p ? ? ? 27. In how many ways the letters of the following words can be arranged: MANAGEMENT, ASSESSMENT, COMMITTEE 28. How many distinct words can be formed from the letters of the word MEERUT? How many of these words start at M and end at T?

Note Unit 2: Permutation and Combinations 66 29. In a random arrangement of letters of the word DROUGHT, find the probability that vowels come together. 30. The letters of the word STUDENT are arranged at random. Find the probability that the word, so formed; (a) starts with S, (b) starts with S and ends with T, (c) the vowels occupy odd positions only, (d) the vowels occupy even positions only. 31. How many triangles can be formed by joining 12 points in a plane, given that 7 points are on one line. 32. In a random arrangement of 10 members of a committee, find the probability that there are exactly 3 members sitting between the president and secretary when the arrangement is done (i) in a row, (ii) in a ring. 33. A six digit number is formed by the digits 5, 9, 0, 7, 1, 3; no digit being repeated. Find the no. of chances that the number formed is (i) divisible by 5, (ii) not divisible by 5. 34. If 30 blankets are distributed at random among 10 beggars, find the chances that a particular beggar receives 5 blankets. 35. In how many ways a committee of 5 persons can be formed out 8 management gurus, 5 artists and 10 mathematicians if any person can be included in the committee. 36. In how many ways one or more of five different goods can be selected? 37. Find the values of n if: (i) 15 11 n

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n C C ? (ii) 30 5 n n C C ? (iii) 18 18 2 n n C C ? ? (iv) 2 3 2 : 44 : 3 n n C C ? (v) 6 4 3 2 n n C

C ? ? ? 38. Out of 16 men, in how many ways a group of 7 men may be selected? 39. Out of 10 questions in an examination paper, in how many ways can a candidate select 5 questions? 40. From 6 boys and 4 girls, 5 are to be selected for admission for B.Sc. course. In how many ways can this be done if there must be exactly 2 girls? 41. In how many ways can 4 books be selected out of 5 Maths and 8 Physics books if (i) there must be exactly one Maths book, (ii) atleast one Maths book?

Business Mathematics Note 67 42. In how many ways can a cricket team of 11 players with atleast 2 bowlers and atleast 2 wicket-keepers be selected out of 3 bowlers, 3 wicket-keepers and 10 other players. 43. Find the number of ways of forming a committee of 2 teachers and 3 students out of 10 teachers and 20 students if (i) a particular teacher has to be included, (ii) a particular student has to be excluded. 44. How many line segments are formed by joining: (i) 10 points in a plane of which 5 are collinear. (ii) 12 points in a plane of which 4 are collinear. (iii) 12 points in a plane of which 3 are collinear. (iv) 20 points in a plane of which 6 are collinear. 45. How many triangles are formed by joining: (i) 6 points in a plane no three of which are collinear (ii) 12 points in a plane of which 4 are collinear. (iii) 8 points in a plane of which 5 are collinear. Answers: Self Assessment 1. Combinatorial 2. n1 × n2 × ...... × nk 3. False 4. True 5. True 6. True 7. combination 8. combination 9. r! 10. ( ) n n r r P n! C r! r! n r ! = = - 11. True 12. False 13. False 14. True 15. True 16. False 17. True 18. True 19. True 20. True 2.7 Further Readings Books Kenneth Rosen, Discrete Mathematics and Its Applications, 7/e, 2012, McGraw-Hill. Bathul, Shahnaz, Mathematical Foundations of Computer Science, PHI Learning. Lambourne, Robert, Basic Mathematics for the Physical Sciences,2008, John Wiley & Sons.

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Business Mathematics Note 69 Unit 3: Set Theory CONTENTS Objectives Introduction 3.1 Set 3.1.1 Elements of a Set 3.2 Notation and Methods of Set Representation 3.3 Types of Sets 3.4 Venn Diagram 3.5 Set Operations 3.6 Union (Set Addition) 3.7 Intersection (Set Multiplication) 3.8 Complement 3.9 Difference 3.10 Laws of Algebra of Sets 3.11 Duality 3.12 Laws of Set Algebra(with proof) 3.13 De-Morgan's Laws 3.14 Theorem on subsets 3.15 Number of Elements of a Set 3.15 Summary 3.16 Keywords 3.17 Review Questions 3.18 Further Readings Objectives After studying this unit, you will be able to: 1. Define the term set and understand methods of set representation , notation and different types of sets 2. Discuss venn diagram, set operations, union (set addition) and intersection (set multiplication), complement and difference of sets 3. Describe laws of algebra of sets 5. Explain duality and number of elements in a set

Note Unit 3: Set Theory 70 Introduction When we say that quantity demanded of a commodity is a function of its price, we mean that the quantity demanded depends upon the price and, therefore, for a given price, we can determine the quantity demanded. Here quantity demanded is a dependent variable while price is an independent variable. The behaviour of the dependent variable with respect to change in independent variable(s) is different in different situations. This fact gives rise to several types of functions. In order to give a formal definition of a function it is necessary to introduce the concept of the Set of Ordered Pairs. In this unit, we will discuss the term set and understand methods of set representation , notation and different types of sets .We will also focus on venn diagram, set operations, union (set addition) and intersection (set multiplication), complement and difference of sets. Finally we will focus on laws of algebra of sets, duality and number of elements in a set. 3.1 Set A set is a collection of distinct objects. These objects may be numbers or individuals or some other items.

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For example, the set of all real numbers, the set of all firms in an industry, the set of all

colleges in Delhi University etc. 3.1.1 Elements of a Set The objects of a set are also termed as its elements. An element, or member, of a set is any one of the distinct objects that make up that set. So, number, letter, point, line, or any other object contained in a se are called elements of a set. Writing A = {1, 2, 3, 4} means that the elements of the set A are the numbers 1, 2, 3 and 4. Also, the elements of the set {a, b, c} are the letters a, b, and c. A set can be written either by enumeration or by description. In an enumeration approach, all the elements of a set are enclosed by { } type of brackets. For example, if A denotes the set of even positive integers less than 9, we can write A = {2, 4, 6, 8}. Theorem: If a set A contains n elements. Then P (A) contains 2 n elements. Proof: Let a 1 , a 2 , a 3 , ... a n , be n elements of a set A. Then the number of subsets of A having one element each of the type {a 1 }, {a 2 },{a 3 }...{a n } is n = n C 1 The number of subsets of A having two element each of the type {a 1, a 2 },{a 1 , a 3 }...{a n } is n = n C 2 The number of subsets of A having three element each of the type {a 1, a 2 , a 3 },{a 2, a 3 , a 4 }... is n = n C 3 ... ... The number of subsets of A having n element each of the type is = n C n Also, the null set ? is the subset of A, its number is = n C 0 . Hence, the total number of subsets of A

Business Mathematics Note 71 P (A) = n C 0 + n C 1 + n C 2 ...+ n C n Or P (A) = (1+1) n [by Binomial theorem] Or P(A) = 2 n . Thus,

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the number of subsets of a set with n elements is 2 n . If the number of

elements in a set is large, then the set is written by its description, enclosed by the same type of brackets, as used in enumeration approach. For example, the set of all real numbers B can be written as B = {x : x is a real number}. Here we give a symbol for the elements of the set and its description separated by a vertical bar. The theory of sets is a fundamental notion. It has a great contribution in the study of different branches of mathematics and has a great importance in modern mathematics. The beginning of set theory was laid down by a German mathematician George Cantor (1845–1918). Thus, a collection of well-defined objects is called a set. The 'objects' are called elements. The elements are definite and distinct. By the term 'well-defined', we mean that it must be possible to tell beyond doubt, whether or not a given object belongs to the collection (set) under consideration. The term 'distinct', means that no element should be repeated. Following are the examples of sets: (i) The set of vowels of English alphabet = {a, e, i, o, u} (ii) The set of all straight lines in a plane. (iii) The set of odd numbers between 3 & 19 is {5, 7, 9, 11, 13, 15, 19} Task Discuss in group, to specify, which of the following are well-defined sets. (a) All the colors in the rainbow. (b) All the points that lie on a straight line. (c) All the honest members in the family. (d) All the consonants of the English alphabet. (e) All the tall boys of the school. (f) All the efficient doctors of the hospital. (g) All the hardworking teachers in a school. (h) All the prime numbers less than 100. (i) All the letters in the word GEOMETRY. Self Assessment Fill in the blanks: 1. Quantity .... is a dependent variable while price is an independent variable. 2. The behaviour of the dependent variable with respect to change in independent variable(s) is .... in different situations

Note Unit 3:

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Set Theory 72 3. A set is a collection of .... objects 4. The objects of a set are also termed as its .... 5. An element, or ...., of a set is any one of the distinct objects that make up that set. 6. A set can be written either by enumeration or by .... 3.2 Notation and Methods of Set Representation Sets are usually denoted by capital letters A, B, C, D, ... and their elements are denoted by corresponding small letters a, b, c, d, ... Note It is not necessary that the elements of a set A are denoted by a. If a is an element of set A, then this fact is denoted by the symbol a ? A and read as "a belongs to A". If a is not an element of A, then we write a ? A and read it as "a does not belong to A."

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Representation of a Set A set can be represented in the following two ways: (

i) Tabular form (or Roster method) and The tabular form is also called the listing method. If all the elements of a set are kept within {} and elements are separated from one another by comma (,), then this form of the set is called tabular form. E.g.= {1, 2, 3,...} = the set of natural numbers. (ii) Set builder form Did u know? Set builder form is also called the rule method In this form, we specify the defining property of the elements of the sets, e.g., if A is the set of all prime numbers, we use a letter, usually x, to represent the elements and we write A = {x : x is a prime number} Caution! When the number of elements in a set is small, we use listing method, But when the number of elements in the set is large or infinite, we use the set builder form. Example Write the set of the letters in the word CALCUTTA.
Business Mathematics Note 73 Solution Since no element enters the set more than once. Therefore A, C and T will occur only once. Hence the required set is {C, A, L, U, T} Example Write the following sets into the tabular form: (i) A = {x | > x > 5, x ?} ;is the set of integers. (ii) A = {x | 3x 2 – 12x = 0, x is a natural number}. Solution (i) The integers satisfying the inequality 0 > x > 5 are 1, 2, 3, 4 ? ?? A = {1, 2, 3, 4} (ii) 3x 2 – 12x = 0 3x(x – 4) = 0 ? x = 0 & x = 4 Since x is a natural number. Therefore x = 4 Hence A = {4} Self Assessment State whether the following statements are true or false: 7. Sets are usually denoted by capital letters 8. Elements are denoted by corresponding small letters 9. The tabular form is also called the rule method. 10. Set builder form is also called the listing method 3.3 Types of Sets 1.

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Singleton set: A set containing only one element is called a singleton set.

E.g. The sets {0}, {x}, {?} all consist of only one element. 2. Empty set: The set having no element is called the empty set or the null set or the void set. It is denoted by ? or {}. A = {x: x 2 + 1 = 0, x is a real number}. A is a null set. Since there is no real number satisfying the equation x 2 + 1 = 0. So the set A is null set. 3. Subset: A set A is called a subset of B
if every element of the set

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A is also an element of the set B. We write it as A ? B, and read as 'A is a subset of B 'or' A is contained in B'. In symbols, if A &

B are two sets such that
Note Unit 3: Set Theory 74 x? A ? x ?

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B. Then A is a subset of B. Note: For every set A, A ? A, i.e., A is itself a subset of A. Also note that the empty set ? is always subset of every set. If A is not a subset of B. We denote A B. E.g.: If A = {a, b, c} and B = {a, b, c, e}. Then clearly A ? B. 4. Proper subset: If A is a subset of B

and if there is at least one element
in
B which is not in

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A, then A is called the proper subset of B. if A = {1, 2, 3} and B = {1, 2, 3, 4}. Then A is a proper subset of B, and is denoted by A ? B, read as 'A is a proper subset of B'.

We can also say, B is a super set of
A, and write it as B ? A. 5. Comparability: Two sets, A & B, are said to be comparable if either of the following conditions is satisfied:

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A ? B or B ? A. I. If A = {1, 2, 3}, B = {1, 2, 3, 4, 5}. Then A ? B so that A and B are comparable. II. If A = {a, b, c} and B = {a}.

Then B ? A, and hence A & B
are
comparable. 6.

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Equal sets: Two sets are said to be equal if they contain the same elements,

i.e.,

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if every element of A is an element of B, and every element of B is also an element of A.

It is denoted by A = B.
Two sets are equal if and only if A ? B and B ? A. =< A = B. If A = {1, 2}, B = {1, 2} and C = {x: x 2 - 3x + 2 = 0}. Then A = B = C. 7. Equivalent sets: Two sets are called equivalent sets, if and only if there is one to one correspondence between their elements. If A = {a, b, c} and B = {1, 2, 3}. Then correspondence in the elements of A and of B is one to one, A is equivalent to B, we write it as A ~ B. Caution! If two sets are equal, they are equivalent but two equivalent sets are not necessarily equal. 8. Finite set: A set is said to be finite set if in counting its different elements, the counting process comes to an end. Thus

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a set with finite number of elements is a finite set. E.g.: I. The set of

vowels = {a, e, i, o, u}.
Business Mathematics Note 75 II. The set of people living in Delhi. 9. Infinite set: A set, which is neither a null set nor a finite set is called an infinite set. The counting process can never come to an end in counting the elements of this set. E.g.: The set of natural numbers= {1, 2, 3, 4...}. 10. Set of sets: If the elements of a set are sets, then the set is called a set of sets. E.g.: {{a}, {a, b}, {a, b, c} 11.
Power set: The set of

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all subsets of a given set A, is called the power set A. The power set A is denoted by P (A). E.g.: If A = {a, b, c}. Then its subsets are ?, {a}, {b}, {c}, {a, b},{a, c},{b, c},{a, b, c} P (A) = {?, {a}, {b}, {c},{a, b}, {a, c},{b, c},{a, b, c}} 12.

Universal set: A set, which contains all sets under consideration as subsets, is called the universal set. It is denoted by U. Note: In the study of different set of letters of English alphabet, the universal set is the set of all letters of English alphabet. 13. Let A r be non – empty set for each r in a set ?. In this case the sets A 1 , A 2 , A 3 ,… A n , are called indexed sets and the set ? = {1,2,3…n} is called index set. Here the suffix r ? ? of A r is called index. Such a family of set is denoted by {A r } r ? ?. E.g.: Let A 1 = {1, 2, 3}, A 2 = {3, 4, 5, 6}, A 3 = {6, 7, 8}, A 4 = {1, 4, 5, 12}, ? = {a, b, c, d, e}. Here we find that E a non-empty set A ? ? ? ?. Hence ? is called index set and the sets A 1, A 2, A 3, A 4, A 5 are called indexed sets. Self Assessment Fill in the blanks: 11. A set containing only one element is called a .... set 12. The set having no element is called the empty set or the null set or the .... set 13. A set A is called a .... of B

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if every element of the set A is also an element of the set B 14.

Two sets are called .... sets, if and only if there is one to one correspondence between their elements. 15.

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The set of all subsets of a given set A, is called the .... set A

Note Unit 3: Set Theory 76 3.4 Venn Diagram Pictorial representations of sets represented by closed figures are called set diagrams or Venn diagrams. Sometimes, relationship between the sets is easily expressed by means of diagrams called Venn Diagrams. Euler was the first mathematician who used circles to represent sets. Universal set U is always shown by a rectangle. Therefore, a Venn diagram or set diagram is a diagram that shows all possible logical relations between a finite collection of sets. Venn diagrams were conceived around 1880 by John Venn. They are used to teach elementary set theory, as well as illustrate simple set relationships in probability, logic, statistics, linguistics and computer. This example involves two sets, A and B, represented here as coloured circles. The orange circle, set A, represents all living creatures that are two-legged. The blue circle, set B, represents the living creatures that can fly. Each separate type of creature can be imagined as a point somewhere in the diagram. Living creatures that both can fly and have two legs—for example, parrots—are then in both sets, so they correspond to points in the area where the blue and orange circles overlap. That area contains all such and only such living creatures. Humans and penguins are bipedal, and so are then in the orange circle, but since they cannot fly they appear in the left part of the orange circle, where it does not overlap with the blue circle. Mosquitoes have six legs, and fly, so the point for mosquitoes is in the part of the blue circle that does not overlap with the orange one. Creatures that are not two-legged and cannot fly (for example, whales and spiders) would all be represented by points outside both circles. The combined area
of sets A and

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B is called the union of A and B, denoted by A ∪ B. The

union in this case contains all living creatures that are either two-legged or that can fly (or both). The area in both A and B, where the two sets overlap, is called the intersection of A and B. Venn diagram has represented some sets in the following different conditions: 1. Universal set U = {1, 2, 3…25} Subset A = {1, 2, 3, 10, 14}.
Business Mathematics Note 77 2. Universal set U = {1, 2, 3…25} A = {1, 2, 3, 10, 14}, B = {2, 3} Clearly B ? A. 3. Universal set U = {1, 2, 3…25} No common elements A = {1, 2, 3} B = {4, 5, 6} 4. Universal set U = {1, 2, 3…25} A = {3, 4, 6, 7} B = {4, 6, 10, 12} The elements 4, 6 of the sets A & B are common. 5.

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Disjoint Sets: Two sets A and B are said to be disjoint, if they have no element in common, i.e., A ? B = ?

Note Unit 3: Set Theory 78 (A ? B = ?) E.g.: The set of all even integers and the set of all odd integers are disjoint sets. Remarks: i. When A and B are two disjoint sets, then A ? B = ?. ii. Each of the

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sets A and B contains A ? B as a subset, i.e., (A ? B) ? A and ¶(A ? B) ? B.

Self Assessment State whether the following statements are true or false: 16. Pictorial representations of sets represented by closed figures are called set diagrams or Venn diagrams. 17. Euler was the second mathematician who used circles to represent sets 18. Universal set U is always shown by a circle. 19. Venn diagrams were conceived around 1880 by John Venn. They are used to teach elementary set theory, as well as illustrate simple set relationships in probability, logic, statistics, linguistics and computer. 3.5 Set Operations There are two basic operations of sets: (a) Union of sets and (b) Intersection of Sets. (a)

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The union of two sets A and B, written as B A? , is another set consisting of elements that belong to A or B (

i.e., to A or B or both). (b)

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The intersection of two sets A and B, written as B A? , is the set of all those elements that belong to A and B (

i.e., to both). Note: ? ? is the set of all
those elements that belong to A but not to B. ??B A is termed as the difference of A and B and is also written as A\B or A–B. 3.6 Union (Set Addition) The union or join of two sets A and B, written as A ? B (read as

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A cup B), is the set of all elements, which are either in A, or in B, or in both.

Business Mathematics Note 79 Thus A ? B = {x: x ? A or x ? B}. Here 'or' means that x is in atleast one of A or B, and may lie in both. The common elements of A and B are taken only once in A ? B. The Venn-diagram for A ? B is given by the shaded area in the above figure. Clearly

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A and B are each a subset of A ? B. E.g. If A = {1, 2, 3, 4, 5} and B = {2, 4, 5, 6, 11, 12} Then A ? B = {1, 2, 3, 4, 5, 6, 11, 12}. Remark: i. When B ? A, then A ? B = A ii. A and B are both subsets of A ? B, i.e., A ? (A ? B) and B ? (A ? B) iii. A ? A = A

iv. If n 1 be

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the number of elements in A and n 2 be the number of elements in B,

then the number of elements of A ? B cannot exceed n 1 + n 2 ; for the elements common to A and B are to be counted only once in A ? B. 3.7 Intersection (Set Multiplication) The intersection or meet of two sets A and B written as A ? B (read as

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A cap B), is the set of all elements that belong to both A and B. Thus A ? B = {x : x ? A and x ? B}.

The Venn-diagram for A ? B is given in the above figure by the shaded area. Clearly

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A ? B is a subset of both A and B. Also A ? A = A. E.g.: If A = {1, 2, 3, 4, 5} Note Unit 3: Set Theory 80 B = (2, 4, 5, 6, 11, 12} Then A ? B = {2, 4, 5}. 3.8 Complement The complement of a set is defined as another set consisting of all elements of the universal set which are not elements of the original set. The complement of the set A for the universal set U is generally denoted by A?, and thus A? = {x : x ? A} or A? = {x | x ? U but x ? A} E.g.: i. If U = {1, 2, 3, 4, 5} and A = {1, 2, 4} Then A? = {3, 5} ii. If U is the set of all letters of English alphabet and A the set of vowels then A? is the set of

letters of the English alphabet other than
the vowels. 3.9
Difference The

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difference of two sets A and B denoted by A – B (read as A minus B), is the set of all elements of A which are not in B. Thus A – B = {x : x ? A, x ? B} Similarly B – A = {x : x ? B, x ? A}

The shaded area in the above Venn – diagram represents the sets A – B and B – A E.g.: If A = {a, b, c, d, e} & B = {d, e, p, q}
Business Mathematics Note 81 Then A – B = {a, b, c}. Symmetric difference:
If
A and B are two sets, then the set (
A – B) ? (B – A)

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is called the symmetric difference of A and B, and is denoted by A ? B

or A ? B. Thus, the symmetric difference

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of the sets A and B is the set of all elements of A and B, which are not common to both A and B. Thus A ? B = {x : x ? A and x ? B or x ? B and x ? A} E.g.: If A = {1, 2, 3}, B = {2, 3, 4, 5} Then A – B ={1, 2, 3} – {2, 3, 4, 5} = {1} and B – A = {2, 3, 4, 5} – {1, 2, 3} = {4, 5} ? ? A ? B = (

A – B) ? (B –
A) = {1} ? {4, 5} = {1, 4, 5}.
Self Assessment Fill in the blanks: 20. The ……………………… of two sets A and B, written as , is another set consisting of elements that belong to A or B 21.

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The ……………………… of two sets A and B, written as , is the set of all those elements that belong to A and B 22.

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The ……………………… of a set is defined as another set consisting of all elements of the universal set which are not

elements of the original set. 23. The ………………………

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of two sets A and B denoted by A – B (read as A minus B), is the set of all elements of A which are not in B. 3.10

Laws of Algebra of Sets 1. Commutative Laws: For any two finite
sets A and B; (i) A U B = B U

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A (ii) A ∩ B = B ∩ A 2. Associative Laws: For any three finite sets A, B and C; (i) (A U B) U C = A U (B U C) (ii) (A ∩ B) ∩ C = A ∩ (B ∩ C)

Thus, union and intersection are associative. 3. Idempotent Laws: For any finite set A;
Note Unit 3: Set Theory 82 (i) A U

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A = A (ii) A ∩ A = A 4. Distributive Laws: For any three finite sets A, B and C; (i) A U (B ∩ C) = (A U B) ∩ (A U C) (ii) A ∩ (B U C) = (A ∩ B) U (A ∩ C)

Thus, union and intersection
are distributive over intersection and union respectively. 5. De

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Morgan's Laws: For any three finite sets A, B and C; (i) A – (B U C) = (A – B) ∩ (A – C) (ii) A - (B ∩ C) = (A – B) U (A – C) De Morgan's Laws can also we written as: For any two finite sets A and B; (i) (A U B)' = A' ∩ B' (ii) (A ∩ B)' = A' U B'

More laws of algebra of sets: 6. For any two
finite sets

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A and B; (i) A – B = A ∩ B' (ii) B – A = B ∩ A' (iii) A – B = A ⇔ A ∩ B = ∅ (iv) (A – B) U B = A U B (v) (A – B) ∩ B = ∅ (vi) A ⊆ B ⇔ B' ⊆ A' (vii) (A – B) U (B – A) = (A U B) – (A ∩ B) 7. For any three finite sets A, B and C; (i) A – (B ∩ C) = (A – B) U (A – C) (ii) A – (B U C) = (A – B) ∩ (A –

C) (
iii)

90%	MATCHING BLOCK 64/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

A ∩ (B - C) = (A ∩ B) - (A ∩ C) (iv) A ∩ (B △ C) = (A ∩ B) △ (A ∩ C) 3.11

Duality The principle of duality for sets states that for any true statement about sets, the dual statement obtained by interchanging unions and intersections, interchanging U and ? and reversing inclusions is also true.
Business Mathematics Note 83 Did u know? A statement is said to be self-dual if it is equal to its own dual. For example ( ) ? ? ?

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A A B A has dual ( ) . ? ? ? A A B A ( ) ? ? ? A A B A has dual ( ) . ? ? ? A A B A ? ? A A ?

has dual statement . ? ? A U A ? ? C A A

U has dual ? ? C A A ?? Notice here that the complement of A, A C + U − A does not become? ? ? but stays A C (= U − A) Set-theoretic union and intersection are dual under the set complement operator C. That is, ( ) .

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C C C A B A B ? ? ? Proof: , , . C C A U A B U B ? ? ? ? ( ) ( ) , ( ) . C x U A U B x U A U B x A B x A B ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ( ) , , . C C C C C x A B x A B x A B x A B x A B ? ? ? ? ? ? ? ? ? ? ? ? ?

The principle of duality for sets, which asserts that for any true statement about sets, the dual statement obtained by interchanging unions and intersections, interchanging U and ∅ and reversing inclusions is also true. 3.12 Laws of Set Algebra(with proof) Laws of union of sets If A, B, C are any three sets, ? the empty set and U the universal set. Then their union follows

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the following laws: 1. Idempotent law: A ? A = A 2. Commutative law: A ? B = B ? A 3. Associative law: (A ? B) ? C = A ? (B ? C) 4. Identity law: (a) A ? ? = A, (b) A ? U =

U Proof: 1. Let x ? A ? A ? x ? A or x ? A ? x ? A i.e., A ?
A ?
A .... (1)
Note Unit 3: Set Theory 84 Let y ∈ A. Then y ∈ A ⇒ y ∈ A or y ∈ A ⇒ y ∈
A ∪ A i.e., A ⊆ A ∪ A .... (2) From (1) & (2), we have
A ∪ A = A. 2. To prove
A ∪ B = B ∪ A Let x ∈ A ∪ B ⇒ x ∈ A or x ∈ B ⇒ x ∈ B or x ∈
A ⇒ x ∈ B ∪ A i.e.,

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A ∪ B ⊆ B ∪ A .... (1) Let y ∈ B ∪ A ⇒ y ∈ B or y ∈ A ⇒ y ∈ A or y ∈ B ⇒ y ∈ A ∪ B i.e., B ∪ A ⊆ A ∪ B .... (2) From (1) & (2), we have A ∪ B = B ∪ A. 3. To prove (A ∪ B) ∪ C = A∪ (B ∪ C) Let x∈ (A ∪ B) ∪ C ⇒ x ∈ (A ∪ B) or x ∈ C ⇒ (x ∈ A or x ∈ B) or x ∈ C ⇒ x ∈ A or (x ∈ B or x ∈ C) ⇒ x ∈ A or (x ∈ B ∪ C) i.e., x ∈ A ∪ (B ∪ C) .... (1) Let y ∈ A ∪ (B ∪ C) ⇒ y ∈ A or (y ∈ B ∪ C) ⇒ y ∈ A or (y ∈ B or y ∈ C) ⇒ (y ∈ A or y ∈ B) or (y ∈ C) ⇒ (y ∈ A ∪ B) or (y ∈ C) ⇒ y ∈ (A ∪ B) ∪ C i.e., A ∪ (B ∪ C) ⊆ (A ∪ B) ∪ C .... (2) From (1) & (2), we have (A ∪ B)∪ C = A ∪ (B ∪ C) 4. To prove A ∪ φ = A and A ∪

U =
U Let x ∈ A ∪ φ
Business Mathematics Note 85 ⇒ x ∈ A or x ∈ φ ⇒ x ∈ A i.e., A ∪ φ ⊆ A .... (1) But A ⊆ A ∪ φ .... (2) From (1) &(2), we have A ∪ φ = A Again let y ∈ A ∪ U ⇒ y ∈ A or y ∈ U ⇒ y ∈ U i.e., A ∪ U ⊆ U .... (1) But U ⊆ A ∪ U .... (2) From (1) and (2), we have A ∪ U = U Laws of Intersection of Sets If A, B, C are any three sets, then their intersection follows the following laws: 1. Idempotent law: A ∩
A = A 2. Commutative law:

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A ∩ B = B ∩ A 3. Associative law: (A ∩ B) ∩ C = A ∩ (B ∩ C) 4. Identity law: A ∩ U = A, A ∩ φ = φ 5. Distributive law: A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) Proof: 1. To prove A ∩ A = A Let x ∈ A ∩ A ⇒ x ∈ A

and x ∈A ⇒ x ∈ A i.e. A ∩ A ⊆ A .... (1) Let y ∈ A ⇒ y ∈ A ⇒ y ∈ A and y ∈ A ⇒ y ∈ A ∩ A i.e. A ⊆
A ∩ A .... (2)
From (1) and (2), we have
Note Unit 3: Set Theory 86
A ? A = A 2.
To prove

A ? B = B ? A Let x ?
A ? B ? x ? A and x ? B ? x ? B and x ? A ? x ?
B ? A i.e.

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A ? B ? B ? A .... (1) Let y ? B ? A ? y ? B and y ? A ? y ? A and y ? B ? y ? A ? B i.e. B ? A ? A ? B .... (2) From (1) and (2), we have A ? B = B ? A 3. To prove (A ? B) ? C = A ? (B ? C) Let x ? (A ? B) ? C ? x ? (A ? B) and x ? C ? (x ? A and x ? B) and x ? C ? x ? A and (x ? B and x ? C) ? x ? A and (x ? B ? C) ? x ? A ? (B ? C) i.e. (A ? B) ? C ? A ? (B ? C) .... (1) Let y ? A ? (B ? C) ? y ? A and y ? (B ? C) ? y ? A and (y ? B and y ? C) ? (y ? A and y ? B) and y ? C ? y ? (A ? B) ? C i.e. A ? (B ? C) ? (A ? B) ? C .... (2) From (1) and (2), we have (A ? B) ? C = A ? (B ? C) 4.

To prove A ? U = A, A ? ? = ?
Let x ? A ? U ? x ? A and x ? U
Business Mathematics Note 87 ? x ? A [Because A ? U] i.e. A ? U ? A .... (1) Again, let y ?A ? y ? A and y ? U [Because A ? U] i.e. A ? A ? U .... (2) from (1) and (2), we have A ? U = A Again to prove A ? ? = ? The set A ? ? is the set of all those elements, which are common to both A and the empty set ?. But ? contains no elements. Therefore the set A ? ? contains no elements, Thus A ? ? = ?. 5. (i) To prove

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A ? (B ? C) = (A ? B) ? (A ? C) Let x ? A ? (B ? C) ?? x ? A or (x ? B ? C) ?? x ? A or (x ? B and x ? C) ?? (x ? A

or x ? B) and (x ? A or x ? C) ?? (x ?
A ? B) and (x ? A ? C) ?? x ? (A ? B) ? (A ? C) i.e.

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A ? (B ? C) ? (A ? B) ? (A ? C) .... (1) Again, let y ? (A ? B) ? (A ? C) ?? (y ? A ? B) and (y ? A ? C) ?? (y ? A or y ?B) and (y ? A or y ? C) ?? y ? A or (y ?B and y ? C) ?? y ? A or (y ? B ? C) ?? y ? A ? (B ? C) i.e., (A ? B) ? (A ? C) ? A ? (B ? C) .... (2) From (1) and (2), we have A ? (B ? C) = (A ? B) ? (A ? C). (ii) To prove A ? (B ? C) = (A ? B) ? (A ? C). Let x ? A ? (B ? C) ?? x ? A and (x ? B ? C) ?? x ? A and (x ? B or x ? C) ?? (x ? A and x ? B) or (x ? A and x ? C) ?? (x ? A ? B) or (x ? A ? C) Note Unit 3: Set Theory 88 ?? x ? (A ? B) ? (A ? C) i.e., A ? (B ? C) ? (A ? B) ? (A ? C) .... (1) Similarly, let y ? (A ? B) ? (A ? C) ?? y ? (A ? B) or y ? (A ? C) ?? (y ? A and y ?B) or (y ? A and y ? C) ?? y ? A and (y ?B or y ? C) ?? y ? A and (y ? B ? C) ?? y ? A ? (B ? C) i.e., (A ? B) ? (A ? C) ? A ? (B ? C) .... (2) From (1) and (2), we have A ? (B ? C) = (A ? B) ? (A ? C). Laws of

Complement of A Set If ? is the empty set and U the universal set and A be any one of its subsets. Then the component of A, i.e., A? follows the following laws: 1.

57%	MATCHING BLOCK 71/248	W

A ? A? = U 2. A ? A? = ? 3. (A?) ? = A 4. (A ? B)? = A? ? B?. 5. (A ? B) ? = A? ? B?. Proof 1: To prove A ? A? =

U Since every set is a subset of a universal set, therefore the set A ? A? ? U .... (1) Again, let x ? U, then x ? U ? x ? A or x ? A?. ? x ? (A ? A?) i.e.. U ? A ? A? .... (2) From (1) & (2), we have A ? A? = U Proof 2: To prove A ? A? = ? Since the null set ? is a subset of every set, therefore it follows that ? ? A ? A? .... (1) Again, let x ? A ? A?
Business Mathematics Note 89 ? x ? A and x ? A? ? x ? A and x ? A ? x ? ? i.e. A ? A? = ? .... (2) From (1) and (2), we have A ? A? = ? Proof 3: To prove (A?)? = A. Let

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x ? (A?)? Then x ? (A?)? ? x ? A? ? x ? A i.e. (A?)? ? A .... (1) Again, let x ? A Then x ? A ? x ? A? ? x ? (A?)? i.e. A ? (A?)? .... (2) From (1) and (2), we have (A?)? = A. Proof 4: Let x ? (A ? B) ? ? x ? A ? B ? x ? A or x ? B. ? x ? A? or x ? B? ? x ? A? ?

B? i.e. (

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A ? B) ?? A? ? B? .... (i) Again, Let y ? A? ? B? y ? A? ? B? ? y ? A? and y ? B?. ? y ? A or y ? B ? y ? (A ? B) ? y ? (A ? B) i.e. A? ? B?? (A ? B) ? .... (ii) From (i) and (ii), we have (A ? B) ?= A? ? B?.

Proof 5: Let x ? (A ? B) ? ?

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x ? A ? B ? x ? A or x ? B. ? x ? A? and x ? B? Note Unit 3: Set Theory 90 ? x ? A? ? B? i.e. (A ? B) ?? A? ? B? .... (i) Similarly, we can show A? ? B?? (A ? B) ? .... (ii) From (i) and (ii), we have (A ? B) ?= A? ? B?. Law of Symmetric Difference If A and B are any two sets. Then A ? B = (A ? B) – (A ? B). Proof: Let x ? A ? B. Then x ?(A – B) ? (B – A) ?? (x ?

A and x ? B) or (x ? B and

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x ? A) ?? (x ? A and x ? B) or x ? B and [x ? A and x ? B] or x ? A ?? [x ? B or (x ? A and x ? B)] and [x ? A or (x ? A and x ? B)] ?? [(x ? B or x ? A) and (x ? B or x ? B)] and [(x ? A or x ? A) and (x ? A or x ? B)] ?? [x ? A ? B and x ? U] and [x ? U and x ? A ? B)] ?? x ? (A ? B) and x ? (A ? B) ?? x ? [(A ? B) – (A ? B)]

i.e.

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A ? B = (A ? B) – (A ? B). 3. A – (B ? C) = (A – B) ? (A – C). 4. A – (B ? C) = (A – B) ? (A – C). 3. To prove A – (B ? C) = (A – B) ?(A – C). Let x ? A – (B ? C) ?? x ? A and x ? (B ? C) ?? x ? A and (x ? B and x ? C) ?? (x ? A and x ? B) or (x ? A and x ? C) ?? x ? (A – B) ? x ? (A – C) ?? x ? (A – B) ? (A – C). i.e., A – (B ? C) ? (A – B) ? (A – C) .... (i) Similarly, we can show (A – B) ? (A – C) ? A – (B ? C) .... (ii) From (i) and (ii), we have A – (B ? C) = (A – B) ? (A – C) Business Mathematics Note 91 4. To prove A – (B ? C) = (A – B) ? (A – C) Let x ? A – (B ? C) ?? x ? A and x ? (B ? C) ?? x ? A and (x ? B and x ? C) ?? (x ? A and x ? B) or (x ? A and x ? C) ?? x ? (A – B) or x ? (A – C) ?? x ? (A – B) ? (A – C) i.e., A – (B ? C) ? (A – B) ? (A – C) .... (i) Similarly, we can show (A – B) ? (A – C) ? A – (B ? C) .... (

ii) From (i) and (
ii), we have A – (B ? C) = (A – B) ? (
A –
C).
Self Assessment State whether the following statements are true or false: 24. For any two finite sets A and B; (i) A U B = B U A 25.
For any three finite sets

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A, B and C (ii) (A ∩ B) ∩ C = A ∩ (B ∩ C) 26. For any finite set A; (ii) A ∩ A = A 27. For any three finite sets A, B and C; (ii) A ∩ (B U C) = (A ∩ B) U (A ∩ C) 28. For any three finite sets A, B and C; (i) A – (B U C) = (A – B) ∩ (A – C) 29. For any two finite sets A and B; (i) A – B = A ∩ B' 30. For any three finite sets A, B and C; A ∩ (B - C) = (A ∩ B) - (A ∩ C) 31. A

statement is said to be self-dual if it is not equal to its own dual.

Note Unit 3: Set Theory 92 3.13 De-Morgan's Laws The complement of the union of two sets is equal to the intersection of their complements and the complement of the intersection of two sets is equal to the union of their complements. These are called De Morgan's laws. For any two finite sets A and B; (i) (A U B)' = A' ∩ B' (which is a De Morgan's law of union). (ii) (A ∩ B)' = A' U B' (which is a De Morgan's law of intersection). Proof of De Morgan's law: (A U B)' = A' ∩ B' Let P = (A U B)' and Q = A' ∩ B' Let x be an arbitrary element of P then x ∈ P ⇒ x ∈ (A U B)' ⇒ x ∉ (A U B) ⇒ x ∉ A and x ∉ B ⇒ x ∈ A' and x ∈ B' ⇒ x ∈ A' ∩ B' ⇒ x ∈ Q Therefore, P ⊂ Q .... (i) Again, let y be an arbitrary element of Q then y ∈ Q ⇒ y ∈ A' ∩ B' ⇒ y ∈ A' and y ∈ B' ⇒ y ∉ A and y ∉ B ⇒ y ∉ (A U B) ⇒ y ∈ (A U B)' ⇒ y ∈ P Therefore, Q ⊂ P .... (ii) Now combine (i) and (ii) we get; P = Q i.e. (A U B)' = A' ∩ B' Proof of De Morgan's law: (A ∩ B)' = A' U B' Let M = (A ∩ B)' and N = A' U B' Let x be an arbitrary element of M then x ∈ M ⇒

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x ∈ (A ∩ B)' ⇒ x ∉ (A ∩ B) ⇒ x ∉ A or x ∉ B ⇒ x ∈ A' or x ∈ B' ⇒ x ∈ A' U B' ⇒ x ∈ N

Therefore, M ⊂ N .... (i) Again, let y be an arbitrary element of N then y ∈ N ⇒ y ∈ A' U B'
Business Mathematics Note 93 ⇒ y ∈ A' or y ∈ B' ⇒ y ∉ A or y ∉ B ⇒ y ∉ (A ∩ B) ⇒ y ∈ (A ∩ B)' ⇒ y ∈ M Therefore, N ⊂ M .... (ii) Now combine (i) and (ii) we get; M = N i.e. (A ∩ B)' = A' U B' Also, for any three sets A, B and C (a) Let ? ? ? A x ? and] and [C x B x ? ? ?] and [B
x A x ? ? and] and [C
x
A

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x ? ? ?)\()\(C A B A ? (b) Let) (\ C B A x ? ? ?) (and C B x A x ? ? ? ?] or [and C x B x A x ? ? ? ?] and [or] and [C x A x B x A x ? ? ? ? ?)\()\() (\ )\()\(C A B A C B A C A B A x ? ? ? ? ? ?

Similarly it can be shown that) (\ )\()\(C B A C A B A ? ? ? Hence)\()\() (\ C
A B A C
B
A ? ? ?
Examples 1. If
U = {j, k, l, m, n}, X = {j, k, m} and Y = {k, m, n}. Proof of De Morgan's law: (

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X ∩ Y)' = X' U Y'. Solution: We know, U = {j, k, l, m, n} X = {j, k, m} Y = {k, m, n} (X ∩ Y) = {j, k, m} ∩ {k, m, n} = {k, m} Therefore, (X ∩ Y)' = {j, l, n} .... (i) Again, X = {j, k, m} so, X' = {l, n} and Y = {k, m, n} so, Y' = {j, l} X' ∪ Y' = {l, n} ∪ {j,

l}
Note Unit 3: Set Theory 94 Therefore, X' ∪ Y' = {j, l, n} .... (ii) Combining (i)and (ii) we get; (X ∩ Y)' = X' U Y'. Hence Proved. 2. Let U = {1, 2, 3, 4, 5, 6, 7, 8},

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P = {4, 5, 6} and Q = {5, 6, 8}. Show that (P ∪ Q)' = P' ∩ Q'. Solution: We know, U = {1, 2, 3, 4, 5, 6, 7, 8} P = {4, 5, 6} Q = {5, 6, 8} P ∪ Q = {4, 5, 6} ∪ {5, 6, 8} = {4, 5, 6, 8} Therefore, (P ∪ Q)' = {1, 2, 3, 7} .... (i) Now P = {4, 5, 6} so, P' = {1, 2, 3, 7, 8} and Q = {5, 6, 8} so, Q' = {1, 2, 3, 4, 7} P' ∩ Q' = {1, 2, 3, 7, 8} ∩ {1, 2, 3, 4, 7} Therefore, P' ∩ Q' = {1, 2, 3, 7} .... (ii) Combining (i)and (ii) we get; (P ∪ Q)' = P' ∩

Q'. Hence Proved. 3.14 Theorem on subsets Theorem: 1. The empty set is the subset of every set. Let A be any set and ? be the empty set. It is clear that ? has no element of A. Thus ? is a subset of A. Proved Theorem: 2. Every set is the subset of itself. Let A be any set. Therefore every element of A is an element of the set A. By definition of the subset, the set A is the subset of the set
A. Hence

A ? A Proved Theorem: 3.
If
A ? B and B ? A. then A = B. Proof: Given

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A ? B. If x ? A ? x ? B .... (1) Also B ? A. If x ? B ? x ? A .... (2) From (1) and (2), x ? A ? x ? B. ? A = B. Proved Business Mathematics Note 95 Theorem: 4. If A ? B and B ? C. then A ? C. Proof: A ? B ? x ? A ? x ? B .... (1) Again B ? C. ? x ? B ? x ? C .... (2) From (1) and (2), it is clear x ? A ? x ? C ?? A ? C Proved Theorem: If a

set A contains n elements. Then P (A) contains 2 n elements. Proof: Let a 1 , a 2 , a 3 , ... a n , be n elements of a set A. Then the number of subsets of A having one element each of the type {a 1 }, {a 2 },{a 3 }...{a n } is n = n C 1 The number of subsets of A having element each of the type {a 1, a 2 },{a 1 , a 3 }...{a n } is n = n C 2 The number of subsets of A having element each of the type {a 1, a 2 , a 3 },{a 2, a 3 , a 4 }... is n = n C 3 3.15 Number of Elements of a Set In a finite set, if operations are made, some new subsets will be formed. In this section we will find the values of these new subjects. Since A is a finite set, we shall denote it by n (A) for the finite elements in A, which may be obtained by actual counting. But for unions of two or more sets, we have different formulae : 1. For union of Two sets : For two sets A and B which are not disjoint. 2. For Union of Three Sets : Let A, B and C be the three sets (not mutually disjoint); then 2. For Union of Three Sets : Let A, B and C be the three sets (no mutually disjoint); then
Note Unit 3: Set Theory 96 Example Use Venn diagrams in different situations to find the following

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sets. (a) A ∪ B (b) A ∩ B (c) A' (d) B - A (e) (A ∩ B)' (f) (A ∪ B)' Solution: ξ = {a, b, c, d, e, f, g, h, i, j} A = {a, b, c, d, f}

B = {d, f, e, g} A ∪ B = {

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elements which are in A or in B or in both} = {

a, b, c, d, e, f, g} A ∩ B = {elements which are common to both A and B} = {d, f} A' = {elements of ξ, which are not in A} = {e, g, h, i, j} B - A = {elements which are in B but not in A} = {e, g} (A ∩ B)' = {elements of ξ which are not in A ∩ B} = {a, b, c, e, g, h, i, j} (A ∪ B)' = {elements of ξ which are not in A ∪ B} = {h, i, j} Example If A and B are two non-empty sets, prove that ")' (B A B A ? ? ? , where A' is compliment of A etc.
Business Mathematics Note 97 Solution Let)' (
B

31%	MATCHING BLOCK 86/248	SA	CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

A x ? ? ? ? B x A x ? ? and ? ' and ' B x A x ? ? ?)"(B A x ? ? ? ")"(B A B A ? ? ? Similarly it can be shown that)"()"(B A B A ? ? ? Hence ")' (B A B A ? ? ? Example If n (A)

and n(B)
denote the number of elements in the finite sets A and B, then prove, by Venn diagram that

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n(A) + n(B) = n(A ? B) + n(A ? B) showing A, B, B A?

etc., as given below Solution. Let A and B be two subsets of the universal set U. We draw venn diagram. From the diagram, we can write A = (

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A ? B') ?(A ? B) Since (A ? B) and (A ? B) are disjoint sets, we have n(A) =) ( )' (B

An B An ? ? ? ...(1) Similarly) ( )'( )(B An B An Bn ? ? ? ? ...(2) Adding (1) and (2), we get)'( ) (2)' ( )( )(B An B An B An Bn An ? ? ? ? ? ? ? ...(3) Further, note that)'( ) ( )' ( ) (B An B An B An B An ? ? ? ? ? ? ? ? Thus) () ( )( )(B A B An Bn An ? ? ? ? ? ? Example Are the sets ?, {?}, {0} different? A A'?B' ? A?B'? A'?B? B A?B

Note Unit 3: Set Theory 98 Solution ? is an empty set, i.e. it contains no element. The set {?} is a singleton set, for it contains an element ?. The set {0} is a singleton set, for it contains an element 0. Example Which of the following sets are equal? S 1 = {1, 2, 2, 3}, S 2 = {

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x| x 2 – 2x + 1 = 0}, S 3 = {1, 2, 3}, S 4 = {x| x 3 – 6x 2 + 11x – 6 = 0}. Solution We have S 1 = {1, 2, 2, 3} = {1, 2, 3}. Since x 2 – 2x + 1 = 0 ? (x – 1) 2 = 0 or x = 1,1. ? S 2 = {x| x 2 – 2x + 1 = 0} ={1, 1} = {1}. Again S 3 = {1, 2, 3} Also x 3 – 6x 2 + 11x – 6 = 0 or (x – 1)(x 2 – 5x + 6) = 0 or (x – 1)(x – 2)(x – 3) = 0 ? x = {1, 2, 3}.

S 4 = {1, 2, 3}. Thus it is clear S 1 = S 3 = S 4 . Answer Example Find the number of proper subsets of the set of letters of the UTTAR PRADESH. Solution The set of letters of the word UTTAR PRADESH is {U, T, A, R, P, D, E, S, H}. Because there are 9 elements in the set. Therefore the total number of subsets is = 2 9 . Since the set consisting of all the 9 elements is not a proper set. Therefore the required number of proper subsets = 2 9 – 1 = 512 – 1 = 511 Answer Example Prove that A ? ? ? A = ?. Solution Let A ? ?. Since ? is a subset of every set.

Business Mathematics Note 99 Therefore, in particular φ is a subset of A, i.e. φ ⊂ A. Now φ ⊂ A & A ⊂ φ ⇒ A = φ. Proved Example Write the tabular form as well as set builder form of set containing the elements 1, 3, 5, 7, 9... Solution Tabular form of the given set is {1, 3, 5, 7, 9...} The set builder form of the given set is {2n – 1: n∈}. Example Is a set A comparable with itself? Solution Since A ⊂ A is true for every set A. By definition, this declares that A is comparable with itself. Example Are the following sets equal:

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A = {x: x is a letter in the word REAP}, B = {x: x is a letter in the word PAPER}, and C = {x: x is a letter in the word RAPE}, Solution A = {x: x is a

letter in the word REAP}, or A = {R, E, A, P} B = {x: x is a letter in the word PAPER}, or B = {P, A, E, R} = {R, E, A, P}. C = {x: x is a letter in the word RAPE}, or C = {R, A, P, E} = {R, E, A, P}. Thus {R, E, A, P} =

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A = B = C. ⇒ A = B = C. Example If A = {1, 2, 3, 4}, B = {2, 3, 5, 6} and C = {4, 5, 6, 7}. Then verify that i). A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C) ii). A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) Note Unit 3: Set Theory 100 Solution (i) B ∩ C = {2, 3, 5, 6} ∩ {4, 5, 6, 7} = {5, 6}. A ∪ (B ∩ C) = {1, 2, 3, 4} ∪ {5, 6} = {1, 2, 3, 4, 5, 6} .... (i) A ∪ B = {1, 2, 3, 4} ∪ {2, 3, 5, 6} = {1, 2, 3, 4, 5, 6} A ∪ C = {1, 2, 3, 4} ∪ {4, 5, 6, 7} = {1, 2, 3, 4, 5, 6, 7}. ∴ (A ∪ B) ∩ (A ∪ C) = {1, 2, 3, 4, 5, 6} ∩ (1, 2, 3, 4, 5, = {1,2,3,4,5,6} .... (ii) From (1) & (2), we have A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C). (ii) B ∪ C = {2, 3, 5, 6} ∪ {4, 5, 6, 7} = {2, 3, 4, 5, 6, 7}. A ∩ (B ∪ C) = {1, 2, 3, 4} ∩ {2, 3, 4, 5, 6, 7} = {2, 3, 4} .... (i) Again A ∩ B = {1, 2, 3, 4} ∩ {2, 3, 5, 6} = {2, 3} .... (ii) A ∩ C = {1, 2, 3, 4} ∩ {4, 5, 6, 7} = {4}. .... (iii) ∴ (A ∩ B) ∪ (A ∩ C) = {2, 3} = {2,3,4} .... (iv) From (1) & (4), we observe A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

Example If is the set of complex numbers and
A = {
x ∈: x 4 – 1 = 0}, B = {x ∈: x 3 – 1 = 0}. Find A – B and A ∪ B. Solution We have

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x 4 – 1 = 0 ⇒ (x 2 + 1) (x 2 – 1) = 0 ⇒ x = ± 1, ± i Since, x ∈⇒ x = ± i ∴ A = {- i, i} Again x 3 – 1 = 0 ⇒ (x – 1)(x 2 + x + 1) = 0 ⇒ x = 1 and x = (–1 ± √1–4) / 2 ⇒ x = 1 and x = (–1 ± √3 i) / 2 Since, x ∈⇒ x = (–1 ± i√3) / 2 ∴ B = {(-1 + i√3) / 2, (–1 – i√3) / 2 A – B ={–i, i} – {(–1 + i√3) / 2, (–1 – i√ Business Mathematics Note 101 = {–i, i}. Answer A ? B = {–i, i} ? {(–1 + i?3) / 2, (–1 – i?3) / = {–i, i, (–1 + i?3) / 2, (–1 - i?3) / 2 }. Answer Example If A = {x: x ? 5, x ?}, B = {x: x ? 5, x ?}, C = {x: 2 > x > 6,

x ? } ,
Then (i) Find the value of
A – (B ? C). (ii) Show that
A – (B ? C) = (A – B) ? (A – C). Solution A = {x: x ? 5, x ? } = {1, 2, 3, 4, 5} B = {x: x ? 10, x ? } = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} C = {x: 2 ? x, x ? } = {3,4,5} Therefore, (i) (

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B ? C) = {3, 4, 5} A – (B ? C) = {1, 2} (ii) A – B = ?. and A – C = {1, 2}. (A – B) ? {A – C} = {1, 2} = A – (B ? C). (A – B) ? {A – C} = {1, 2} = A – (B ? C). Hence A – (B ? C) = (A – B) ? (A – C).

Example Sets A and B
have 3 and 6 elements respectively. What

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is the least number of elements in A ? B? Solution♥ The number of elements in the set A ? B

is least, when A ? B. Then all the 3 elements of A are in B. Since B has 6 elements. Therefore the least number of elements is 6. It is clear from the fact that A ? B ? A ? B ? B ? n (A ? B) = n (B) = 6. Answer Example In a group of athletic teams in a certain institute, 21 are in the basket ball team, 26 in the hockey team, 29 in the foot ball team. If 14 play hockey and basketball, 12 play foot ball and basket ball, 15 play hockey and foot ball, 8 play all the three games.
Note Unit 3: Set Theory 102 (i) How many players are there in all? (ii) How many play only foot ball? Solution Given: No. of basket ball players B = 21, No. of hockey players H = 26, No. of foot ball players F = 29, No. of players playing hockey & basket ball both H ? B = 14 No. of players playing foot ball & basket ball both F ? B = 12 No. of players playing hockey & foot ball both F ? H = 15. Therefore (i) The no. of total players = n (B ? H ? F) = n (B) + n(H) + n(F) – n(B ? H) – n(H ? F) – n(B ? F) + n(F ? B ? H) = 21 + 26 + 29 – 14 – 12 – 15 + 8 = 43 Answer (ii) No of players playing football, basket ball but not hockey = 12 – 8 = 4. No of players playing foot ball, hockey but not basket ball = 15-8=7 No of players playing football only = No. playing foot ball – (no. Playing football and hockey + No. playing foot ball, basket ball & hockey) = 29 – (7 + 4 + 8) = 10. Example If P and Q are two sets such that P ᴜ Q has 40 elements, P has 22 elements and Q has 28 elements, how many elements does P ∩ Q have? Solution: Given n(P ᴜ Q) = 40, n(P) = 18, n(Q) = 22 We know that n(P U Q) = n(P) + n(Q) - n(P ∩ Q) So, 40 = 22 + 28 - n(P ∩ Q) 40 = 50 - n(P ∩ Q) Therefore, n(P ∩ Q) = 50 – 40 = 10
Business Mathematics Note 103 Example In a class of 40 students, 15 like to play cricket and football and 20 like to play cricket. How many like to play football only but not cricket? Solution: Let C = Students who like cricket F = Students who like football C ∩ F = Students who like cricket and football both C - F = Students who like cricket only F - C = Students who like football only.

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n(C) = 20 n(C ∩ F) = 15 n (C U F) = 40 n (F) = ? n(C ᴜ F) = n(C) + n(F) - n(C ∩ F) 40 = 20 + n(

F) – 15 40 = 5 + n(F) 40 – 5 = n(F) Therefore, n(F)= 35 Therefore, n(F - C) = n(F) - n (C ∩ F) = 35 – 15 = 20 Therefore, Number of students who like football only but not cricket = 20 More problems on cardinal properties of sets Example There is a group of 80 persons who can drive scooter or car or both. Out of these, 35 can drive scooter and 60 can drive car. Find how many can drive both scooter and car? How many can drive scooter only? How many can drive car only? Solution: Let S = {Persons who drive scooter} C = {Persons who drive car} Given,

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n(S ∪ C) = 80 n(S) = 35 n(C) = 60 Therefore, n(S ∪ C) = n(S) + n(C) - n(S ∩ C) 80 = 35 + 60 - n(S ∩ C) 80 = 95 - n(S ∩ C) Therefore, n(S∩C) = 95 – 80 = 15

Therefore, 15 persons drive both scooter and car. Therefore, the number of persons who drive a scooter only = n(S) - n(S ∩ C)
Note Unit 3: Set Theory 104 = 35 – 15 = 20 Also, the number of persons who drive car only = n(C) - n(S ∩ C) = 60 – 15 = 45 Example It was found that out of 45 girls, 10 joined singing but not dancing and 24 joined singing. How many joined dancing but not singing? How many joined both? Solution: Let S = {Girls who joined singing} D = {Girls who joined dancing} Number of girls who joined dancing but not singing = Total number of girls - Number of girls who joined singing 45 – 24 = 21 Now,

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n(S - D) = 10 n(S) =24 Therefore, n(S - D) = n(S) - n(S ∩ D) ⇒ n(S ∩ D) = n(S) - n(S -

D) = 24 - 10 = 14 Therefore, number of girls who joined both singing and dancing is 14. Self Assessment Fill in the blanks: 32. The complement of the union of two sets is equal to the intersection of their complements and the complement of the intersection of two sets is equal to the union of their complements. These are called .... 33. The .... set is the subset of every set. 34. ....set is the subset of itself. 35.
If A ? B and B ? A. then .... 36. If A ? B and B ? C. then .... 3.15 Summary

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if A and B are two non – empty intersecting sets, then ? n (A ? B) = n (A) + n (B) – n(A ? B) ? n (A ? B) = n (A – B) + n (A ? B) + n (B – A) ? n (A – B) + n (A ? B) = n (A)

Business Mathematics Note 105 z n (B – A) + n (A ∩ B) =
n (B). 3.16
Keywords Set: A collection of well-defined objects is called a set Elements: The 'objects' are called elements. Tabular form (or Roster method): The tabular form is also called the listing method. If all the elements of a set are kept within {} and elements are separated from one another by comma (,), then this form of the set is called tabular form. Set builder form (rule method): Set builder form is also called the rule method. In this form, we specify the defining property of the elements of the sets, e.g., if A is the set of all prime numbers, we use a letter, usually x, to represent the elements and we write

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Singleton set: A set containing only one element is called a singleton set.

E.g. The sets {0}, {x}, {φ} all consist of only one element. Empty set: The set having no element is called the empty set or the null set or the void set. It is denoted by φ or {}. Subset: A set
A is called a subset of
B

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if every element of the set A is also an element of the set B.

We
write it as A ⊆ B, Proper subset:
If A

51%	MATCHING BLOCK 100/248	W	

is a subset of B and if there is at least one element in B which is not in A, then A is called the proper subset of

B.

96%	MATCHING BLOCK 103/248	SA	UNIT -1 -ECE18R316 -PTAND RP.ppt (D108941599)

Equal sets: Two sets are said to be equal if they contain the same elements,

i.e.,

92%	MATCHING BLOCK 102/248	W	

if every element of A is an element of B, and every element of B is also an element of A.

It is denoted by A = B.
Equivalent sets: Two sets are called equivalent sets, if and only if there is one to one correspondence between their elements. Finite set: A set is said to be finite set if in counting its different elements, the counting process comes to an end. Thus

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a set with finite number of elements is a finite set.

Infinite set: A set, which is neither a null set nor a finite set is called an infinite set. The counting process can never come to an end in counting the elements of this set. Set of sets: If the elements of a set are sets,

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then the set is called a set of sets. Power set: The set of all subsets of a given set A, is called the power set A. The power set A is denoted by P (A). Universal set: A set, which contains

all sets under consideration as subsets, is called the universal set. It is denoted by U. Index set and Indexed set: Let A r be non – empty set for each r in a set Δ. In this case the sets A 1 , A 2 , A 3 ,... A n , are called indexed sets and the set Δ = {1,2,3...n} is called index set. Venn diagrams: Pictorial representations of sets represented by closed figures are called set diagrams or Venn diagrams

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Union of sets: The union of two sets A and B,

written as B A∪ , is another set consisting of elements that belong to A or B (i.e., to A or B or both).
Note Unit 3: Set Theory 106

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Intersection of two sets : The intersection of two sets A and B, written as B A? , is the set of all those elements that belong to A and B (

i.e., to both).

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Complement of a set : The complement of a set is defined as another set consisting of all elements of the universal set which are not

elements of the original

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set. Difference of two sets : The difference of two sets A and B denoted by A – B (read as A minus B), is the set of all elements of A which are not in B.

De Morgan's laws: The complement of the union of two sets is equal to the intersection of their complements and the complement of the intersection of two sets is equal to the union of their complements. These are called De Morgan's laws. 3.17 Review Questions 1. If
A = {1, 2, 3, 4}, B = {2, 3, 4, 5} & C = {4, 5, 6, 7}, find A – (B – C). 2. If A = {1, 3, 6, 10, 15, 21}, & B = {15, 3, 6}, find (A – B) ? (B – A). 3. If X = {1, 2, 3, 4, 5} & Y = {1, 3, 5, 7, 9}, find the values of X ? Y and (X – Y) ? (Y – X). 4. If A = {1, 2, 3, 4, 5} & B = {1, 3, 5, 7, 9}, find the symmetric difference of

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A &B. 5. If A = {a, b, c, d}, & B = {e, f, c, d}, find A ? B. 6. If A = A ? B, show B = A ? B. 7. If A & B are two sets, find the value of A ? (A ? B). 8. If A, B are subsets of a set S 1 and A', B' are the complements of A& B respectively. Prove that A ? B ? B' ? A'. 9. Prove that for any two sets A & B, (A – B) ? (B – A) = (A ? B) – (A ?

B). 10.
The number of students and the subjects offered by them are shown by a rectangle and three circles, in Venn diagram. M represents Mathematics, P? Physics and C ??Chemistry. Given total no. = 175 n(M) = 100, n(P) = 70, n(C) = 46, n(MP) = 30, n(MC) = 28, n(PC) = 23 n (MPC) = 18 Find i). n MPC ii). n PMC iii). n CMP iv). n MPC 11. Is the set A = {x: x + 5 = 5} null? Business Mathematics Note 107 12. Write down all the subsets of the set {1, 2, 3}. 13. How many subsets of the letters of the word ALLAHABAD will be formed? 14. Are the following sets equal? (i) A = {x: x is a letter in the word WOLF}. (ii) B = {x: x is a letter in the word FOLLOW}. 15. If A ? B , B ? C and C ? A. show that B = A. Answers: Self Assessment 1. demanded 2. different 3. distinct 4. elements. 5. member 6. description 7. True 8. True 9. False 10. False 11. Singleton 12. Void 13. Subset 14. equivalent 15. power 16. True 17. False 18. False 19. True 20. union 21. intersection 22. complement 23. difference 24. True 25. True 26. True 27. True 28. True 29. True 30. True 31. False 32. De Morgan's laws. 33. Empty 34. Every 35. A = B 36. A ? C. 3.18 Further Readings Books Kenneth Rosen, Discrete Mathematics and Its Applications, 7/e, 2012, McGraw-Hill. Bathul, Shahnaz, Mathematical Foundations of Computer Science, PHI Learning. Lambourne, Robert, Basic Mathematics for the Physical Sciences,2008, John Wiley & Sons. Bari, Ruth A.; Frank Harary. Graphs and Combinatorics, Springer.
Note Unit 3: Set Theory 108 F. Ernest Jerome, Connect for Jerome, Business Mathematics in Canada, 7e, Canadian Edition. S Rajagopalan and R Sattanathan, Business Mathematics, 2 edition, 2009, Tata McGraw Hill Education. Garrett H.E. (1956), Elementary Statistics, Longmans, Green & Co., New York. Guilford J.P. (1965), Fundamental Statistics in Psychology and Education, Mc Graw Hill Book Company, New York. Hannagan T.J. (1982), Mastering Statistics, The Macmillan Press Ltd., Surrey. Lindgren B.W (1975), Basic Ideas of Statistics, Macmillan Publishing Co. Inc., New York. Selvaraj R., Loganathan C., Quantitative Methods in Management. Sharma J.K., Business Statistics, Pearson Education Asia Walker H.M. and J. Lev, (1965), Elementary Statistical Methods, Oxford & IBH Publishing Co., Calcutta. Wine R.L. (1976), Beginning Statistics, Winthrop Publishers Inc., Massachusetts. Online links en.wikipedia.org/wiki/Set_(mathematics) ltcconline.net/greenl/courses/102/.../sets_and_set_operations.htm www.cs.odu.edu/~toida/nerzic/level-a/set/set_operations.html

Business Mathematics Note 109 Unit 4: Logarithm CONTENTS Objectives Introduction 4.1 Definition of Logarithm 4.2 Laws of Logarithm 4.3 Common Logarithm and Natural Logarithm 4.5 Antilogarithm 4.6 Summary 4.7 Keywords 4.8 Review Questions 4.9 Further Readings Objectives After studying this unit, you will be able to: 1. Define Logarithm 2. Discuss laws of logarithm 3. Relate common logarithm and natural logarithm 4. Explain Antilogarithm with suitable examples Introduction In mathematics logarithms were developed for making complicated calculations simple. For example, if a right circular cylinder has radius r = 0.375 meters and height h = 0.2321 meters, then its volume is given by: V = A = πr 2 h = 3.146 × (0.375) 2 × 0.2321. Use for logarithm tables makes such calculations quite easy. However, even calculators have functions like multiplication; power etc.

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still, logarithmic and exponential equations and functions are very common in mathematics.

In this unit, we will discuss .We will also focus on logarithm and laws of logarithm. Further, we will focus on common logarithm and natural logarithm, Antilogarithm with suitable examples. 4.1 Definition of Logarithm f a x = M (M < 0, a < 0, a ≠ 1), then x (i.e., index of the power) is called the logarithm of the number M to the base a and is written as x = log a M. Hence, if a x = M then x = log a M; conversely, if x = log a M then a x = M.
Note Unit 4: Logarithm 110 If 'a' is

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a positive real number (except 1), n is any real number and a n = b, then n is called

the

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logarithm of b to the base a. It is written as log

a b (read as log of b to the base a). Thus, a n = b ⇔ log a b = n. Did u know? a n is called the exponential form and log a b = n

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is called the logarithmic form. For example: ? 3 2 = 9 ⇔ log 3 9 = 2 ? 5 4 = 625 ⇔ log 5 625 = 4 ? 7 0 = 1 ⇔ log 7 1 = 0 ? 2 -3 = 1 / 8 ⇔ log 2 (1 / 8) = -3 ? 10 -2 = 0.01 ⇔ log 10 0.01 = -2 ? 2 6 = 64 ⇔

log 2 64 = 6 ? 3 - 4 = 1/3 4 = 1/81 ⇔ log 3 1/81 = -4 ? 10 -2 = 1/100 = 0.01 ⇔ log 10 0.01 = -2 Note Logarithmic functions are important largely because of their relationship to exponential functions. Logarithms can be used to solve exponential equations and to explore the properties of exponential functions. They will also become extremely valuable in calculus, where they will be used to calculate the slope of certain functions and the area bounded by certain curves. In addition, they have practical applications in economics too. Basic Logarithm Facts 1. Since a < 0 (a ≠ 1), a n < 0 for any rational n. Hence logarithm is defined only positive real numbers. From the definition it is clear that the logarithm of a number has no meaning if the base is not mentioned. 2. The above examples shows that the logarithm of a (positive) real number may be negative, zero or positive. 3. Logarithmic values of a given number are different for different bases. 4. Logarithms to the base a 10 are called common logarithms. Also, logarithm tables assume base 10.

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If no base is given, the base is assumed to be 10. For example: log 21 means log 10 21. 5.

Logarithm to the base 'e' (where e = 2.7183 approx.) is called natural logarithm, and is usually written as ln. Thus ln x means log e x. 6. If a x = - M (a < 0, M < 0), then the value of x will be imaginary i.e., logarithmic value of a negative number is imaginary.

Business Mathematics Note 111 7. Logarithm of 1 to any finite non-zero base is zero. Proof: We know, a 0 = 1 (a ≠ 0). Therefore, from the definition, we have, log a 1 = 0. 8. Logarithm of a positive number to the same base is always 1. Proof: Since a 1 = a. Therefore, log a a = 1. From 7 and 8 we say that, log a 1 = 0 and log a a = 1 for any positive real 'a' except 1. 9. If

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x = log a M then a log a M = a Proof: x = log a M. Therefore, a x = M or, a log a M = M [Since, x = log a

M]. Did U Know? "Logarithm" is a word made up by Scottish mathematician John Napier (1550- 1617), from the Greek word logos meaning "proportion, ratio or word" andarithmos meaning "number", ... which together makes "ratio-number" ! Logarithmic Functions The logarithm of a positive number is defined as a power to which a base (< 1) must be raised to get that number. Logarithmic functions are inverse to exponential functions. The inverse of an exponential function x b y ? can be written as x y b ? log , where log is an abbreviation of logarithm. We note that domain of this function is (0, ?) and the range is (??, ?). Since an inverse function is obtained by mere algebraic manipulation, its geometrical properties are similar to the properties of an exponential function. Did u know? Like an exponential function, a logarithmic function is also monotonic. Following the convention of keeping x as independent variable and y as dependent variable, we can write a logarithmic function as y x a ? log , where a < 1 is a constant. The graph of this function and that of x a y ? are symmetric with respect to the line y = x. Thus their graph will be mirror image of each other with reference to the line y = x as shown in Fig. 15 Figure 4.1: Logarithmic Functions Important features of a logarithmic function are: (i) The domain of the function is and the range is .
Note Unit 4: Logarithm 112 (ii) It is a monotonically increasing function. (iii) Since the curve passes through the point , it implies that logarithm of unity is (iv) always zero. (v) The logarithm of a number lying between 0 and 1 is negative. (vi) The logarithm of a negative number is not defined. Task Discuss in group, the applicability of logarithm in Coach and Player game. Exponentials and Logarithms Here, we will mainly discuss how to change the logarithm expression to Exponential expression and conversely from Exponential expression to logarithm expression. To discus about convert Exponentials and Logarithms we need to first recall about logarithm and exponents. The logarithm of any number to a given base is the index of

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the power to which the base must be raised in order to

equal the given number. Thus, if a x = N, x is called the logarithm of N to the base a. For example: 1. Since 3 4 = 81, the logarithm of 81 to base 3 is 4. 2. Since 10 1 = 10, 10 2 = 100, 10 3 = 1000, ... The natural number 1, 2, 3, ... are respectively the logarithms of 10, 100, 1000, ... to base 10. The logarithm of N to base a is usually written as log a N, so that the same meaning is expressed by the two equations a x = N; x = log a N Examples on convert Exponentials and Logarithms

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Convert the following exponential form to logarithmic form: (i) 10 4 = 10000 Solution: 10 4 = 10000 ⇒ log 10 10000 = 4 (ii) 3 -5 = x Solution: 3 -5 = x ⇒ log 3 x = -5 (iii) (0.3) 3 = 0.027 Business Mathematics Note 113 Solution: (0.3) 3 = 0.027 ⇒ log 0.3 0.027 = 3 Convert the following logarithmic form to exponential form: (i) log 3 81 = 4 Solution: log 3 81 = 4 ⇒ 3 4 = 81,

which is the required

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exponential form. (ii) log 8 32 = 5/3 Solution: log 8 32 = 5/3 ⇒ 8 5/3 = 32 (iii) log 10 0.1 = -1 Solution: log 10 0.1 = -1 ⇒ 10 -1 = 0.1. By converting to exponential form, find the values of following: (i) log 2 16 Solution: Let log 2 16 = x ⇒ 2 x = 16 ⇒ 2 x = 2 4 ⇒ x = 4, Therefore, log 2 16 = 4. (ii) log 3 (1/3) Solution: Let log 3 (1/3) = x ⇒ 3 x = 1/3 ⇒ 3 x = 3 -1 ⇒ x = -1, Therefore, log 3 (1/3) = -1. (iii) log 5 0.008 Note Unit 4: Logarithm 114 Solution: Let log 5 0.008 = x ⇒ 5 x = 0.008 ⇒ 5 x = 1/125 ⇒ 5 x = 5 -3 ⇒ x = -3, Therefore, log 5 0.008 = -3. Solve the following for x: (i) log x 243 = -5 Solution: log x 243 = -5 ⇒ x -5 = 243 ⇒ x -5 = 3 5 ⇒ x -5 = (1/3) -5 ⇒ x = 1/3. (ii) log √5 x = 4 Solution: log √5 x = 4 ⇒ x = (√5) 4 ⇒ x = (5 1/2) 4 ⇒ x = 5 2 ⇒ x = 25. (iii) log √x 8 = 6

Solution: log √x 8 = 6 ⇒ (√
x) 6 = 8 ⇒ (x 1/2) 6 = 2 3 ⇒ x 3 = 2 3 ⇒ x = 2.
Logarithmic Form Vs. Exponential Form The logarithm function with base a has domain all positive real numbers and is defined by log a M = x ⇔ M = a x
Business Mathematics Note 115 where M < 0, a < 0, a ≠ 1 Logarithmic Form Exponential Form log a M = x ⇔ M = a x Log 7 49 = 2 ⇔ 7 2 = 49 Write the exponential equation in logarithmic form. Exponential Form Logarithmic Form M = a x ⇔ log a M = x 2 4 = 16 ⇔ log 2 16 = 4 10 -2 = 0.01 ⇔ log 10 0.01 = -2 8 1/3 = 2 ⇔ log 8 2 = 1/3 6 -1 = 1/6 ⇔ log 6 1/6 = -1 Write the logarithmic equation in exponential form. Logarithmic Form Exponential Form log a M = x ⇔ M = a

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x log 2 64 = 6 ⇔ 2 6 = 64 log 4 32 = 5/2 ⇔ 4 5/2 = 32 log 1/8 2 = -1/3 ⇔ (1/8) -1/3 = 2 log 3 81 = x ⇔ 3 x = 81 log 5 x = -2 ⇔ 5 -2 = x log x = 3 ⇔ 10 3 = x Solve for x: 1. log 5 x = 2 x = 5 2 = 25 2. log 81 x = ½ x = 81 1/2 ⇒ x= (9 2) 1/2 ⇒ x = 9 3. log 9 x = -1/2 x = 9 -1/2 ⇒ x = (3 2) -1/2 ⇒ x = 3 -1 ⇒ x= 1/3 Note Unit 4: Logarithm 116 4. log 7 x = 0 x = 7 0 ⇒ x = 1 4.2

Laws of
Logarithm There are following four logarithm formulas which explain laws of logrithm : Product Rule Law: log a (MN) = log a M + log a N Quotient Rule Law: log a (M/N) = log a M - log a N Power Rule Law: log a M n = n log a M Change of base Rule Law: log a M = log b M × log a b Let's observe the detailed step-by-step explanation of mathematical proof of logarithm rules or log rules. 1. Proof of Product Rule Law: log a (MN) = log a M + log a N Let log a M = x ⇒ a sup<x = M and log a N= y ⇒ a y = N Now a x · a y = MN or, a x + y = MN Therefore from definition, we have, log a (MN) = x + y = log a M + log a N [putting the values of x and y] Corollary: The law is true for more than two positive factors i.e., log a (MNP) = log a M + log a N + log a P since, log a (MNP) = 1og a (MN) + log a P = log a M+ log a N+ log a P Therefore in general, log a (MNP ....)= log a M + log a N + log a P + ... Hence, the logarithm of the product of two or more positive factors to any positive base other than 1 is equal to the sum of the logarithms of the factors to the same base. 2. Proof of Quotient Rule Law: log a (M/N) = log a M - log a N Let log a M = x ⇒ a x = M and log a N = y ⇒ a y = N Business Mathematics Note 117 Now a x /a y = M/N or, a x - y = M/N Therefore from definition we have, log a (M/N) = x - y = log a M- log a N [putting the values of x and y] Corollary: log a [(M × N × P)/R × S × T)] = log a (M × N × P) - log a (R × S × T) = log a M + log a N + log a P - (log a R + log a S + log a T) The formula of quotient rule [log a (M/N) = log a M - log a N] is stated as follows: The logarithm of the quotient of two factors to any positive base other than I is equal to the difference of the logarithms of the factors to the same base. 3. Proof of Power Rule Law: log a M n = n log a M Let log a M n = x ⇒ a x = M n and log a M = y ⇒ a y = M Now, a x = M n = (a y) n = a ny Therefore, x = ny or, log a M n = n log a M [putting the values of x and y]. 4. Proof of Change of base Rule Law: log a M = log b M × log a b Let log a M = x ⇒ a x = M, log b M = y ⇒ b y = M, and log a b = z ⇒ a z = b. Now, a x = M= b y - (a z)y = a yz Therefore x = yz or, log a M = log b M × log a b [putting the values of x, y, and z]. Corollary: (i) Putting M = a on both sides of the change of base rule formula [log a M = log b M × log a b] we get,
log

47%	MATCHING BLOCK 120/248	W

a a = log b a × log a b or, log b a × log a b = 1 [since, log a a = 1] or, log b a = 1/log

a b

i.e., the logarithm of a positive number a with respect to a positive base b (≠ 1) is equal to the reciprocal of logarithm of b with respect to the base a. (ii) From the log change of base rule formula we get, log b M = log a M/log a b i.e., the logarithm of a positive number M with respect to a positive base b (≠ 1) is equal to the quotient of the logarithm of the number M and the logarithm of the number b both with respect to any positive base a (≠ 1).
Note Unit 4: Logarithm 118 Note: (i) The logarithm formula log a M = log b M × log a b is called the formula for the change of base. (ii) If bases are not stated in the logarithms of a problem, assume same bases for all the logarithms. Summarisation of logarithm rules or log rules: If M < 0, N < 0, a < 0, b < 0 and a ≠ 1, b ≠ 1 and n is any real number, then (i) log a 1 = 0 (ii) log a a = 1 (iii) a log a M =

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M (iv) log a (MN) = log a M + log a N (v) log a (M/N) = log a M - log a N (vi) log a M n = n log a M (vii) log a M = log a M × log a b (viii) log b a × log a b = 1 (

ix) 10g b a = 1/log a b (x) log b M = 1og a M/log a b 1. Find the logarithms of: (i) 1728 to the base 2√3 Solution: Let x denote the required logarithm. Therefore, log 2√3 1728 = x or, (2√3) x = 1728 = 2 6 · 3 3 = 2 6 · (√3) 6 or, (2√3) x = (2√3) 6 Therefore, x = 6. (ii) 0.000001 to the base 0.01. Solution: Let y be the required logarithm. Therefore, log 0.01 0.000001 = y or, (0.01 y = 0.000001 = (0.01) 3 Therefore, y = 3. 2. Proof that,

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log 2 log 2 log 2 16 = 1. Business Mathematics Note 119 Solution: L. H. S. = log 2 log 2 log 2 2 4 = log 2 log 2 4 log 2 2 = log 2 log 2 2 2 [since log 2 2 = 1] = log 2 2 log 2 2 = 1 · 1 = 1.

Proved. 3. If logarithm of 5832 be 6, find the base. Solution: Let x be the required base. Therefore, log x 5832 = 6 or, x 6 = 5832 = 3 6 · 2 3 = 3 6 · (√2) 6 = (3 √2) 6 Therefore, x = 3√2 Therefore, the required base is 3√2 4. If 3 +
log 10

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x = 2 log 10 y, find x in terms of y. Solution: 3 + log 10 x = 2 log 10 y or, 3 log 10 10 + log 10 x= 1og 10 y 2 [since log 10 10 = 1] or. log 10 10 3 + log 10 x = log 10 y 2 or, log 10 (10 3 · x) = log 10 y 2 or, 10 3 x = y 2

or, x = y 2 /1000, which gives x in terms y. 5. Prove that, 7 log (10/9) + 3 log (81/80) = 2

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log (25/24) + log 2. Solution: Since,7 log (10/9) + 3 log (81/80) — 2 log (25/24) = 7(log 10 – log 9)+ 3(1og 81 - log 80)- 2(1og 25 - 1og 24) = 7[log(2 · 5) - log3 2] + 3[1og3 4 - log(5 · 2 4)] - 2[log5 2 - log(3 · 2 3)] = 7[log 2 + log 5 – 2 log 3] + 3[4 log 3 - log 5 - 4 log 2] - 2[2 log 5 – log 3 – 3 log 2] = 7 log 2+ 7 log 5 - 14 log 3 + 12 log 3 – 3 log 5 – 12 log 2 – 4 log 5 + 2 log 3 + 6 log 2 = 13 log 2 – 12 log 2 + 7 log 5 – 7 log 5 – 14 log 3 + 14 log 3 = log 2 Therefore 7 log(10/9) +3 log (81/80) = 2 log (25/24) + log 2. Proved. 6. If log 10 2 = 0.30103, log 10 3 = 0.47712 and log 10 7 = 0.84510, find the values of (i) log 10 45 Note Unit 4: Logarithm 120 (ii) log 10 105. (i) log 10 45 Solution: log 10 45= log 10 (5 × 9) = log 10 5 + log 10 9 = log 10 (10/2) + log 10 3 2 = log 10 10 - log 10 2 + 2 log 10 3 = 1 - 0.30103 + 2 × 0.47712 = 1.65321. (ii) log 10 105 Solution: log 10 105 = log 10 (7 x 5 x 3) = log 10 7 + log 10 5 + log 10 3 = log 10 7 + log 10 10/2 + log 10 3 = log 10 7 + log 10 10 -

log 10 2 + log 10 3 = 0.845
l0 + 1 - 0.30103 + 0.47712 = 2.02119. 7. Prove that,

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log b a × log c b × log d c = log d a. Solution: L. H. S. = log b a × log c b × log d C = log c a × log d c [since log b

M × log a b = log a M] = log d a. (using the same formula) Alternative Method: Let, log b a = x Since, b x = a, log c b = y Therefore, c y = b and log d c = z Therefore, d z = c. Now, a = b x = (c y) x = c xy = (d z) xy = d xyz Therefore

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log d a = xyz = log b a × log c b × log d c. (putting the value of x, y, z) 8. Show that, log 4 2 × log 2 3= log 4 5 × log 5 3. Solution: L. H. S. = log 4 2 × log 2 3 = log 4 3 Business Mathematics Note 121 = log 5 3 × log 4 5. Proved. 9. Show that, log 2 10 - log 8 125 = 1. Solution: We have, log 8 125 = log 8 5 3 = 3 log 8 5 = 3 · (1/log 5 8) = 3 · (1/log 5 2 3) = 3 · (1/3 log 5 2) = log 2 5 Therefore, L.H. S. = log 2 10 - log 8 125 = log 2 10 - log 2 5 = log 2 (10/5) = log 2 2 = 1.

Proved. 10. If log

19%	MATCHING BLOCK 127/248	W

x/(y - z) = log y/(z - x) = log z/(x – y) show that, x x y y z z = 1 Solution: Let, log x/(y - z) = log y/(z - x) = log z/(x – y) = k Therefore, log x = k(y - z) ⇔ x log x = kx(y - z) or, log x x = kx(y -

z) ... (1) Similarly, log y y = ky (z - x) ... (2) and log z z = kz(x - y) ... (3) Now, adding (1), (2) and (3) we get, log x x + log y y + log z z = k (xy - xz + yz - xy + zx - yz) or, log (x x y y z z) = k × 0 = 0 = log 1 Therefore, x x y y z z = 1 Proved. 11. If a 2 - x · b 5x = a x + 3 · b 3x show that, x log (
b/a) = (1/2) log a.
Solution:

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a 2 - x · b 5x = a x + 3 · b 3x Therefore, b 5x /b 3x = a x + 3 /a 2 - x or, b 5x - 3x = a x + 3 – 2 + x or, b 2x = a 2x + 1 or, b 2x =a 2x · a or, (b/a) 2x = a

or, log (b/a) 2
x = log a (
taking logarithm both sides) or, 2x log (b/a) =log a or, x log (b/a) = (1/2) log a Proved. 12. Show that, a log a 2 x × b log b 2 y × c log c 2 z = √xyz Solution: Let, p = a log a 2 x Now, taking logarithm to the base a of both sides we get,
Note Unit 4: Logarithm 122
log a p =

25%	MATCHING BLOCK 129/248	W

log a a log a 2 x ⇒ log a p = log a 2 x · log a a ⇒ log a p = log a 2 x [since, log a a = 1] ⇒ log a p = 1/(log x a 2) [since, log n m = 1/(log m n)] ⇒ log a p = 1/(2 log x a) ⇒ log a p = (1/2) log a x ⇒ log a p = log a x ½ ⇒ log a p = log a √x Therefore, p = √x or, a log a 2

x = √x
Similarly, b log b 2 y = √y and c log c 2 z = √z L.H.S = √x · √y · √z = √xyz Proved. 13. If y = a 1/(1 – log a x) and z = a 1/(1 – log a y) show that, x = a 1/(1 – log a z) Solution: Let, log a x = p, log a y = q and log a z = r Then, by problem, y = a 1/(1 - p) .... (1) and z = a 1/(1 - q) .... (2) Now, taking logarithm to the base a of both sides of (1) we get, log a y = log a a 1/(1 - p) or, q = 1/(1 – p), [since log a a = 1] Again, taking logarithm to the base a of both sides of (2) we get, log a z = log a a 1/(1 - q) or, r = 1/(1 – q) or, 1 - q = 1/r or, 1 - 1/(1 – p) = 1/r or, 1 - 1/r = 1/(1 – p) or, (r – 1)/r = 1/(1 – p) or, 1 - p = r/(r – 1) or, p = 1- r/(r – 1) = 1/(1 – r) or, log a x = 1/(1-log a z) or, x = a 1/(1 – log a z) Proved. 14. If x, y,z are in G. P., prove that, log a x+ log a z = 2/(log y a)[x, y, z, a < 0). Solution: By problem, x, y, z are in G. P.
Business Mathematics Note 123 Therefore, y/x = z/y or, zx = y 2 Now, taking logarithm to the base a (< 0) of both sides we get, log a zx = log a y 2 [since x, y, z < 0] or, log a

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x + log a z = 2 log a y = 2/(log y a) [since log a y × log y a = 1] Proved. 15. Solve log x 2 · log x/16 2 = log x/64 2. Solution: Let, log 2 x = a ; then, log x 2 = 1/ (log 2 x) = 1/a and log x/16 2 = 1/ [log 2 (x/16)] = 1/(log 2 x — log 2 16) = 1/(log 2 x — log 2 24) = 1/(a – 4) [since, log 2 2 = 1] Similarly, log x/64 2 = 1/[log 2 (x/64)] = 1/(log 2 x – log 2 64) = 1/(a - log 2 2 6) = 1/(

a – 6) Therefore, the given equation becomes, 1/

48%	MATCHING BLOCK 142/248	SA	CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

a · 1/(a - 4) = 1/(a – 6) or, a 2 - 4a = a - 6 or, a 2 - 5a + 6 = 0 or, a 2 - 2a - 3a + 6 = 0 or, a(a - 2) - 3(a - 2) = 0 or, (a - 2)(a - 3) = 0 Therefore, either, a - 2 = 0 i.e., a = 2 or, a - 3 = 0 i.e., a = 3 When a = 2

then, log 2 x = 2 therefore, x = 2 2 = 4 Again, when a = 3 then, log 2 x = 3 , therefore x = 2 3 = 8 Therefore the required solutions are x = 4, x = 8. Self Assessment Fill in the blanks: 1. In mathematics .... were developed for making complicated calculations simple 2. A right circular cylinder has volume, that is given by formula : V = ... 3. a n is called the .... form 4. log a b = n is called the .... form. 5. 10 -2 = ... 6. The logarithm of a positive number is defined as a power to which a base .... must be raised to get that number. Note Unit 4: Logarithm 124 7. Logarithmic functions are .... to exponential functions 8. log a (MN) = ... 9. log a (M/N) = ... 10. log a M n = ... 11. log a M = ... 4.3 Common Logarithm and Natural Logarithm In Logarithm we have already seen and discussed that the logarithmic value of a positive number depends not only on the number but also on the base; a given positive number will have different logarithmic values for different bases. In practice, however, following two types of logarithms are used: (i) Natural or Napierian logarithm (ii) Common logarithm The logarithm of a number to the base e is known as Napierian or Natural logarithm after the name of John Napier; here the number e is an incommensurable number and is equal to the infinite series: 1 + 1 / 1! + 1 / 2! + 1 / 3! + ... • Did u Know? Natural logarithms are very useful in analytical work. Natural logarithm of a number is often written without indication of the base. For example, u e log is often written as logu or lnu. When e is taken as base of an exponential function, it is termed as natural exponential function. The logarithm of a number to the base 10 is known as common logarithm. This system was first introduced by Henry Briggs. This type is used for numerical calculations. The base 10 in common logarithm is usually omitted. For example, log 10 2 is written as log 2. The rest of the part deals with the method of determining common logarithms of positive numbers. Characteristic and Mantissa: Now, consider a number (say 6.72) between 1 and 10. Clearly, 1 > 6.72 > 10 Therefore, log 1 > log 6.72 > log 10 Business Mathematics Note 125 or, 0 > log 6.72 > 1 [Since log 1 = 0 and log 10 = 1] Therefore, the logarithm of a number between 1 and 10 lies between 0 and 1. That is, log 6.72 = 0 + a positive decimal part = 0· .... We now consider a number (say 58.34) between 10 and 100. Clearly, 10 > 58.34 > 100 Therefore, log 10 > log 58.34 > log 100 or, 1 > log 58.34 > 2 [Since log 10 = 1 and log 100 = 2] Therefore, the logarithm of a number between 10 and 100 lies between 1 and 2. That is, log 58.34 = 1 + a positive decimal part = 1· ...... Similarly, the logarithm of a number (say 463) between 100 and 1000 lies between 2 and 3 (since log 100 = 2 and log 1000 = 3). That is, log 463 = 2 + a positive decimal part = 2· .... In like manner the logarithm of a number between 1000 and 10000 lies between 3 and 4 and so on. Now, consider a number (say .54) between 1 and .1. Clearly, .1 > .54 > 1 Therefore, log .1 > log .54 > log 1 or, - 1 > log .54 > 0, [Since log 1 = 0 and log .1 = - 1] Therefore, the logarithm of a number between .1 and 1 lies between - 1 and 0. That is, log .54 = -0· .... = - 1 + a positive decimal part. We now consider a number (say .0252) between .1 and ·01. Clearly, .01 > .0252 > .1 log 0.1 > log .0252 > log .1 or, -2 > log .0252 > - 1 [since log .1 = - 1 and log .01 = -2] Therefore, the logarithm of a number between .01 and .1 lies between -2 and - 1 . That is, log .0252 = - 1· ..... = - 2+ a positive decimal part. Similarly, the logarithm of a number between .001 and .01 lies between - 3 and -2 and so on.

Note Unit 4: Logarithm 126 Note The common logarithm of a positive number consists of two parts. One part is integral which may be zero or any integer (positive or negative) and the other part is non-negative decimal. The integral part of a common logarithm is called the characteristic and the non-negative decimal part is called the mantissa. Suppose, log 39.2 = 1.5933, then 1 is the characteristic and 5933 is the mantissa of the logarithm. If log .009423 = - 3 + .9742, then - 3 is the characteristic and .9742 is the mantissa of the logarithm. Since log 3 = 0.4771 and log 10 = 1, so the characteristic of log 3 is 0 and the mantissa of log 10 is 0. Determination of Characteristic and Mantissa: The characteristic of the logarithm of a number is determined by inspection and the mantissa by logarithmic table. (i) To find the characteristic of the logarithm of a number greater than 1: Since, log 1 = 0 and log 10 = 1 , hence the common logarithm of a number between 1 and 10 (i.e., whose integral part consists of one digit only) lies between 0 and 1. For example, each of the numbers 5, 8.5, 9.64 lies between 1 and 10 (see that the integral part of each of them consists of one digit only); hence their logarithms lie between 0 and 1 i.e., log 5 = 0 + a positive decimal part = 0· ...... log 8.5 = 0 + a positive decimal part = 0· ..... log 9.64 = 0 + a positive decimal part = 0· ..... Therefore, the characteristic of each of log 5, log 8.5 or log 9.64 is 0. Again, the common logarithm of a number whose integral part consists of two digits only (i.e., of a number between 10 and 100) lies between 1 and 2 (log 10 = 1 and log 100 = 2). For example, the integral part of each of the numbers 36, 86.2, 90.46 consists of two digits; hence their logarithms lie between 1 and 2, i.e., log 36 = 1 + a positive decimal part = 1· ...... log 86.2 = 1 + a positive decimal part = 1· ...... log 90.46 = 1 + a positive decimal part = 1· ...... Therefore, the characteristic of each of log 36, log 86.2 or log 90.46 is 1. Similarly, the characteristic of the logarithm of a number whose integral part consists of 3 digits is 2. In general, the characteristic of the logarithm of a number whose integral part consists of n digits is n - 1. Accordingly, we have the following rule:
Business Mathematics Note 127

53% **MATCHING BLOCK 132/248** **W**

The characteristic of the logarithm of a number greater than 1 is positive and is one less than the number of digits in the integral part of the number. Example: (ii) To find the characteristic of the logarithm of a number

lying between 0 and 1: Since, log .1 = -1 and log 1 = 0, hence the common logarithm of a number between .1 and 1 lies between -1 and 0. For example, each of .5, .62 or .976 lies between .1 and 1; hence their logarithms lie between -1 and 0, i.e., log .5 = -0· ..... = -1 + a positive decimal part = 1· ..... log .62 = -0· .... = -1 + a positive decimal part = 1· ..... log .976 = -0· ..... = - 1 + a positive decimal part = 1· ..... [See that a number between (- 1) and 0 is of the form (-0· ......), such as (-0.246), (-0.594) etc. But (- 0.246) can be expressed as follows: - 0.246 = -1 + 1 -0.246 = -1 + 0.754 = -1+ a positive decimal part. It is the convension to represent the mantissa of the logarithm of a number as positive. For this reason a number lying between (- 1) and 0 is expressed in the above form. Again, (-1) + .754 is written as 1.754. Clearly, the integral part in1.754 is negative [i.e., (- 1)] but the decimal part is positive. 1.754 is read as bar 1 point 7, 5, 4. Note that, (- 1.754) and (1.754) are not the same. 1.754 = - 1 + .754 but (-1.754) = - 1 - .754] Therefore, the characteristic of each of log .5, log .62 or log .976 is (- 1). Again, a number having one zero between the decimal sign and the first significant figure lies between .0l and .1. Hence, its logarithm will lie between (-2) and (- 1) [Since, log .01 = - 2 and log .1 = - 1]. For example, each of .04, .056, .0934 lies between .01 and .1 (see that there is one zero between the decimal sign and the first significant digit in all the numbers) hence, their logarithms will lie between (-2) and (- 1), i.e., log .04 = - 1· .... = -2 + a positive decimal part = 2· .... log .056 = -1· .... = -2 + a positive decimal part = 2· .... 1og.0934= -1· .... = -2 + a positive decimal part = 2· .... Similarly, the characteristic of the logarithm of a number having two zeroes between the decimal sign and the first significant figure is (- 3). In general, the characteristic of the logarithm of a number having n zeroes between the decimal sign and the first significant figure is - (n + 1). Accordingly, we have the following rule:
Note Unit 4: Logarithm 128

50% **MATCHING BLOCK 133/248** **W**

The characteristic of the logarithm of a positive number less than 1 is negative and is numerically greater by 1 than the number of zeroes between the decimal sign and the first significant figure of the number. Example: (iii) To find the mantissa [using log-table]: After determining the characteristic of the logarithm of a positive number by inspection, its mantissa is determined by the logarithmic table. At the end of the book both four-figure and five-figure tables are given. A four-figure table gives the value of mantissa correct to 4 decimal places. Similarly, a five-figure or a nine-figure log-table gives the value of mantissa correct to five or nine decimal places. Using any one of them we can find the mantissa f the common logarithm of a number lying between 1 to 9999, If the number contains more than 4 significant digits then to find the mantissa by the table either we can approximate it upto 4 significant figures for rough calculations or else we can utilize the principle of proportional parts for more precise calculations. In tables mantissa correct to certain places of decimals are given without the decimal point. It should be remembered that the mantissa of common logarithm of a number is independent of the position of the decimal point in the number. Caution! The decimal point of the number is discarded when the mantissa is determined by the log-table. For example, the mantissa of each of the numbers 6254, 625.4, 6.254 or, 0.006254 is the same. Observing the log-table given at the end of the book we see that it is divided into following four parts: (a) n the extreme left-hand column numbers ranging from 10 to 99; (b) umbers ranging from 0 to 9 in the top-most row; (c our-digit numbers (in a four-figure log-table) below each figure of the top-most row; (d) ean difference column. Suppose we are to find the mantissa of (i) log 6 (ii) log 0.048 (iii) log 39.2 and (iv) log 523.4 by log-table. (i) log 6

Business Mathematics Note 129 Since mantissa of log 6 and log 600 are same, we shall have to see the mantissa of log 600. Now we find the figure 60 in the column of part (a) of the table; next we

76%	MATCHING BLOCK 134/248	W

move horizontally to the right to the column headed by 0 of part (b) and read the number 7782

in part (c) of the table (see four-figure log-table). Thus the mantissa of log 6 is .7782. (ii) log 0.048 Since the mantissa of common logarithm is independent of the position of the decimal point, hence to find the mantissa of log 0.048 we shall find the mantissa of log 480. As in (i) we first-find the figure 48 in the column of part (a) of the table ; next we

76%	MATCHING BLOCK 135/248	W

move horizontally to the right to the column headed by 0 of part (b) and read the number 6812

in part (c) of the table. Thus the mantissa of log 0.048 is .6812. (iii) log 39.2 Similarly, to find the mantissa of log 39.2 we shall find the mantissa of log 392. As in (i), we find the figure 39 in the column of part (a); next we

76%	MATCHING BLOCK 136/248	W

move horizontally to the right to the column headed by 2 of part (b) and read the number 5933

in part (c) of the table. Thus the mantissa of log 39.2 is .5933 (iv) log 523.4 In like manner we first discard the decimal point in 523.4. Now we find the figure 52 in the column of part (a); next we

69%	MATCHING BLOCK 137/248	W

move horizontally to the right to the column headed by 3 of part (b) and read the number 7185 in part (c) of the table. Again we move along the same horizontal line further right to the column headed by 4 of mean difference and read the number 3 there. If this 3

be added with 7185, then we shall get the mantissa of log 523.4. Thus the mantissa of log 523.4 is .7188. Clearly, the characteristics of log 6, log 0.048, log 39.2 and log 523.4 are 0, (-2), 1 and 2 respectively. Hence, we have, log 6 = 0.7782, log 0.048 = 2.68l2, log 39.2 = 1.5933 and log 523.4 = 2.7188. Self Assessment State whether the following statements are true or false: 12. The logarithm of a number to the base e is known as Napierian or Natural logarithm 13. The number e is an incommensurable number and is equal to the infinite series 14. The logarithm of a number to the base 10 is known as common logarithm. common logarithm was first introduced by Henry Briggs. 15. log 10 2 is written as 10log 2

Note Unit 4: Logarithm 130 16. The common logarithm of a positive number consists of two parts. One part is integral which may be zero or any integer (positive or negative) and the other part is non-negative decimal. 17. The integral part of a common logarithm is called the characteristic and the non- negative decimal part is called the mantissa. 18. The characteristic of the logarithm of a number is determined by inspection and the mantissa by logarithmic table. 4.5 Antilogarithm

98%	MATCHING BLOCK 138/248	W

If log M = x, then M is called the antilogarithm of x and is written as M = antilog x. For example, if log 39.2 = 1.5933, then antilog 1.5933 = 39.2. If the logarithmic value of a number be given then the number can be determined from the antilog-table. Antilog-table is similar to log-table; only difference is in the extreme left-hand column which ranges from .00 to .99. Example Find antilog 2.5463. Solution: Clearly, we are to find the number whose logarithm is 2.5463. For this consider the mantissa .5463. Now find .54 in the extreme left-hand column of the antilog-table (see four-figure antilog-table) and then move horizontally to the right to the column headed by 6 of the top-most row and read the number 3516. Again we move along the same horizontal line further right to the column headed by 3 of mean difference and read the number 2 there. This 2 is now added to the previous number 3516 to give 3518. Since the characteristic is 2, there should be three digits in the integral part of the required number. Therefore, antilog 2.5463 = 351.8. Example If log x = -2.0258, find x. Solution: In order to find the value of x using antilog-table, the decimal part (i.e., the mantissa) must be made positive. For this we proceed as follows: log x = -2.0258 = - 3 + 3 - 2.0258 = - 3 + .9742 =3.9742 Therefore, x = antilog 3.9742. Now, from antilog table we get the number corresponding to the mantissa .9742 as (9419 + 4) = 9423. Again the characteristic in log x is (- 3).

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Hence, there should be two zeroes between the decimal point and the first significant digit in the value of x. Therefore, x = .009423.

Self Assessment Fill in the blanks: 19.

97%	MATCHING BLOCK 140/248	W

If log M = x, then M is called the .... of x and is written as M = antilog x. 20.

Antilog-table is similar to .... 21. The only difference between log and antilog tables

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is in the extreme left-hand column which ranges from 4.6

Summary ? If 'a' is

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a positive real number (except 1), n is any real number and a n = b, then n is called

the

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logarithm of b to the base a. It is written as log a b (read as log of b to the base a). ? Logarithmic values of a given number are different for different bases. ? Logarithmic functions are important largely because of their relationship to exponential functions. ? Logarithms can be used to solve exponential equations and to explore the properties of exponential functions. ? Two kinds of logarithms are often used: common logarithms and natural (or Napierian) logarithms. ? The power to which a base of 10 must be raised to obtain a number is called the common logarithm (log) of the number. ? The power to which the base e (e = 2.718281828.......) must be raised to obtain a number is called the natural logarithm (ln) of the number. ?

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If log M = x, then M is called the antilogarithm of x and is written as M = antilog x. 4.7

Keywords Common Logarithmic Function: The function f (x) = log 10 x , often written f (x) = log x . Logarithmic Function: The inverse of the exponential function f (x) = a x . y = log a x means a y = x , where a < 0 and a≠1 . Natural Logarithmic Function: The function f (x) = log e x , often written f (x) = ln x . 4.8 Review Questions 1. Find

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the logarithm of 2025 to the base 3 5 2. The logarithm of a number to the base 2 is k. What is its logarithm to the base 2 2 ? Note

Unit 4: Logarithm 132 3. Show that 4. Show that log b a

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x log c b x log a c = 1. 5. Find the value of log 2 [log 2 {log 3 (log 3 27 3)}]. 6. If log 2 x + log 4 x + log 16 x=21/4 then, find x 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. Business Mathematics Note 133 18. 19. 20 21. If log 3 = 0.4771, find the number of digits in 3 43 . 22. Given log 10 2 = 0.30103, find log 10 (1000/256) 23. Find the value of : (

i) 0.8176 × 13.64, (ii) (789.45) 1/8 24. Find, from tables, the antilogarithm of – 2.7080 25. Find the number of zeros between the decimal point and the first significant figures in: 26.

33%	MATCHING BLOCK 149/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

Given log 8 = 0.931, log 9 = 0.9542 ; find the value of log 60 correct to 4 decimal 4 places. 27. Given log 2 = 0.30103, log 3 = 0.47712 ; find the value of : (i) log 4500 (ii) log 0.015 (iii) log 0.1875. 28. Using tables find the value of : (i) 19.66 / 9.701 (ii) (

iii) (iv) 29. Find the value of log 64 with base 4 30. Find the value of log 125 with base 5 5 31. Find the value of log 144 with base 2 3 32. Show that log (1 + 2 + 3) = log1 + log2 + log3 33 . Show that log 2 log 2 log 2 16 = 1 34. Show that = 1 35. If log x + log y = log (x + y) then express x in terms of y. Answers: Self Assessment 1. logarithms 2. πr2h 3. exponential 4. logarithmic

Note Unit 4: Logarithm 134 5. -2 6. 1 7. Inverse 8. loga M + loga N 9. loga M - loga N 10. n loga M 11. logb M × loga b 12. True 13. True 14. True 15. False 16. True 17. True 18. True 19. antilogarithm 20. log-table 21. .00 to .99. 4.9 Further Readings Books Kenneth Rosen, Discrete Mathematics and Its Applications, 7/e, 2012, McGraw-Hill. Bathul, Shahnaz, Mathematical Foundations of Computer Science, PHI Learning. Lambourne, Robert, Basic Mathematics for the Physical Sciences,2008, John Wiley & Sons. Bari, Ruth A.; Frank Harary. Graphs and Combinatorics, Springer. F. Ernest Jerome, Connect for Jerome, Business Mathematics in Canada, 7e, Canadian Edition. S Rajagopalan and R Sattanathan, Business Mathematics, 2 edition, 2009, Tata McGraw Hill Education. Garrett H.E. (1956), Elementary Statistics, Longmans, Green & Co., New York. Guilford J.P. (1965), Fundamental Statistics in Psychology and Education, Mc Graw Hill Book Company, New York. Hannagan T.J. (1982), Mastering Statistics, The Macmillan Press Ltd., Surrey. Lindgren B.W (1975), Basic Ideas of Statistics, Macmillan Publishing Co. Inc., New York. Selvaraj R., Loganathan C., Quantitative Methods in Management. Sharma J.K., Business Statistics, Pearson Education Asia Walker H.M. and J. Lev, (1965), Elementary Statistical Methods, Oxford & IBH Publishing Co., Calcutta. Wine R.L. (1976), Beginning Statistics, Winthrop Publishers Inc., Massachusetts.

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Note Unit 5: Binomial Theorem 136 Unit 5: Binomial Theorem CONTENTS Objectives Introduction 5.1 Concept of Binomial Theorem 5.1.1 General Term and Equidistant term 5.1.2 Middle Term (s) of (a+x)n 5.2 Some Particular form of Expansion 5.2.1 Binomial Coefficients 5.2.2 Greatest Binomial Coefficient (s) 5.3 Properties of Binomial Coefficient (s) 5.4 Pascal's Triangle 5.7 Summary 5.8 Keywords 5.9 Review Questions 5.10 Further Readings Objectives After studying this unit, you will be able to: 1. Define binomial theorem 2. Discuss general theorem of (a+x)n and middle term (s) of (a+x)n 3. Describe binomial coefficients, equidistant terms and coefficients 4. Explain greatest binomial coefficient (s) 5. State various properties of binomial coefficient (s) Introduction We are already familiar with algebraic expressions which are very common in mathematics. An algebraic expression having two terms is called a binomial expression. One can easily find the square or cube of binomial expressions like a+b or a-b . But it will be difficult to expand (a+b)^7 by repeated multiplication. Binomial theorem gives an easier way to expand (a+b)^n, where n is an integer or a rational number. In this unit, we will consider to be always a positive integer. In mathematics, the binomial coefficient n Ck is the coefficient of the x ^k term in the polynomial expansion of the binomial power (1 + x) ^n. In this unit, We will focus on binomial theorem. Further, will study general theorem of (a+x) n and middle term (s) of (a+x)n. Finally, we will turn to binomial coefficients, equidistant terms and coefficients, greatest binomial coefficient (s) and properties of binomial coefficient (s). Business Mathematics Note 137 5.1 Concept of

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Binomial Theorem A binomial is a polynomial with two terms. Binomial

Theorem is a theorem that specifies the expansion of a binomial of the form (x + y) n as the sum of n + 1 terms of which the general term is of the form where k takes on values from 0 to n. Did u Know? The Binomial Theorem was first discovered by Sir Isaac Newton. Note

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The binomial theorem describes the algebraic expansion of powers of a binomial, hence it is referred to

as binomial expansion.

93%	MATCHING BLOCK 151/248	W

According to the theorem, it is possible to expand the power (x + y) n into a sum involving terms of the form

ax b y c ,

100%	MATCHING BLOCK 152/248	W

where the exponents b and c are nonnegative integers with b + c = n, and the coefficient a of each term is a specific positive integer depending on n and b.

When an exponent is zero, the corresponding power is usually omitted from the term. Statement

Note Unit 5: Binomial Theorem 138 Did u know? The binomial theorem shows how to calculate a power of a binomial -- (a + b) n -- without actually multiplying out. 5.1.1 General Term and Equidistant term Equidistant Terms: In the expansion of (a + x) n , the (r + 1) th term from the beginning is

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n C r a n−r x r . As the (r + 1) th term from the end is the (n – r + 1) th term from the beginning and this term is C n-r a r x n–r Did u Know? In combinatorics, nCk is interpreted as the number of k-element subsets of an n- element set, that is the number of ways that k things can be 'chosen' from a set of n things. Caution! For the case when the number n is not a positive integer the binomial theorem becomes, for –1 > x > 1,

Business Mathematics Note 139 5.1.2 Middle Term (s) of (a+x) n If the number of terms in the expansion of (a+x) n is odd then there is exactly one middle term, but the expansion will have two middle terms if the number of terms is even. Case 1: When n is even, say n=2m , then the number of terms is equal to 2m+1 which is odd. Hence the middle term is the (m+1)th term, that is,5. 5.2 Some Particular form of Expansion

Note Unit 5: Binomial Theorem 140 5.2.1 Binomial Coefficients are called binomial coefficients and in short they can be written as: The binomial coefficient

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can be interpreted as the number of ways to choose k elements from an n-element set. This is related to binomials for the following reason: if we write (x + y) n as a product

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then, according to the distributive law, there will be one term in the expansion for each choice of either x or y from each of the binomials of the product. For example, there will only be one term

x n ,

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corresponding to choosing x from each binomial. However, there will be several terms of the form

x n–2

100%	MATCHING BLOCK 157/248	W

y 2 , one for each way of choosing exactly two binomials to contribute a y. Therefore, after combining like terms, the coefficient of

x n–2

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y 2 will be equal to the number of ways to choose exactly 2 elements from an n-element set. 5.2.3

Greatest Binomial Coefficient (s)

Business Mathematics Note 141

Note Unit 5: Binomial Theorem 142

Business Mathematics Note 143 Did u Know? Where the sum involves more than two numbers, the theorem is called the Multinomial Theorem. Self Assessment Fill in the blanks: 1. A binomial is a .... with two terms 2. Binomial Theorem is a theorem that specifies the expansion of a binomial of the form (x + y)n as the sum of .... terms of which the general term is of the form where k takes on values from 0 to n. 3. The Binomial Theorem was first discovered by .... Newton. 4. The binomial theorem shows how to calculate a power of a binomial -- (a + b) n -- .... actually multiplying out.

Note Unit 5: Binomial Theorem 144 5. For the case when the number n is not a positive integer the binomial theorem becomes, for .... 6. If the number of terms in the expansion of (a+x) n is .... then there is exactly one middle term 7. The expansion will have two middle terms if the number of terms is .... 5.3 Properties of Binomial Coefficient (s) Following are some basic properties of binomial coefficients.

Business Mathematics Note 145 5.4 Pascal's triangle

Note Unit 5: Binomial Theorem 146 This triangular array is called Pascal's Triangle. Each row gives the combinatorial numbers, which are the binomial coefficients. That is, the row 1 2 1 are the combinatorial numbers 2 C k , which are the coefficients of (a + b) 2 . The next row, 1 3 3 1, are the coefficients of (a + b) 3 ; and so on. Caution! Pascal's triangle can be difficult to use if the exponent is very high. To construct the triangle, write 1, and below it write 1 1. Begin and end each successive row with 1. To construct the intervening numbers, add the two numbers immediately above. Thus to construct the third row, begin it with 1, then add the two numbers immediately above: 1 + 1. Write 2. Finish the row with 1. To construct the next row, begin it with 1, and add the two numbers immediately above: 1 + 2. Write 3. Again, add the two numbers immediately above: 2 + 1 = 3. Finish the row with 1. The most basic example of the binomial theorem is the formula for the square of x + y: The binomial coefficients 1, 2, 1 appearing in this expansion correspond to the third row of Pascal's triangle. The coefficients of higher powers of x + y correspond to later rows of the triangle: Notice that 1. the powers of x go down until it reaches 0 ( ), starting value is n (the n in .) 2. the powers of y go up from 0 ( ) until it reaches n (also the n in .) 3. the nth row of the Pascal's Triangle will be the coefficients of the expanded binomial. (Note that the top is row 0.) 4. for each line, the number of products (i.e. the sum of the coefficients) is equal to . 5. for each line, the number of product groups is equal to .

Business Mathematics Note 147 Did u know? The binomial theorem can be applied to the powers of any binomial. For example, Note For a binomial involving subtraction, the theorem can be applied as long as the opposite of the second term is used. This has the effect of changing the sign of every other term in the expansion: Another useful example is that of the expansion of the following square roots: Example Example

Note Unit 5: Binomial Theorem 148 Example: Example: Example:

Business Mathematics Note 149 Example Example Example

Note Unit 5: Binomial Theorem 150 Example Example Expand (a – b) 5 . Solution. We found the binomial coefficients to be 1 5 10 10 5 1. The difference with (a – b) is that the signs of the terms will alternate: (

78%	MATCHING BLOCK 159/248	W

a – b) 5 = a 5 – 5a 4 b + 10a 3 b 2 – 10a 2 b 3 + 5ab 4 – b 5 . For, a – b = a + (–b),

therefore each term will have the form a 5 – k (–b) k . When k is even, (–b) k will be positive. But when k is odd, (–b) k will be negative. Each odd power of b will have a minus sign. Example In the expansion of (x – y) 15 , calculate the coefficients ofx 3 y 12 and x 2 y 13 . Solution: The coefficient of x 3 y 12 is positive because the exponent of y is even. That coefficient is 15 C 12 . But 15 C 12 = 15 C 3 , and so we have 15· 14· 13 1· 2· 3 = 455. The coefficient of x 2 y 13 , on the other hand, is negative because the exponent of y is odd. The coefficient is – 15 C 13 = – 15 C 2 . We have – 15· 14 1· 2 = –105. Example Write the first four terms of (a + b) n . Do not use factorials.

Business Mathematics Note 151 Solution. (

72%	MATCHING BLOCK 163/248	SA	unit 2.docx (D32968061)

a + b) n = n C 0 a n + n C 1 a n – 1 b + n C 2 a n – 2 b 2 + n C 3 a n – 3 b 3 = a n + na n – 1 b + a n – 2 b 2 + a n – 3 b 3 .

Notice: Each coefficient is a factor of the next coefficient. The coefficient n is a factor of . That in turn is a factor of . To construct the next coefficient, then, multiply the present coefficient by the exponent of a in that term -- a n – 3 b 3 -- namely n – 3: n(n – 1)(n – 2)(n – 3) 1· 2· 3· 4 And divide it by 1 more than the exponent of b. That is the coefficient of a n – 4 b 4 . Example Use the binomial theorem to expand (a + b) 8 . Solution The expansion will begin : (a + b) 8 = a 8 + 8a 7 b The first coefficient is always 1. The second is always the exponent of the expansion, in this case 8. The next coefficient can be constructed as described above. It will be the present coefficient, 8 -- (a + b) 8 = a 8 + 8a 7 b + 28a 6 b 2 -- times the exponent of a in that term , 7, divided by 1 more then the exponent of b. It will be 8· 7/2 = 28. The next coefficient -- (a + b) 8 =

31%	MATCHING BLOCK 160/248	W

a 8 + 8a 7 b + 28a 6 b 2 + 56a 5 b 3 -- is 28· 6, divided by 3: Note Unit 5: Binomial Theorem 152 28· 6 / 3 = 28· 2 = 56. The next -- (a + b) 8 = a 8 + 8a 7 b + 28a 6 b 2 + 56a 5 b 3 + 70a 4 b 4 -- is 56· 5, divided by 4: 56· 5 / 4 = 14· 5 = 70. We have now come to the point of symmetry. For, the coefficient ofa 3 b 5 is equal to the coefficient of a 5 b 3 , which is 56. And so on for the remaining coefficients. Here is the complete expansion: (

85%	MATCHING BLOCK 161/248	W

a + b) 8 = a 8 + 8a 7 b + 28a 6 b 2 + 56a 5 b 3 + 70a 4 b 4 + 56a 3 b 5 + 28a 2 b 6 + 8ab 7 + b 8 .

Example Write the 5th term in the expansion of (a + b) 10 . Solution. In the 1st term, k = 0. In the 2nd term, k = 1. And so on. The index k -- the exponent of b -- of each term is one less than the ordinal number of the term. Thus in the 5th term, k = 4. The exponent of b is 4. The 5th term is 10 C 4 a 6 b 4 = 10· 9· 8· 7 1· 2· 3· 4 a 6 b 4 = 210 a 6 b 4 (1) Show that the middle – term in the expansion of (1+x) 2n is 1. 3. 5 ------- (2n-1) / (n!) . 2 n x n , 'n' being a positive integer. Sol.: The no. of terms in (1+x) 2n = 2n+1 (odd). It's ,middle-term = (2
n + 1) / 2 = (n+1)th term. ? T
n+1 = 2
n C
n
x n = 2n! / (n!
x n!). x
n = 2n (2n-1) ------ 4.3.2.1 / (n! x

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n!). x n = [{(2n-1) (2n-3) ----- 3.1.} { 2n (2n-2) ------ 4.2.}] / (n! x n!). x n = [{1. 3. 5. ---- (2n-1)} 2 n {1.2 ---- n}] / (n! x n!) . x n = [{1.3.5----(2n-1)}. 2 n] / (n! x n!). x n = T n+1 = 1. 3. 5 – (2n-1) / (n!). 2 n x n Example Find the term independent of 'x' in the expansion of (i) (1+x+2x 3) [(3 x 2 / 2) – (1/3x)] 9 (ii) [(x 1/3 / 2) + x -1/5] 8 Business Mathematics Note 153 Sol.: (i) (1+x+2x 3) [(3/2)x 2 - (1/3x)] 9 = (1+x+2x 3) { [(3/2)x 2] 9 - 9 C 1 [(3/2)x 2] 8 1/3x + ---------- + + 9 C 6 [(3/2)x 2] 3 (1/3x) 6 - 9 C 7 [(3/2)x 2] 2 (1/3x) 7 ---} = (1+x+2x 3) { [(3/2)x 2] 9 – 9 C 1 (3 7 / 2 8)x 15 + ---- + 9 C 6 (1 x 1 / 2 3 x 3 3) – 9 C 7 1/ (2 2 x 3 5) 1/ x 3 + ----} Term independent of 'x' : 9 C 6 x 1/ (2 3 x 3 3) - 9 C 7 2 / (2 2 x 3 5) = 9! / (6! x 3!) . 1/ (8 x 27) - 9!/(7! x 2!) . 1/ (2 x 243) = (9. 8. 7. 6!) / (6!. 3. 2. 1.) x 1/(8. 27) - (9. 8. 7!) / (7! . 2). 1/ (2.243) = 7 / 18 - 2 / 27 = 17 / 54 (ii) [(1 / 2) x 1/3 + x -1/5] 8 Sol.: General Term T r+1 = n C r [(1/2) x 1/3] n-r . (x -1/5) r n-r -r = n C r [(1/2) n-r] x 3 x 5 Here n = 8 = 8 C r (1/2) 8-r x (8-r)/3 -r/5 40 -8r T r+1 = 8

C r (1/2) 8-r
x 15 ---------------< (i) Putting (40 – 8r) / 15 = 0, we have r = 5 From (i), Term independent of 'x' : T 6 = 8 C 5 (1/2) 8-5 = 8! / (5! X 3!) . 1 / 2 3 = (8. 7. 6. !5) / (5!. 3.2.1) . 1 / 8 = T 6 = 7 Example

66%	MATCHING BLOCK 164/248	SA	unit 2.docx (D32968061)

Find the coefficient of 'x' in the expansion of (1-2x 3 + x 5) [1 + (1/x)] 8 Sol.: (1-2x 3 + 3x 5) [1 + (1/x)] 8 = (1-2x 3 + 3x 5) [1 + 8 C 1 (1/x) + 8 C 2 (1/ x 2) + 8 C 3 (1/ x 3) + 8 C 4 (1/ x 4)+ 8 C 5 (1/ x 5)+ --- + 8 C 8 (1/ x 8) coefficient of x = -2. 8

C 2 + 3 8 C 4 = -2. 8! / (2! x 6!) + 3. 8! / (4! x 4!) = -2. (8. 7) / 2 + 3 (8. 7. 6. 5.) / (4.3.2.1) = -56 + 210 = 154
Note Unit 5: Binomial Theorem 154 Example Prove that the ration of the coefficient of x 10 in (1-x 2) 10 and the term independent of 'x' in [x – (2/x)] 10 is 1 : 32. Sol.: In (1-x) 2 : T r+1 = 10 C r (-1) r (x 2) r Putting r = 5 T 6 = - 10 C 5 x 10 ? Coefficient of x 10 = - 10 C 5 In [x – (2/x)] : T r+1 = 10 C r (-1) r (x) 10-r (2/x) r = (-1) r 10 C r . 2 r .
x 10-2r Putting 10 – 2r = 0 ? r = 5 So term independent of x : T 6 = (-1) 5 10 C 5 . 2 5 Hence their ratio = (- 10 C 5) : (-32. 10 C 5) = 1 : 32 Example If third term in the expansion of (

24%	MATCHING BLOCK 168/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

x + x logx) 5 is 10,00,000. Find the value of 'x'. Sol.: Putting log 10 x = z in the given expression : We have : (x + x z) 5 T 3 = T 2+1 = 5 C 2 (x) 5-2 (x z) 2 = 5 C 2 x 3 . x 2z = 5! / (2! x 3!) x 2z+3 = (5 x 4) / 2! x 2z+3 = T 3 = 10x 2z+3 ? 10,00,000 = 10. x 2

z+3 Or x 2z+3 = 10 5 ? (10 z) 2z+3 = 10 5 or 10 2z2+3z = 10 5 ? 2z 2 + 3z = 5 [Log 10 x = z] or 2z 2 + 3z – 5 = 0 or (z-1) (2z+5) = 0 ? z = 1, - 5 / 2
Business Mathematics Note 155 or log 10 x = 1 or log 10 x = - 5 / 2 since x = 10 or 10 -5/2 Example: If in the expansion of (1+x) m (1-x) n , the coefficients of 'x' and 'x 2 ' are '3' and '-6' res. Find the value of 'm'. Sol.: (1+x) m (1-x) n = [m C 0 + m C 1 x + m C 2 x 2 + ---- + m C m

25%	MATCHING BLOCK 165/248	W

x m] [n C o – n C 1 x + n C 2 x 2 + ------- + (-1) n n C n x n] Coefficient of x = m C 1 x n C o – m C 0 . n C 1 = m! / (1! x m-1!) x 1 – 1 x n! / (1! x n-1!) = m – n = 3 ---------------< (i) Coefficient of x 2 = - m C 1 x n C 1 + n C 0 x m C 2 + m C 0 x n C 2 = - m!/ (1! x m-1!) x n! / (1! x n-1!)+ 1 x m! / (2! x m-2!) + 1 x n! / (2! x

n-2!) = -mn + m (m-1) / 2 + n(n-1) / 2 = -6
or – 2mn + m(m-1) + n(n-1) = -12 or -2mn + m 2 – m + n 2 – n = 12 or (m-n) 2 – (m+n) = -12 From (i), putting the value of (m-n) : - 9 + (m + n) = 12 or m + n = 21 -----------< (ii) eg n (i) + eg n (ii) = 2m = 24 m = 12 Example If the coefficients of (2r + 1)th term and (

23%	MATCHING BLOCK 166/248	W

r + 2)th term in the expansion of (1 + x) 43 are equal, find 'r'. Sol.: In (1 + x) 43 : T 2r+1 = 43 C 2r . x 2r Coefficient = 43 C 2r And T r+2 = 43 C r+1 x r+1 Coefficient = 43 C r According to the questions: 43 C 2r = 43 C r+1 2r + r + 1 = 43

Note Unit 5: Binomial Theorem 156 or 3r = 42 r = 14 Example If the coefficient of '4'th and '13'th terms in the expansion of [x 2 + (1/x)] n be equal, then find the term which independent of 'x'. Sol.: T 4 = T 3+1 = n C 3 (x 2) n-3 . 1/ x 3 Coefficient = n C 3 T 13 = T 12+1 = n C 12 (x 2) n-12 1 / x 12 Coefficient = n C 12 According to the question:
n C 3 = n C 12 ? n = 12 + 3 n = 15 ? Expansion = [x 2 + (1/x)] 15 Now T r+1 = 15

52%	MATCHING BLOCK 167/248	W

C r . (x 2) 15-r . 1/ x r T r+1 = 15 C r . x 30-3r -------------< (i) Putting : 30 – 3r = 0 ? r = 10 From (i) T 11 = 15 C 10 = 15!/(10! x 5!) = (15 x 14 x 13

x 12 x 11) / (5 x 4 x 3 x 2 x 1) = 3003.
Example In the
expansion of (a – b) n , n >= 5, if the sum of the 5 th and 6 th terms is zero. Find (a / b) in terms of 'n'. Sol.: T 5 = T 4+1 =
n
C 4 (

67%	MATCHING BLOCK 169/248	SA	unit 2.docx (D32968061)

a) n-4 (-b) 4 T 5 = n C 4 a n-4 b 4 T 6 = T 5+1 = n C 5 (a) n-5 (-b) 5 = - n C 5 a n-5 b 5 T 5 + T 6 = 0 ? n C 4 a n-4 b 4 – n C 5 a n-5 b 5 = 0 or n C 4 a n-4 b 4 = n C 5 a n-5 b 5

or n!/(4!
x
n-4!) a n-4 = n!/(5! x n-5!) a n-5 b
Business Mathematics Note 157 or

50%	MATCHING BLOCK 170/248	SA	unit 2.docx (D32968061)

a n-4 / (n-4)(n-5!) = a n-5 / 5(n-5!) b or a n-4 / a n-5 = b / 5 (n-4) or a (n-4)-(n-5) = (n – 4) / 5 .b or a = (n – 4)/5 . b or a/b = (n – 4) / 5

Example

83%	MATCHING BLOCK 171/248	SA	unit 2.docx (D32968061)

Find the coefficient of x r in the expansion of [x + (1/x)]

n , if it occurs. Sol.: General term : T p+1 = n C p (x) n-p (1/x) p T p+1 = n C p x n-2p ---------------< (i) Putting n-2p = r ? p = (n - r) / 2 From: (i) T (n-r) / 2 +1 = n C (n-r) / 2 x r Coefficient of x r = n C (n – r) / 2 Example Prove that the coefficient of the term independent of 'y' in the expansion of [(y + 1)/(y 2/3 – y 1/3 + 1) - (y – 1) / (y – y 1/2)] 10 is 210. Sol.: We have (y + 1) / (y 2/3 – y 1/3 + 1) Putting y = t 3 , we have = (t 3 + 1 3) / (t 2 – t + 1) = (t + 1) (t 2 – t + 1) / (t 2 – t + 1) = t + 1 (y + 1) / (y 2/3 – y 1/3 + 1) = y 1/3 + 1 and Putting y = a 2 in (y – 1) / (y – y 1/2) : = (

100%	MATCHING BLOCK 172/248	SA	CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

a 2 – 1) / (a 2 – a) = (a+1) (a-1) / [a (a-1)] = (a + 1) / a = 1 + 1 / a ? (

82%	MATCHING BLOCK 173/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

y – 1) / (y – y 1/2) = 1 + 1 /vy ? (y + 1) / (y 2/3 – y 1/3 +1) - (y – 1) / (y – y 1/2)] 10 = [y 1/3 + 1 – 1 – (1/ y 1/2)] 10 = (y 1/3 – y -1/2) 10 In (y 1/3 – y -1/2) 10 ,

T r+1 = 10 C r (y 1/3) 10-r . (-y -1/2) r = (-1) r 10 C r . (10-r) / 3 - r/2 T r+1 = (-1) r 10 C r . y (20-5r) / 6 Putting (20 – 5r) / 6 = 0
Note Unit 5: Binomial Theorem 158 or r = 4 Putting this value in (1) T 5 = (-1) 4 10 C 4 = 10!/ (6!

58%	MATCHING BLOCK 174/248	SA	unit 2.docx (D32968061)

x 4!) = (10 x 9 x 8 x 7) / (4 x 3 x 2 x 1) T 5 = 210 Example x 4-r occurs in the expansion of [x + (1/ x 2)] 4

n , prove that its coefficients is: = (4
n!) /[(4/3)n-r]! x [(4/3)(2n+r)]! Sol.: In [x + (1/x 2)] 4n , T p+1 = 4n C p (x) 4n-p (1/ x 2) p T p+1 = 4n C p x 4n-3p ----------< (i) Putting : 4n – 3p = 4r or 4 (n-r) / 3 = p From (i) T p+1 = 4
n C 4(n-r) / 4 . x 4r Coefficient of
x 4
r = 4n C 4 (n-r) / 3 = (4

60%	MATCHING BLOCK 178/248	SA	unit 2.docx (D32968061)

n!) / [(4/3)n-r]! x [(4n/1) – 4(n-r)/3]! = (4n!) / [(4/3)n-r]! x [(4/3) 2(n+r)]! Example Find the coefficient of x 50 in (1+x) 41 (1-x+x 2) 40 . Sol.: (1+x) 41 (1-x+x 2) 40 = (1+x) (1+x) 40 (1-x+x 2) 40 = (1+x) [(1+x) (1-x + x 2)] 40 = (1+

x) (1+x 3) 40
General Term = T r+1 = (1+x) [40 C r (x 3) r] = 40 C r (1+x) x 3r = 40 C r (x 3r + x 3r + 1) Here either 3r = 50 or 3r+1 = 50 r = (50 / 3)
or (49 / 3) The value of 'r' is a fraction, so it doesn't contain the term x 50 . So coefficient of x 50 is '0'. Example Show that that

40%	MATCHING BLOCK 175/248	W	

the term independent of 'x' in the expansion of [x + (1/x)] 2n is [1. 3. 5. ---- (2n-1) / (n!)] 2 n Sol.: General Term T r+1 = 2n C r (x) 2n-r (1/x) r = 2n C r . x 2n-2r ---------< (

i)
Business Mathematics Note 159 Here 2n – 2r = 0 or n = r From (i) Tr+1 = 2

70%	MATCHING BLOCK 176/248	W

n C n = 2n! / (n! x n!) = [2n (2n-1) ------ 3. 2. 1] / (n! x n!) = { 2n (2n-2) ---- 4. 2 } { (2n-1) (2n-3) ----- 3.1.} / (n! x n!) = [2 n {n (n-1) -----2.1.}] { (2n-1) ------- 4.3.1.} / (n! x n!) = 2n. n!{(2n-1) ---- 5. 3. 1. / (n! x n!) = {1. 3. 5. ----- (2n -1)} 2 n / n!

Example: The 3 rd , 4 th and 5 th terms in the expansion of (x+a) n are respectively '84', '280' and '560', find the value of 'x', 'a' and 'n'. Sol.: T r+1 = n C r x n-r . a r Putting r = 2, 3 and 4 respectively T 3 =

27%	MATCHING BLOCK 177/248	W

n C 2 x n-2 . a 2 = 84 ------------<(i) T 4 = n C 3 x n-3 a 3 = 280----------<(ii) and T 5 = n C 4 x n-4 a 4 = 560 --------<(iii) egn (i) x egn(iii) : [n C 2 x n-2 a 2] [n C 4 x n-4 a 4] = 84 x 560 = n!/[2! x (n-2)!] x n! / [4! x (n-4)!] . x 2n-6 a 6 = 84 x 560 or n (n-1) / 2 x n(n-1) (n-2) (n-3) / 4! x x 2n-6 a 6 = 84 x 560 -------< (iv) Squaring of eg n (ii), we have : (n C 3 x n-3 a 3) 2 = 280 2 ? n C 3 x n C 3 x x 2n-6 x a 6 = 280 2 = n! / [3! x (n-3)!] x n! / [3! x (n-3)!] x x 2n-6 a 6 = 280 2 or n (n-1)(n-2) / 6 x n(n-1) (n-2) (n-3) / 3! x x 2n-6 a 6 = 280 x 280 -------< (v) egn (v) egn(iv) : ? n 2 (n-1) 2 (n-2) 2 / (6 x 3!) x 2 x 4! / [n 2 (n-1) 2 (n-2)(n-3)] = (280 x 280) / (84 x 560) or 4 (n-2) / 3 (n-3) = 5 / 3

or 4n – 8 = 5n – 15 n = 7
Putting this value in (i), (ii) and (iii) : 7 C 2 x 5 a 2 = 84 ---------------< (vi) 7 C 3 x 4 a 3 = 280 ----------------<(vii) Note Unit 5: Binomial Theorem 160 7 C 4 x 3 a 4 = 560-----------------<(viii) eqn (vii) eqn (vi): (7 C 3 x 4 a 3) / (7 C 2

22%	MATCHING BLOCK 180/248	SA	unit 2.docx (D32968061)

x 5 a 2) = 280 / 84 [7! / (3! x 4!)a] / [7! / (2! x 5!)x] = 10 / 3 or 7! / (3! x 4!) x (2! x 5!) / 7! x a / x = 10 / 3 or 5 / 3 x a / x = 10 / 3 or a = 2x Putting this value in eqn (vi): 7 C 2 . x 5 . 4x 2 = 84 or 7! / (2! x 5!)x 7 = 21 ? (7 x 63) / 2 x 7 = 21 x 7 = 1 ? x = 1 Putting this value in (ix) = a = 2 Example The 6 th term in the expansion of [(1/ x 8/3) + x 2 log 10 x] 8 is 5600. Prove that x =10. Sol.: T 6 = T 5+1 = 8 C 5 (1/ x 8/3) 8-5 (x 2 log 10 x) 5 or 8 C 5 x (1 / x 8) x c 10 x (log 10 x) 5 = 5600 ? 8! / (5! x 3!) x c 2 (log 10 x) 5 = 5600 ? 8. 7. 6. / 6 x c 2 (log 10 x) 5 = 5600 or x 2 (

log 10 x) 5 = 100 = 10 2
Clearly x = 10
satisfied as log 10 10 = 1. If x < 10
or > 10, the result will change in inequality. Self Assessment State whether the following statements are true or false: 1. The triangular array is called Pascal's Triangle. 2. Each row gives the combinatorial numbers, which are the binomial coefficients. 3. Pascal's triangle can be easily used even if the exponent is very high. 4. The most basic example of the binomial theorem is the formula for the square of x + y: 5. The powers of x go down until it reaches 0 (x 0 = 1), starting value is n (the n in (x + y) n .) 6. The powers of y go up from 0 (y 0 = 1) until it reaches n (also the n in (x + y) n .) Business Mathematics Note 161 7. The nth row of the Pascal's Triangle will be the coefficients of the expanded binomial. for each line, the number of products (i.e. the sum of the coefficients) is equal to 2 n +1 8. The binomial theorem can be applied to the powers of any binomial. 5.6 Summary ? A binomial is a polynomial with two terms.

64%	MATCHING BLOCK 179/248	W

The binomial theorem describes the algebraic expansion of powers of a binomial, hence it is referred to

as binomial expansion. ?

93%	MATCHING BLOCK 181/248	W

According to the theorem, it is possible to expand the power (x + y) n into a sum involving terms of the form

ax b y c ,

100% **MATCHING BLOCK 182/248** W

where the exponents b and c are nonnegative integers with b + c = n, and the coefficient a of each term is a specific positive integer depending on n and b. ?

When an exponent is zero, the corresponding power is usually omitted from the term. ? The coefficient a in the term of ax b y c

73% **MATCHING BLOCK 183/248** W

is known as the binomial coefficient n b ? ? ? ? ? ? or n c ? ? ? ? ? ? (the two have the same value). ? The coefficients for varying n and b can be arranged to form Pascal's triangle. ? Numbers also arise in combinatorics, where n b ? ? ? ? ? ? gives the number of different combinations of b elements that can be chosen from an n-element set.

Task Discuss in group, the significance of Pascal's triangle. 5.7 Keywords Binomial : A binomial is a polynomial with two terms. Binomial coefficient: The coefficient a in the term of ax b y c is known as the binomial coefficient. Pascal's triangle : Pascal's triangle is a triangular array of the binomial coefficients. Binomial theorem : Binomial Theorem is a theorem that specifies the expansion of a binomial of the form (x + y) n as the sum of n + 1 terms of which the general term is of the form where k takes on values from 0 to n.
Note Unit 5: Binomial Theorem 162 5.8 Review Questions
Business Mathematics Note 163 Answers: Self Assessment 1. polynomial 2. n + 1 3. Sir Isaac 4. Without 5. –1 > x > 1 6. Odd 7. even 8. True 9. True 10. False 11. True 12. True 13. True 14. False 15. True 5.9 Further Readings Books Kenneth Rosen, Discrete Mathematics and Its Applications, 7/e, 2012, McGraw-Hill. Bathul, Shahnaz, Mathematical Foundations of Computer Science, PHI Learning. Lambourne, Robert, Basic Mathematics for the Physical Sciences,2008, John Wiley & Sons. Bari, Ruth A.; Frank Harary. Graphs and Combinatorics, Springer. F. Ernest Jerome, Connect for Jerome, Business Mathematics in Canada, 7e, Canadian Edition. S Rajagopalan and R Sattanathan, Business Mathematics, 2 edition, 2009, Tata McGraw Hill Education. Garrett H.E. (1956), Elementary Statistics, Longmans, Green & Co., New York. Guilford J.P. (1965), Fundamental Statistics in Psychology and Education, Mc Graw Hill Book Company, New York. Hannagan T.J. (1982), Mastering Statistics, The Macmillan Press Ltd., Surrey. Lindgren B.W (1975), Basic Ideas of Statistics, Macmillan Publishing Co. Inc., New York. Selvaraj R., Loganathan C., Quantitative Methods in Management. Sharma J.K., Business Statistics, Pearson Education Asia Walker H.M. and J. Lev, (1965), Elementary Statistical Methods, Oxford & IBH Publishing Co., Calcutta. Wine R.L. (1976), Beginning Statistics, Winthrop Publishers Inc., Massachusetts.
Note Unit 5: Binomial Theorem 164 Online links en.wikipedia.org/wiki/Binomial_theorem www.mathsisfun.com/algebra/binomial-theorem.html www.examsolutions.net/maths-revision/core.../binomial/.../tutorial-1.php www.sparknotes.com › ... › Math Study Guides › Binomial Expansion www.examsolutions.net/maths-revision/core.../binomial/.../tutorial-1.php
Business Mathematics Note 165 Unit 6: Compound Interest and Annuities CONTENTS Objectives Introduction 6.1 Simple Interest 6.2 Compound Interest 6.3 Interest Compounded Continuously 6.4 Rate of Interest 6.4.1 Nominal and Effective Rate of Interest 6.5 Annuity 6.5.1 Types of Annuity 6.6 Amortization 6.7 Sinking Fund 6.8 Summary 6.9 Keywords 6.10 Review Questions 6.11 Further Readings Objectives After studying this unit, you will be able to: 1. Explain simple interest, compound interest and interest compounded continuously 2. Discuss rate of Interest, nominal and effective rate of interest 3. Describe immediate Annuity or ordinary Annuity, deferred annuity, perpetual annuity or perpetuity 4. Explain amortization and sinking fund Introduction Individuals or institutions borrow or lend money. The use of money bears a cost in the form of interest paid or foregone and thus has a time value. When we borrow some amount of money as loan from somebody for a fixed period of time, an extra amount charged is called the interest. In this unit, we will explain simple interest, compound interest and interest compounded continuously .We will also focus on rate of Interest, nominal and effective rate of interest. Further, we will focus on immediate annuity or ordinary Annuity, deferred annuity, perpetual annuity or perpetuity. Finally, we will focus on amortization and sinking fund.

Note Unit 6: Compound Interest and Annuities 166 6.1 Simple Interest Interest can be simply defined as the price for borrowing or the premium for lending money This interest depends on three factors: (a) Amount borrowed (b) The period of time, and (c) The rate of interest. The amount borrowed is called the principal and is denoted by P. The sum of the principal and the interest is called the amount and is denoted by A. Thus, we have Principal : P Amount : A Period of time : T or N Rate of Interest : R (per annum) A = Principal + Interest = P + N The simple interest on the principal P for the number of years N at the rate of interest R is calculated by using the formula: 100 PNR I ? For solving Examples on the simple interest, we have the following formulae: 1. To find I when P, N, R are given 100 PNR I ? 2. To find P when I, N, R are given 100I P NR ? 3. To find N when I, P, R are given 100I N PR ? 4. To find R when I, P, N are given 100I R PN ? 5. To find A when P and I are given A P I ? ?
Business Mathematics Note 167 6. To find P when A, R, N are given 100 100 A P RN ? ?

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Example Find the simple interest on $2,000 for 10 years at the rate of 10% per annum.

Solution: Here 2000, 10, 10 P N R ? ? ? 100 2000 10 10 100 2000 PNR I ? ? ? ? ? ? ? Simple interest is $ 2000

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Example Calculate the simple interest on $ 4,000 for 5 years at the rate of 6% per annum.

Solution: Here 4000, 5, 6 P N R ? ? ? 100 4000 5 6 100 1200 PNR SI ? ? ? ? ? ? ? Simple interest is $1200 Example Find the amount on $15000 at 5% for 10 years. Solution: Here 15000, 10, 5 P N R ? ? ? . . 1000 15000 10 5 $ 7500 100 PNR S I ? ? ? ? ? Example If a principal amount is charged the simple interest for $750 for 5 years at the rate of 6% per annum, what is the principal?
Note Unit 6: Compound Interest and Annuities 168 Solution: 100 5 6 750 100 750 100 5 6 i.e., 25 100 $2500 PNR SI P P P ? ? ? ? ? ? ? ? ? ? ? Example If a person who takes a loan of $3600 is asked to repay $ 6000 at the end of 3 years, what is the rate of interest charged as per simple interest? Solution: Here 6000, 3600, 3 A P N ? ? ? 6000 3600 2400 SI A P ? ? ? ? ? ? Now, 100 3600 3 2400 100 PNR SI R ? ? ? ? 2400 100 3600 3 2400 36 3 2 22 % 9 R ? ? ? ? ? ? ? Example An amount of loan borrowed from a bank totals to $5000 at the end of 4 years as per the rate of 1 6 % 4 simple interest. What is the loan amount borrowed? Solution: Let the loan amount be $ 100 100 (100 5000) 100 (25/4) 4 500000 125 4000 A P RN ? ? ? ? ? ? ? ? ?? Loan amount = $ 4000
Business Mathematics Note 169 Example A man deposits $100 on the first of every month in a savings bank account in which simple interest is 2% per annum on the minimum monthly balances, what is the amount accumulated at the end of 1 2 2 years? Solution: TotalInterest intereston $100for30months + intereston $100for 29 months +... + intereston $100for1month ? 2 3 2 29 2 1 100 100 .... 100 100 12 100 12 100 12 ? 1 30 29 .... 1 6 1 30 1 30 (Using the formula for the sum to 6 2 terms of an A.P. [ ]) 2 77.50 n n n S a l ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? The amount deposited = 100 × 30 = 3000 ? the amount accumulated at the end of 1 2 2 years is $3000 $77.50 $3077.50 ? ? Example A money lender lent $ 250 and demanded $340 towards the amount due at the end of 4 years. Find the rate of interest (simple). Solution: $340 $250 110 I ? ? ? 100 100 110 250 4 11 I R PN ? ? ? ? ? ? the rate of interest is 11%
Note Unit 6: Compound Interest and Annuities 170 Self Assessment Fill in the blanks: 1. The amount borrowed is called the .... and is denoted by .... 2. The sum of the principal and the interest is called the .... and is denoted by .... 3. The simple interest on the principal P for the number of years N at the rate of interest R is calculated by using the formula: .... 4. Formula to find A when P and I are given is : 6.2 Compound Interest If r% per annum is the rate of simple interest for a sum of $ at the end of 1 year, the amount 1 A will be 1 1 100 r A P ? ? ? ? ? ? ? ? ? Now, if 1 A is taken as the principal amount and interest calculated on this amount at r% interest, at the end of 2 years, the amount will become: 2 1 1 100 100 100 r r r A P P ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? ? ? 2 1 1 (bytaking 1 ascommonfactor) 100 100 100 1 100 r r r P P r P Similarly, the amount at the end of 3 years will be 3 3 1 etc., 100 r A P ? ? ? ? ? ? ? ? ? Continuing this way, we get the amount at the end of n years is 1 100 n n r A P ? ? ? ? ? ? ? ? ? total interest gained on $is 1 100 n n r A P P p ? ? ? ? ? ? ? ? ? ? For solving Examples concerning compound interest we use the formula: 1 100 n r A P ? ? ? ? ? ? ? ? ?

Business Mathematics Note 171 Example Find the compound interest on $4000 at the end of 5 years at the rate of 8% per annum. Solution: 1 100 n r A P ? ? ? ? ? ? ? ? ? 5 5 8 4000 1 100 4000 (1.08) ? ? ? ? ? ? ? ? ? ? ? Taking log on both sides, log log4000 5log1.08 3.6021 5 0.0334 A ? ? ? ? ? 3.6021 0.1670 3.7691 Antilog3.7691 5876.00 A ? ? ? ? ? ? ? at the end of 5 years, the amount $ 5876.00 A ? The principal .00 $ 4000 P ? Compound interest $5876 $4000 ? ? $1876.00 ? Example What sum amounts to $6000 after 4 years at 5% compound interest? Solution: ? ? ? ? 4 4 4 1 100 5 6000 1 100 6000 1.05 6000 1.05 n r A P P P P ? Taking logs on both sides

Note Unit 6: Compound Interest and Annuities 172 log log6000 4log1.05 3.7782 4 0.0212 3.7782 0.0348 3.6934 antilog3.6934 4937 P P ? ? ? ? ? ? ? ? ? ? ? ? ? Principal $ 4937 ? Example In how many years, $1000 will be doubled at 6% compound interest? Solution: Here 2000, 1000, 6 A P r ? ? ? 1 100 n r A P ? ? ? ? ? ? ? ? ? ? ? 6 2000 1000 1 100 2 1.06 n n ? ? ? ? ? ? ? ? ? Taking logs, log2 log1.06 0.3010 (0.0212) 0.3010/0.0212 i.e., 3010/212 n n n n ? ? ? ? ? Taking logs on both sides, ? ? ? ? ? ? ? log log3010 log212 3.4786 2.3263 1.1523 antilog1.1523 14.20 n Hence in 14.2 years, $1000 will be doubled. Example A man borrowed $6250 from a bank and after 2 years paid $6760 in full settlement of his debt. Find the rate of compound interest charged by the bank?

Business Mathematics Note 173 Solution: 6760, 6250, 2 A P n ? ? ? 2 2 1 100 6760 6250 1 100 6760 1 100 6250 n r A P r r ? 676 1 100 625 r ? ? ? Taking log on both sides, 1 log 1 [log676 log625] 100 2 r ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2.8299 2.7959 2 1 0.0340 2 0.0170 ? ? ? ? 1 antilog0.0170 100 1.040 r ? ? ? ? 1.040 1 100 0.040 r ? ? ? ? 0.040 100 4 r ? ? ? ? ? Rate of compound interest 4%p.a. ? Example Find the difference between compound interest and simple interest on $6000 for 5 years at 10%? Solution: 6000, 5, 10 P n R ? ? ? 100 6000 5 10 100 3000 PNR I ? ? ? ? ?

Note Unit 6: Compound Interest and Annuities 174 ? S.I. $3000 ? ...(1) Here 6000, 5, 10 P n R ? ? ? 5 5 10 6000 1 100 . ., 6000(1.01) A i e A ? ? ? ? ? ? ? ? ? Taking log on both sides, log log6000 5log1.1 3.7782 5(0.0414) 3.7782 0.2070 3.9852 A ? ? ? ? ? ? ? antilog3.9852 9665.00 A ? ? ? i. e., . . $ 9665 $ 6000 C I ? ? ? . . $3665 C I ? The difference between C.I. and S.I. is $ 3665 $3000 $ 665 ? ? Example What principal will amount to $1352 in two years at 4% compound interest? Solution: Let P be the principal amount 1 100 n r A P ? ? ? ? ? ? ? ? ? ? ? ? ? 2 2 2 4 1352 1 100 1352 1.04 1352/ 1.04 P P P ? ? ? ? ? ? ? ? ? ? ? ? Taking logs on both sides, log log1352 2log(1.04) 3.1310 2 0.0170 3.1310 0.0340 3.0970 antilog3.0970 1250.00 P P ? ? ? ? ? ? ? ? ? ? ?

Business Mathematics Note 175 ? the principal $1250.00 ? Example Find the compound interest on $2400 for 1 2 2 years at 5% per annum interest being compounded annual. Solution: 1 100 n r A P ? ? ? ? ? ? ? ? ? ? ? 5/2 5/2 5 2400 1 100 2400 1.05 5 log log2400 log1.05 2 A ? ? ? ? ? ? ? ? ? ? ? ? ? 5 3.3802 0.0212 2 3.3802 5 9.0106 3.3802 0.0530 3.4332 ? ? ? ? ? ? ? ? ? ? ? ? ? A antilog 3.4332 2711.00 . . $2711 $2400 $311 C I ? ? ? ? 6.3 Interest Compounded Continuously Let $ P be invested at a rate of interest of 100r% per annum. The amount after one year is ? ?, 1 r P A ? ? when interest is compounded annually, 2 2 1 ? ? ? ? ? ? ? ? r P when interest is compounded half yearly, ... n n r P ? ? ? ? ? ? ? ? ? 1 , when interest is compounded n times a year, , 1 lim ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? n n r P interest compounded continuously,

Note Unit 6: Compound Interest and Annuities 176 ? e n r Pe r n n r 1 lim ? Further, if $ P is invested for t years at the rate of interest 100r% p.a., we can write the expression for A as: ? ? t r P A ? ? 1 [annual compounding] t r P 2 2 1 ? ? ? ? ? ? ? ? ? [half yearly compounding] .... nt n r P ? ? ? ? ? ? ? ? ? 1 [n times a year compounding] rt Pe ? [continuous compounding] Remarks: Here r denotes the interest on a rupee in one year. Note Economic Meaning of e Let a rupee be invested at 100% rate of interest per annum for one year. The amount A after one year is given by ? ?1 1 100 100 11 ? ? ? ? ? ? ? ? ? ? ?A = $ 2 [interest compounded annually] = ? ? ? ? ? ? ? ? ? 4 9 2 1 1 2 $ 2.25 [interest compounded half yearly] = n n ? ? ? ? ? ? ? 1 1 [interest compounded n times a year]. = ? ? ? ? ? ? ? ? ? ? ? e n n n 1 1 lim $ 2.72 (approx.) [interest compounded continuously] Thus, a rupee invested at 100% rate of interest will become $ 2.72 (= e) at the end of the one year, when interest is compounded continuously. Hence, e can be interpreted as the amount of a rupee invested at 100% rate of interest p.a. for one year, when compounded continuously. Example If $ 2,000 is invested at 10% rate of interest per annum, what will be the amount after 3 years if the interest is compounded: (i) annually, (ii) quarterly, (iii) continuously.

Business Mathematics Note 177 Solution. 1. ? ? 3 3 10 A 2,000 1 2,000 1.1 100 ? ? ? ? ? ? ? ? ? ? ? ? $ 2,662 2. ? ? 12 12 10 A 2,000 1 2,000 1.025 400 ? ? ? ? ? ? ? ? ? ? ? ? $ 2,689.79 3. 0.1 3 0.3 A 2,000e 2,000e ? ? ? ? $ 2,699.72 Example Mr. X deposited $10,000 in a bank for 3 years offering interest at the rate of 6% p.a. compounded half yearly during first year, at the rate of 12% p.a. compounded quarterly during second year and at 10% p.a. compounded continuously during third year. Find his balance after three years. Solution. The balance after three years is ? ? ? ? 2 4 0.1 A 10000 1.03 1.03 e $ 13196 ? ? Self Assessment Fill in the blanks 5. If r% per annum is the rate of simple interest for a sum of $at the end of 1 year, the amount 1 A will be .... 6. if 1 A is taken as the principal amount and interest calculated on this amount at r% interest, at the end of 2 years, the amount will become: .... Caution! Compound interest is great when it works in your favor in investments, but it can also be your biggest enemy when it works against you in loans and other debts. 6.4 Rate of Interest The rate of interest, 100r%, for continuous compounding of money, considered above, can also be interpreted as the rate of growth of other characteristics like growth of population, growth of petrol consumption, growth of sales, growth of timber in a wood, growth of bacteria in a culture etc., when rate of change in the characteristics is a constant percentage of its magnitude. Let N be the population of a country at time t and it increases (continuously) at a rate of 100r% per unit of t. Then we can write Nr dt dN .? . This equation can be rewritten as rdt N dN ? Integrating both sides, we get , log c rt N ? ? where c is constant of integration.

Note Unit 6: Compound Interest and Annuities 178 When t = 0, we have 0 logN c ? (say) ? 0 log log log N e N rt ? ? or rt e N N . 0 ? Did u Know? When the exponent of e is ? rt, then r is interpreted as the negative rate of growth or rate of decay. Example Population of India was 68 crores in 1981 and 84 crores in 1991. Assuming that this growth continues and the rate of increase is proportional to population (i.e. exponential growth), estimate the population in the year 2001. Solution Let 68 0 ? N , N = 84 and r be the annual rate of proportional increase. ? r e 10 68 84 ? (t = 10) or 68 84 10 ? r e Taking log of both sides, we get 68 log 84 log 10 ? ?r or e r 10 10 10 log 10 68 log 84 log 10 68 log 84 log ? ? ? ? = 0211 .0 343 .4 0918 .0 4343 .0 10 8325 .1 9243 .1 ? ? ? ? or 2.11% p.a. Thus the estimated population in 2001 is N = 84 73. 103 211 .0 ? e crores. Example Assume that in 1990 the annual world use of natural gas was 50 trillion cubic feet. The annual consumption of gas is increasing at a rate of 3%, compounded continuously. How long will it take to use all available gas, if it is known that in 1990 there were 2,200 trillion cubic feet of proven gas reserves? Assume that no new discoveries are made. Solution. The rate of use of gas in the ith year is given by t e A 03.0 50 ? . If x denotes the number of years required to use all available gas, then ? ? x t dt e 0 03.0 50 200 ,2 or x t e 0 03.0 03.0 1 44 ? or 1 32.1 03.0 ? ? x e or 32.2 03.0 ? x e or 0.03x =log 2.32 ? 05.28 03.0 32.2 log ? ?x years.

Business Mathematics Note 179 Example A company has current sales of $ 1,000 per month and profit to the company averages 10% of the sale. The company's past experience with a certain advertising strategy is that sales increase by 2% per month continuously over the length of advertising campaign (12 months). If the advertising costs $ 130 in a year, determine if the company should embark on a similar campaign when market rate of interest is 12% p.a. Solution. Total increase in sales due to the advertising campaign 000 ,1 12 000 ,1 12 0 02.0 ? ? ? ? dt e t = ? ? 12000 1 000 ,50 024 .0 ? ? e = 13,562.46 ? 12,000 = $ 1,562.46. ? Profit due to advertising = 100 10 46. 1562 ? = $ 156.25. Since this profit is available after one year, its present value is 12.1 25. 156 ?P = $ 139.51 The cost of advertisement = $ 130. Since profit is more than cost of advertisement, the firm should embark on the advertising campaign. 6.4.1 Nominal and Effective Rate of Interest When interest is compounded half yearly, quarterly or monthly etc., the interest earned during a year is greater than the interest earned from annual compounding. When compounding is more frequent than annual, an effective annual interest rate can be determined. This is an annual compounding interest rate which is equivalent to a nominal rate compounded more frequently. The announced annual rate of interest is termed as the nominal rate of interest, while the actual rate by which the money grows during an year is called the effective rate of interest. Did u know? The binomial theorem shows how to calculate a power of a binomial -- (a + b) n -- without actually multiplying out. Relation between Nominal and Effective Rates of Interest Let 100i% be the nominal rate of interest p.a. at which a sum of $ P is invested for t years when compounded n times a year. If 100r% is the effective rate of interest p.a., we have ? ? nt t n i P r P ? ? ? ? ? ? ? ? ? 1 1 or 1 1 ? ? ? ? ? ? ? ? ? ? n n i r

Note Unit 6: Compound Interest and Annuities 180 Thus effective rate of interest is 100r% = % 100 1 1 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? n n i Note If interest compounding is continuous, we have 1? ? i e r . Example The rate of interest on a term deposit is 12% p.a. compounded quarterly. What is the effective rate of interest? Solution. Let 100r% be the effective rate of interest p.a., then we can write 1 400 12 1 4 ? ? ? ? ? ? ? ? ? ?r = 1255 .0 1 400 412 4 ? ? ? ? ? ? ? ? ? Thus the effective rate of interest is = 100r% = 12.55% p.a. Example A moneylender charges interest at the rate of 10% per month payable in advance. What is the effective rate of interest per annum? Solution. Let us assume that the moneylender gives a loan of $ 100. As per his terms of loan, he gives $ 90 after deducting $ 10 as interest and gets $ 100 at the end of the month. Thus $ 10 can be regarded as interest for one month on $ 90. ? Monthly (nominal) rate of interest per rupee i.e. i = 9 1 90 10 ? If the effective rate of interest is 100r% p.a., we can write 1 9 10 1 9 1 1 12 12 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?r = ? ? ? ? 1 9 log 10 log 12 log Anti ? ? = 2.547 Hence the effective rate of interest is 100r% = 254.7%. Did u know? For a given rate of interest, the more frequent compounding will yield more interest during a given period and consequently the higher will be the effective rate of interest. Task Discuss in group, the meaning , constituting factors and demerits of compound interest.

Business Mathematics Note 181 Self Assessment Fill in the blanks: 7. Rate of Interest is also known as .... 8. When the exponent of e is ? rt, then r is interpreted as the .... rate of growth or rate of decay 9. When interest is compounded half yearly, quarterly or monthly etc., the interest earned during a year is .... than the interest earned from annual compounding 10. When compounding is .... than annual, an effective annual interest rate can be determined. 11. The announced annual rate of interest is termed as the....................., 12. The actual rate by which the money grows during an year is called the .... 6.5 Annuity "An annuity is a fixed sum paid at a regular interval under certain stated conditions", the period may be one year or half year or one month. An annuity is a series of payments or receipts, of equal value, made at regular (or equal) time intervals. Did u Know? The term of an annuity is the time between beginning and last payments. Examples: Interest, House rent, salaries, pension etc. 6.5.1 Types of Annuity 1. Annuity Certain: An annuity payable for a fixed number of years is called Annuity Certain. 3. Annuity Contigent: An annuity payment or duration of annuity payments or happening of any event is known as Annuity Contigent. 4. Perpetuity: Annuity which is for ever (i.e., for an infinite period) is called perpetuity. 5. Deferred Annuity: When the Annuity payment starts after lapse of a certain specific period, the annuity is called Deferred Annuity. 6. Annuity Due: If the annuity payments are made in advance at the beginning of each stipulated time then it is called Annuity due. 7. Ordinary Annuity is an annuity for which each payment is made at the end of the each period.

Note Unit 6: Compound Interest and Annuities 182 Self assessment Fill in the blanks: 13. An .... is fixed sum paid at a regular interval under certain stated conditions, the period may be one year or half year or one month. 14. Annuity which is for ever (i.e., for an infinite period) is called .... 15. An annuity payable for a fixed number of years is called .... 16. An Annuity payments are made at the end of each period. Then the annuity is called Annuity .... 17. An annuity payment or duration of annuity payments or happening of any event is known as Annuity .... 6.6 Amortization Amortisation of Loan A loan is said to be amortised if the principal and interest on it are repaid by a sequence of equal installments at regular interval of time. Let us assume that a loan of $ P is taken at a rate of interest of 100r% p.a. for n years. If we decide to repay this loan by equal installments of, say, $ x per year, then the value of x should be such that the present value of annuity is equal to P. Thus, we have ? ? ? ? . 1 1 n r r x P ? ? ? ? . From here we can write ? ? n r rP x ? ? ? ? 1 1 ... (8) Note Total interest on loan = ? ? .P x n ? ? If the repayment of loan is done by equal monthly installments, then 12 annum per interest of Rate % 100 ? r and n = 12 ? number of years of loan. Similar type of adjustment can be done for quarterly, half-yearly payments etc. Balance of Loan after k Installments The balance of loan (i.e. principal) that remains to be repaid after the payment of kth installment, to be denoted by k P , is given by the present value of the remaining (n ? k) payments. Thus ? ? ? ? ? ? k n k r r x P ? ? ? ? ? ? 1 1 = ? ? ? ? n k r r x ? ? ? 1 1 ... (9) Component of Principal in kth Installment The component of principal in kth installment, to be denoted by ? ? ? ? ? ? ? ? n k n k k k k r r x r r x P P V ? ?? ? ? ? ? ? ? ? ? ? ? 1 1 1 1 1 1 = ? ? ? ? ? ? n k n k r r r x ?? ? ? ? ? 1 1 1

Business Mathematics Note 183 = ? ? ? ? ? ? n k n k r x r r r x ?? ?? ? ? ? ? ? 1 1 1 1 1 1 ... (10) Component of Interest in kth Installment The component of interest in kth installment , to be denoted by k I , is given by the difference of the installment and the component of principal. Thus k k V x I ? ? ... (11) Example Find the amortised monthly payment necessary to pay-off a house building loan of $ 1,50,000 at 12% p.a. in 10 years. Further, find the amount of loan paid after the payment of 60 installments. What is the component of principal and interest in 60 th installment? Solution. Here r = 01.0 1200 12 ? and 120 12 10 ? ? ?n ? ? 120 01.1 1 000 ,50,1 01.0 ? ? ? ?x ? ? ? ? 303 .0 1 000 ,50,1 01.0 $ 2,152.08 Also, ? ? ? ? 120 60 60 01.1 1 01.0 08. 2152 ? ? ? P = $ 96,746.84 Thus, the amount of loan paid after the payment of 60 th installment ? ? ? ? ? ? ? ? ?? 61 120 1 60 60 1.1 08. 152 ,2 1.1 08. 152 ,2 V $ 1,172.88 Component of interest, 60 60 V x I ? ? = 2,152.08 ? 1,172.88 = $ 979.20. 6.7 Sinking Fund A sinking fund is an annuity that is created for accumulating money to pay an obligation at some future designated date. For example, a company may plan to replace an existing machine after 10 years by saving certain sum every year (or month etc.) in an account that pays 100r% rate of interest p.a. The crucial question is, what sum should be saved at the end of each year so as to accumulate $ A at the end of n years? Assuming that the interest compounding is annual, the above question can be answered if we solve equation (1) for x. ? ? ? 1 1 ? ? ? n r Ar x ... (2) Remarks: The expression ? ? ? ? ? ? ? ? ? ? 1 1 n r r = r a n ? 1 is often called the Sinking fund factor.
Note Unit 6: Compound Interest and Annuities 184 Example How much money must be set aside each year so as to replace a machine that will cost $ 15000 after 8 years? The rate of interest on saving being 12% p.a. compounded annually . Solution. Let $ x be set aside at the end of each year. It is given that A = 15000, r = 0.12 and n = 8. ? ? ? ? ? ? ? ? 1 12.1 12.0 15000 8 x $ 1219.54 Example A machine bought for $ A has to be replaced in n years. Machine prices are increasing at an annual rate of 100a%. The cost of replacement is covered by investing each year (beginning at the end of present year) an amount of $ x at an annual rate of interest of 100r%. What should be the value of x. Solution. The cost of machine after n years will be = ? ? n a A ?1 . In order to accumulate this sum, the value of x is given by x = ? ? ? ? ? 1 1 . 1 ? ? ? n n r r a A Example The cost of a machine is 1,50,000 and its effective life is 10 years, after which its salvage value will be $ 10,000. What sum should be set aside, at the end of each year, to replace this machine after 10 years? It is known that price of machine increases by 4% p.a. and the rate of interest on saving is 9% p.a. compounded annually? Solution. The cost of replacement after 10 years = 150000 10)04.1(? 10000 = $ 212036.64 If x $ be set aside every year, then ? ? 1 09.1 09.0 64. 212036 10 ? ? ?x = $ 13956.27 Amount of an Ordinary Deferred Annuity Let us assume that the payments begin after a lapse of m years. The sum of an annuity for n annual payments of $ x at the end of (m + n) years is r ax A n ? ? . = ? ? ? ? ? ? ? ? ? ? r r x n 1 1
Business Mathematics Note 185 Example Determine the sum of an annuity of 5 annual payments of $ 350 each, if the first payment is made at the end of 2 years. The rate of interest is 8% p.a. effective. Solution. Here x = 350, r = 0.08 and n = 5 ? ? ? ? 08.0 1 08.1 350 5 ? ?A = $ 2053.31. Amount of an Annuity Due As defined earlier, the payments are made at the beginning of each period in case of an annuity due. Let D A denote the amount of an annuity due, then we can write ? ?, 1 r A A D ? ? where A is the amount of an ordinary annuity ? ? ? ? ? ?r r r x n ? ? ? ? 1 1 1 ? ? x r r x n ? ? ? ? ? ? ? ? ? ? ? ? 1 1 1 = ? ? x r xa n ? ? ?1 = ? ? ? ? 1 1 ? ? ? r ax n ... (3) Remarks: ? ? 1 1 ? ? ? r a n is the sum of an annuity due of Re 1 and termed as annuity due amount factor. The same formula is applicable for a deferred annuity due. The amount of an annuity of n annual payment when the interest is compounded continuously is ? ? ? ? 1 1 – – r rn e ex A ? which is approximately equal to ? n rt dt ex 0 . ... (4) A D = A.e r , when compounding is continuous. Example A person deposits $ 200 at the beginning of every month in a saving bank account that pays interest @ 8% p.a. compounded monthly. Find the sum in his account at the end of 5 years. Solution. Here x = 200, 150 1 1200 8 ? ?r and n = 5 ? 12 = 60 x r ax A n D ? ? ? ?1 . = 200 ? 150 200 1 150 1 1 61 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? = $ 14793.34
Note Unit 6: Compound Interest and Annuities 186 Example If $ 2000 is deposited at the beginning of each year in a saving bank account paying 4% p.a. compounded continuously, find sum after 3 years. Solution. Here x = 2000, 04.0 100 4 ? ?r , n = 3 ? ? ? ? ? ? ? ? ? ? 04.1 04.0 13.0 2000 . 1– 1– 2000 . 1– 1– 04.0 04.0 12.0 ? ? ? ? ? e e e e e ex A r r rn D = $ 6760. Example Harish deposited $ 400 every month in an account paying 3% interest p.a. compounded continuously. Find the sum after 5 years. Solution. Here x = 400, r = 400 1 1200 3 ? , n = 5 ? 12 = 60 0025 .0 16.0 400 1– 1– 400 400 1 400 60 ? e e A = $ 25,600 Note: When nothing is mentioned, the payment be regarded at the end of the period. Self Assessment State True/False 18. A sinking fund is an annuity that is created for accumulating money to pay an obligation at some future designated date. 19. The amount of an annuity of n annual payment when the interest is compounded continuously is ? ? ? ? 1 1 – – r rn e ex A ? 20. The payments are made at the last of each period in case of an annuity due. 21. Let D A denote the amount of an annuity due, then we can write ? ?, 1 r A A D ? ? where A is the amount of an ordinary annuity. 22. The expression ? ? ? ? ? ? ? ? ? ? 1 1 n r r = r a n ? 1 is often called the Sinking fund factor.

Business Mathematics Note 187 6.8 Summary ? The simple interest on the principal p for the number of years N at the rate of interest R is calculated as 100 PNR I ? ? Formula for compound interest is r P 1 100 n A ? ? ? ? ? ? ? ? ? The rate of interest, 100r%, for continuous compounding of money, considered above, can also be interpreted as the rate of growth of other characteristics like growth of population, growth of petrol consumption, growth of sales, growth of timber in a wood, growth of bacteria in a culture etc., ? When interest is compounded half yearly, quarterly or monthly etc., the interest earned during a year is greater than the interest earned from annual compounding. When compounding is more frequent than annual, an effective annual interest rate can be determined ? The announced annual rate of interest is termed as the nominal rate of interest ? The actual rate by which the money grows during an year is called the effective rate of interest. ? An annuity is a fixed sum paid at a regular interval under certain stated conditions ? An annuity is a series of payments or receipts, of equal value, made at regular (or equal) time intervals ? A loan is said to be amortised if the principal and interest on it are repaid by a sequence of equal installments at regular interval of time. ? The component of interest in kth installment , to be denoted by k I , is given by the difference of the installment and the component of principal. ? A sinking fund is an annuity that is created for accumulating money to pay an obligation at some future designated date. 6.9 Keywords Amount: The sum of the principal and the interest is called the amount. Principal: The amount borrowed is called the principal. Annuity: A fix sum paid at a regular interval under certain stated conditions. Perpetuity: Annuity which is forever, i.e., for an infinite period. 6.10 Review Questions 1. A TV originally cost $400. If the price of the TV increases by 15%, what is the new price?
Note Unit 6: Compound Interest and Annuities 188 2. Mr Kumar bought a car for $5,000. During the first year, its value depreciated by 20%. What was the value of the car after one year? 3. The population of a city in the year 2000 was 500,000. Over the following decade the population grew by 8%. What was the population of the city in 2012? 4. 15% of the cost of a computer was tax. If the tax was $180, what was the cost of the computer? 5. In a sale, the price of a coat was reduced by 25%. If Joe paid $90 for the coat, what was its original price? 6. Calculate the present value and future value of annuity of $5,000 per annum for 12 years, the interest being 4% compounded annually. 7. If Mr. X borrows $20,000 at 4% compound interest and agrees to pay both principal and the interest in 10 equal instalments at the end of year. Find the amount of each instalment. 8. Find

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the simple interest on $7000 for 8 years at the rate of 7% per annum. 9. Find the

principal if the simple interest is $750 for 10 years

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at the rate of 5% per annum. 10. Find the rate of interest

if $4000 earns a simple interest of $600 for 4 years. 11. Find the amount after 8 years if the rate of interest 6% is charged on $10000. 12. A person who took a loan of $6000 was asked to repay a sum of $8200 after 6 years. What is the rate of interest charged on simple interest. 13. At what rate of compound interest will $400 amount to $441 in 2 years? 14. A manufacturer buys a machine for $44000 and writes off 1 12 % 2 depreciation per annum. At the end of 3 years, he sells the machine for $33,013.75. What is the percentage gain he has made on the book-value of the machine? 15. In how many years a principal will become three times if the rate of interest charged is 5% compound interest. 16. What is the amount of an annuity consisting of annual payments of $ 500 each, the first being made at the end of 5 years and the last at the end of 10 years, if rate of interest is 8% effective? 17. An annuity consists of 15 quarterly payments of $ 800 each, the first payment being made at the end of first year. What is the amount of this annuity if rate of interest is 12% p.a. converted quarterly? 18. Sohan is considering two different saving plans. In the first plan he has to deposit $ 500 at the end of every six months with a rate of interest 7% p.a. compounded half yearly. Under the second plan he has to deposit $ 1000 at the end of every year with a rate of interest of 7.5% p.a. compounded annually. Which plan should he chose so as to get larger savings at the end of 10 years? Would your answer change if the rate of interest on the second plan were 7%? 19. The cost of a machine is $ 1,00,000 and its effective life is 12 years. If the scrap realises only $ 5,000, what amount should be retained out of profits at the end of

Business Mathematics Note 189 each year to accumulate $ 1,00,000? The rate of compound interest is given to be 5% p.a. 20. A machine costs a company $ 65,000 and its effective life is estimated to be 25 years. A sinking fund is created for replacing the machine at the end of its lifetime, when its scrap realises a sum of $ 2,500 only. What (equal) amount should be provided every year, out of profits, for sinking fund if it accumulates at 3.5% p.a. compound? 21. A sinking fund is to be created for the redemption of debentures of $ 2,00,000 at the end of 20 years. How much money must be set aside every year for the sinking fund if the rate of interest is 10% p.a.? 22. A sinking fund is formed by investing $ x at the end of each year for n years. Show that the final amount of the fund is ? ? ? ? 1 1 1 ? ? ? ? ? ? ? n r x r x x ...... , when interest is added yearly at 100r% p.a. Show that this sum will be equal to A if ? ? 1 1 ? ? ? n r Ar x . 23. If $ 1,500 is deposited each year in a saving bank account paying 6% interest p.a. compounded continuously, find the sum after 4 years. 24. If $ 500 is deposited every month in a saving bank account paying 4% interest p.a. compounded continuously, find the sum after three years Answers: Self Assessment 1. Principal, P 2. Amount, A 3. 100 PNR I ? 4. A P I ? ? 5. 1 1 100 r A P ? ? ? ? ? ? ? ? ? 6. 2 1 1 100 100 100 r r r A P P ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 7. Rate of Growth or Decay 8. Negative 9. Greater 10. more frequent 11. nominal rate of interest 12. effective rate of interest 13. Annuity 14. Annuity uncertain 15. Annuity Certain 16. Inmidiate 17. Contingent 18. True 19. True 20. False 21. True 22. True

Note Unit 6: Compound Interest and Annuities 190 6.11 Further Readings Books Kenneth Rosen, Discrete Mathematics and Its Applications, 7/e, 2012, McGraw-Hill. Bathul, Shahnaz, Mathematical Foundations of Computer Science, PHI Learning. Lambourne, Robert, Basic Mathematics for the Physical Sciences,2008, John Wiley & Sons. Bari, Ruth A.; Frank Harary. Graphs and Combinatorics, Springer. F. Ernest Jerome, Connect for Jerome, Business Mathematics in Canada, 7e, Canadian Edition. S Rajagopalan and R Sattanathan, Business Mathematics, 2 edition, 2009, Tata McGraw Hill Education. Garrett H.E. (1956), Elementary Statistics, Longmans, Green & Co., New York. Guilford J.P. (1965), Fundamental Statistics in Psychology and Education, Mc Graw Hill Book Company, New York. Hannagan T.J. (1982), Mastering Statistics, The Macmillan Press Ltd., Surrey. Lindgren B.W (1975), Basic Ideas of Statistics, Macmillan Publishing Co. Inc., New York. Selvaraj R., Loganathan C., Quantitative Methods in Management. Sharma J.K., Business Statistics, Pearson Education Asia Walker H.M. and J. Lev, (1965), Elementary Statistical Methods, Oxford & IBH Publishing Co., Calcutta. Wine R.L. (1976), Beginning Statistics, Winthrop Publishers Inc., Massachusetts. Online links
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Business Mathematics Note 191 Unit 7: Other Useful Mathematics Devices CONTENTS Objectives Introduction 7.1 Rounding of Numbers 7.2 Absolute, Relative and Percentage Errors 7.2.1 Absolute Error 7.2.2 Relative Error or fractional error 7.2.3 Percentage Error 7.3 Significant Figures 7.3.1 Meaning 7.3.2 Importance 7.3.5 Significant Figures - Mathematical Operations 7.4 Short Processes of Calculation 7.5 Roots and Reciprocals Expressed as Power 7.6 A.P. Series and G.P. Series 7.6.1 Arithmetic Progression 7.6.2 Geometric progression 7.6.3 Arithmetic – Geometric Series 7.6.4 Sum of First N Natural Numbers 7.6.5 The Sum of The Squares of First N Natural Numbers 7.7 Inequalities 7.7.1 Method of Drawing 7.8 Concept of ‘Function’ 7.8.1 Classification of Functions 7.9 Polynomial 7.9.1 Shifting of Graphs 7.10 Sigma (?) Notation 7.11 Interpolation 7.12 Summary 7.13 Keywords 7.14 Review Questions 7.15 Further Readings

Note Unit 7: Other Useful Mathematics Devices 192 Objectives After studying this unit, you will be able to: 1. Describe rounding of numbers, absolute, relative and percentage Errors 2. Discuss significant figures and short processes of calculation 3. Focus on roots and reciprocals expressed as power, A.P. Series and G.P. Series 4. Understand sum and sum of the squares of numbers 5. Explain inequalities, concept of 'function, polynomial, sigma notation and Interpolation Introduction A mathematical instruments and devices are tools or devices used in the study or practice of mathematics. In geometry, construction of various proofs was done using only a compass and straight edge; arguments in these proofs relied only on idealized properties of these instruments and literal construction was regarded as only an approximation. In applied mathematics, mathematical instruments were used for measuring angles and distances, in astronomy, navigation, surveying and in the measurement of time. Instruments such as the astrolabe, the quadrant, and others were used to measure and accurately record the relative positions and movements of planets and other celestial objects. The sextant and other related instruments were essential for navigation at sea. The astrolabe was an early mathematical instrument used in astronomy and navigation. Most instruments are used within the field of geometry, including the ruler, dividers, protractor, set square, compass, ellipsograph, T-square and opisometer. Others are used in arithmetic (for example the abacus, slide rule and calculator) or in algebra (the integraph). In astronomy, many have said the pyramids (along with Stonehenge) were actually instruments used for tracking the stars over long periods or for the annual planting seasons. In this unit, we will discuss rounding of numbers, absolute, relative and percentage Errors .We will also focus on significant figures and short processes of calculation, roots and reciprocals expressed as power, A.P. Series and G.P. Series, sum and sum of the squares of numbers. Finally, we will study inequalities, concept of 'function, polynomial, sigma notation and Interpolation. 7.1 Rounding of Numbers Rounding digit – means to round to the closest prescribed position. For example when asked to round to nearest tens, then your rounding digit is the second number to the left (ten's place) when working with whole numbers. When asked to round to the nearest hundred, the third place from the left is the rounding digit (hundreds place). To round a decimal number to a whole number is easy. If the tenth is five or more, round up to the next whole number. If it is less than five, round down to the previous whole number. Consider an example, while rounding 4685 to the nearest hundred,

Business Mathematics Note 193 take the digit in the place being rounded off and highlight it. Here it is 6, because it is in the hundreds place When rounding whole numbers then, the rules to remember are: Rule One. Determine what your rounding digit is and look to the right side of it. If the digit is 0, 1, 2, 3, or 4 do not change the rounding digit. All digits that are on the right hand side of the requested rounding digit will become 0. Rule Two. Determine what your rounding digit is and look to the right of it. If the digit is 5, 6, 7, 8, or 9, your rounding digit rounds up by one number. All digits that are on the right hand side of the requested rounding digit will become 0. Rounding with decimals: When rounding numbers involving decimals, there are 2 rules to remember: Rule One: Determine what your rounding digit is and look to the right side of it. If that digit is 4, 3, 2, or 1, simply drop all digits to the right of it. Rule Two: Determine what your rounding digit is and look to the right side of it. If that digit is 5, 6, 7, 8, or 9 add one to the rounding digit and drop all digits to the right of it. Rule Three: This rule provides more accuracy and is sometimes referred to as the 'Banker's Rule'. When the first digit dropped is 5 and there are no digits following or the digits following are zeros, make the preceding digit even (i.e. round off to the nearest even digit). E.g., 2.315 and 2.325 are both 2.32 when rounded off to the nearest hundredth. Note: The rationale for the third rule is that approximately half of the time the number will be rounded up and the other half of the time it will be rounded down. An example: 765.3682 becomes: 1000 when asked to round to the nearest thousand (1000) 800 when asked to round to the nearest hundred (100) 770 when asked to round to the nearest ten (10) 765 when asked to round to the nearest one (1) 765.4 when asked to round to the nearest tenth (10th) 765.37 when asked to round to the nearest hundredth (100th.) 765.368 when asked to round to the nearest thousandth (1000th) Self Assessment Fill in the blanks: 1. .... digit – means to round to the closest prescribed position 2. When asked to round to nearest tens, then your rounding digit is the .... number to the left (ten's place) when working with whole numbers.

Note Unit 7: Other Useful Mathematics Devices 194 3. When asked to round to the nearest hundred, the ……………………. place from the left is the rounding digit (hundreds place). 4. To round a decimal number to a ……………………. number is easy 5. If the tenth is five or more, round up to the next whole number. If it is less than five, round down to the ……………………. whole number. 6. If the digit is 0, 1, 2, 3, or 4 ……………………. change the rounding digit. 7.2 Absolute, Relative and Percentage Errors 7.2.1 Absolute Error Absolute error is defined as the magnitude of difference between the actual and the individual values of any quantity in question. Say we measure any given quantity for n number of times and a 1 , a 2 , a 3 …..a n are the individual values then Arithmetic mean a m = [a 1 +a 2 +a 3 + …..a n]/n a m = [Σ i=1 i=n a i]/n Now absolute error formula as per definition Absolute Error = Actual Value - Measured Value Δa 1 = a m – a 1 Δa 2 = a m – a 2 Δa n = a m – a n Thus, absolute error is an error which is written in terms of the units of the measurement. So for example, let's say you take a measurement of something that is around 40cm with an error on that measurement of (+-) 1cm. This would be an absolute error; the error (1cm) is in cm, just as the measurement (40cm) is in cm. Mean Absolute Error= Δamean= [Σi=1i=n |Δai|]/n Caution! While calculating absolute mean value, we dont consider the +- sign in its value. 7.2.2 Relative Error or fractional error It is defined as the ratio of mean absolute error to the mean value of the measured quantity δa =mean absolute value/mean value = Δa mean /a m So, in the above example, the relative error would be 1cm/40cm = 0.025. It's commonly written as a percentage, so in this case, the relative error would be (+-) 2.5%.

Business Mathematics Note 195 7.2.3 Percentage Error It is the relative error measured in percentage. So Percentage Error =mean absolute value/mean value X 100= Δa mean /a m X100 An example showing how to calculate all these errors is solved below: The density of a material during a lab test is 1.29, 1.33, 1.34, 1.35, 1.32, 1.36 1.30 and 1.33 So we have 8 different values here so n=8 Mean value of density u= [1.29+1.33+1.34+1.35+1.32+1.36+1.30+1.33] / 8 = 1.3275 = 1.33 (rounded off) Now we have to calculate absolute error for each of these 8 values Δu 1 = 1.33 – 1.29 = 0.04 Δu 2 = 1.33 – 1.33= 0.00 Δu 3 = 1.33 – 1.34= -0.01 Δu 4 = 1.33 – 1.35= -0.02 Δu 5 = 1.33 – 1.32= 0.01 Δu 6 = 1.33 – 1.36= -0.03 Δu 7 = 1.33 – 1.30= 0.3 Δu 8 = 1.33 – 1.33= 0.00 Now remember we don’t take +- signs in calculating Mean absolute value So mean absolute value = [0.04+0.00+0.01+0.02+0.01+0.03+0.03+0.00]/8 = 0.0175 = 0.02 (rounded off) Relative error = +- 0.02/1.33 =+- 0.015 = +- 0.02 Percentage error = +- 0.015*100 = +- 1.5% Self Assessment State whether the following statements are true or false: 7. Absolute error is defined as the magnitude of sum between the actual and the individual values of any quantity in question. 8. Absolute error is an error which is written in terms of the units of the measurement 9. While calculating absolute mean value, we dont consider the +- sign in its value. 10. Relative Error or fractional error is defined as the ratio of mean absolute error to the mean value of the measured quantity 11. Percentage Error is the relative error measured in percentage 7.3 Significant Figures All measurements are approximations—no measuring device can give perfect measurements without experimental uncertainty. By convention, a mass measured to

Note Unit 7: Other Useful Mathematics Devices 196 13.2 g is said to have an absolute uncertainty of plus or minus 0.1 g and is said to have been measured to the nearest 0.1 g. In other words, we are somewhat uncertain about that last digit—it could be a "2"; then again, it could be a "1" or a "3". A mass of 13.20 g indicates an absolute uncertainty of plus or minus 0.01 g. 7.3.1 Meaning A significant figure is that digit that carries meaning, contributing to its precision. This includes all digits except leading zeros where they serve merely as placeholders to indicate the scale of the number and spurious digits that are introduced. The number of significant figures in a result is simply the number of figures that are known with some degree of reliability. Therefore, significant figures of a number are the digits that we can confidently say we know have meaning in an answer, or a measurement. If we measure something in millimeters on a ruler and find it to be a little over 1mm, we can estimate how much over it is, but only the 1 mm is significant. Significant figures or digits of a measured number are the digits that indicate how precise it is. Example: The number 13.2 is said to have 3 significant figures. The number 13.20 is said to have 4 significant figures. Did u know? The purpose of significant figures is to indicate to the reader how accurate the measurement is considered to be. 103 has three significant figures, indicating that all three digits are accurate and precise. 7.3.2 Importance It is an indication of the accuracy of the original measurements and therefore an indication of the accuracy of any derived measurements, and statistics or inferences that may be drawn from the measurements. Note Significant Figures – Rules Significant figures are critical when reporting scientific data because they give the reader an idea of how well you could actually measure/report your data. Before looking at a few examples, let's summarize the rules for significant figures. 1. ALL non-zero numbers (1,2,3,4,5,6,7,8,9) are ALWAYS significant. 2. ALL zeroes between non-zero numbers are ALWAYS significant. 3. ALL zeroes which are SIMULTANEOUSLY to the right of the decimal point AND at the end of the number are ALWAYS significant. 4. ALL zeroes which are to the left of a written decimal point and are in a number <= 10 are ALWAYS significant. A helpful way to check rules 3 and 4 is to write the number in scientific notation. If you can/must get rid of the zeroes, then they are NOT significant.

Business Mathematics Note 197 Examples: How many significant figures are present in the following numbers? Number # Significant Figures Rule(s) 48,923 5 1 3.967 4 1 900.06 5 1,2,4 0.0004 (= 4 E-4) 1 1,4 8.1000 5 1,3 501.040 6 1,2,3,4 3,000,000 (= 3 E+6) 1 1 10.0 (= 1.00 E+1) 3 1,3,4 This gives you some idea of how to determine the number of significant figures in a single number. 7.3.5 Significant Figures - Mathematical Operations Addition and subtraction When adding or subtracting numbers, count the NUMBER OF DECIMAL PLACESto determine the number of significant figures. The answer cannot CONTAIN MORE PLACES AFTER THE DECIMAL POINT THAN THE SMALLEST NUMBER OF DECIMAL PLACES in the numbers being added or subtracted. Example: 23.112233 (6 places after the decimal point) 1.3324 (4 places after the decimal point) + 0.25 (2 places after the decimal point) 24.694633 (on calculator) 24.69 (rounded to 2 places in the answer) There are 4 significant figures in the answer Multiplication and division: When multiplying or dividing numbers, count the number of significant figures. the answer cannot contain more significant figures than the number being multiplied or divided with the least number of significant figures. Example: 23.123123 (8 significant figures) x 1.3344 (5 significant figures) 30.855495 (on calculator) 30.855 (rounded to 5 significant figures)

Note Unit 7: Other Useful Mathematics Devices 198 Self Assessment Fill in the blanks: 12. A significant figure is that digit that carries meaning, contributing to its ……………………… 13. All measurements are ……………………… 14. ……………………… measuring device can give perfect measurements 15. ……………………… of a number are the digits that we can confidently say we know have meaning in an answer, or a measurement 16. The purpose of significant figures is to indicate to the reader how ……………………… the measurement is considered to be. 17. Significant figures are ……………………… when reporting scientific data because they give the reader an idea of how well you could actually measure/report your data 18. When multiplying or dividing numbers, count the number of significant figures. the answer ……………………… contain more significant figures than the number being multiplied or divided with the least number of significant figures. 7.4 Short Processes of Calculation Finding Square of a two-digit number without a calculator and/or multiplication if you have a two digit number (A)that starts with 5 and the second number is n, put 25 at the beginning and put (n) *(n+1) at the end Examples 15*15= 2 25 (2*1=2) 25*25= 6 25 (2*3=6) 35*35= 12 25 (3*4=12) 45*45=202 5 (4*5=20) …….. and so on. Finding cube of a two-digit number We have (a + b) 3 = a 3 + 3a 2 b + 3ab 2 + b 3 For finding the cube of a two-digit number with the tens digit = a and the units digit = b, we make four columns, headed by a 3 , (3a 2 × b), (3a × b 2) and b 3 The rest of the procedure is the same as followed in squaring a number by the column method. We simplify the working as; a 2 × a = a 3 ; a 2 × 3b = 3a 2 b; b 2 × 3a = 3ab 2 ;

Business Mathematics Note 199 b 2 × b = b 3 ; Example Find the value of (29) 3 by the short-cut method.

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Solution: Here, a = 2 and b =9. a 2 × a = a 3 ; a 2 × 3b = 3a 2 × b; b 2 × 3a = 3a × b 2 ; b 2 × b =

b 3 Therefore, (29) 3 = 24389 Example Find the value of (71) 3 by the short-cut method.

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Solution: Here, a = 7 and b = 1 a 2 × a = a 3 ; a 2 × 3b = 3a 2 × b; b 2 × 3a = 3a × b 2 ; b 2 × b =

b 3 Therefore, (71) 3 = 357911 Finding square root of a number without a calculator Example Find Square root of 676 Solution: Method 1 Factorisation 676 = 4 × 169 = $2^2 \times 13^2$ √676 = 2 × 13 = 26 Method 2 By division method 2---|--676| 2 --------------- No. is divided in groups of 2 from right (6 & 76) 2---|--4 ----------------- 6 divide by a no whose square is > 6 or = 6 (so 2) -----|---- ------------- add divisor 2 +2 = 4 -4--|----276 ------------- subtract 6 - 4 = 2 and take next Grp 76 down choose a digit e.g. 1 then 41 × 1 = 1 a no which gives multiplication = 276 or less than 276 46 × 6 = 276 so keep 6 as sub quotient. continued division 46--|----276 --6--| 276 ------------- 52 | --000 -------------- the addition obtained is twice the quotient 26 You answer is 26

Note Unit 7: Other Useful Mathematics Devices 200 Finding cube root of a number The cube root of a number is denoted by ∛ The cube root of a number x is that number whose cube gives x. We denote the cube root of x by ∛x Thus, 3√64 = cube root of 64 = 3∛4 × 4 × 4 = ∛4 3 = 4 For example: (i) Since (2 × 2 × 2) = 8, we have ∛8 = 2 (ii) Since (5 × 5 × 5) = 125, we have ∛125 = 5 Method of finding the cube root of a given number by factorization To find the cube root of a given number, proceed as follows: Step I. Express the given number as the product of primes. Step II. Make groups in triplets of the same prime. Step III. Find the product of primes, choosing one from each triplet. Step IV. This product is the required cube root of the given number. Caution! If the group in triplets of the same prime factors cannot complete, then the exact cube root cannot be found. Example Evaluate the cube root: ∛216 Solution: By prime factorization, we have 216 = 2 × 2 × 2 × 3 × 3 × 3 = (2 × 2 × 2) × (3 × 3 × 3) Therefore, ∛216 = (2 × 3) = 6 Example Find the cube root of (-1000). Solution: We know that ∛-1000 = -∛1000 Resolving 1000 into prime factors, we get

Business Mathematics Note 201 1000 = 2 × 2 × 2 × 5 × 5 × 5 = (2 × 2 × 2) × (5 × 5 × 5) Therefore, ∛1000 = (2 × 5) = 10 Therefore, ∛-1000 = -(∛1000) = -10 7.5 Roots and Reciprocals Expressed as Power The SYMBOL , as we have seen, symbolizes one number, which is the square root of a. By this symbol we mean the cube root of a. It is that number whose third power is a. The reciprocal of a number is found by dividing 1 by the number, for example the reciprocal of 2 is 0.5 and the reciprocal of 0.5 is 2, while the reciprocal of 2/3 is 3/2. Reciprocals are very useful in many areas. For example, because 8 = 2 3 . In this symbol ("cube root of 8"), 3 is called the index of the radical. In general, means a = b n . Equivalently, Read "The nth root of a." For example, -- The sixth root of 64 -- is 2, because 64 is the 6th power of 2. If the index is omitted, as in , the index is understood to be 2. Examples = 11. = 2, because 25 = 32. = 10, because 104 = 10,000.

Note Unit 7: Other Useful Mathematics Devices 202 = –2, because (–2) 5 = –32. We see that, if the index is odd, then the radicand may be negative. But if the index is even, the radicand may not be negative. There is no such real number, for example, as . Fractional exponent What sense can we make of the symbol ? It turns out that we must identify with . = . Why? Because must obey the rules of exponents. And when it does, it obeys the same formal rule that defines , namely ( ) 2 = a. For, according to the power of a power rule: ( ) 2 = · 2 = a 1 = a. Therefore we must identify with . In general, = The denominator of a fractional exponent is equal to the index of the radical. Example 8 means The cube root of 8, which is 2. 81 means The fourth root of 81, which is 3. (–32) means The fifth root of –32, which is –2. 8 is the exponential form of the cube root of 8. is its radical form. Negative exponent A number with a negative exponent is defined to be the reciprocal of that number with a positive exponent. a –v = 1 a v a –v is the reciprocal of a v . Therefore,

Business Mathematics Note 203 1 = 1 = Example Evaluate It is the reciprocal of 16/25 with a positive exponent. So it is the square root of 25/16, which is 5/4, raised to the 3rd power: 125/64. The rules of exponents An exponent may now be any rational number. Rational exponents u, vwill obey the usual rules. a u a v = a u + v Same Base = a u – v (ab) u = a u b u Power of a product (a u) v = a uv Power of a power = Power of a fraction 7.6 A.P. Series and G.P. Series 7.6.1 Arithmetic Progression An arithmetic progression is a sequence of numbers in which each term after the first term is obtained by adding a constant quantity to its previous term. This constant quantity is called common difference. In general, an Arithmetic Progression (A.P.) is given by , , 2 , 3 ....

89%	MATCHING BLOCK 189/248	W
a a d a d a d ? ? ? Here a is the first term and ‘d’ is the common difference.		

To find the common difference of an A.P., subtract any term from its next term. i.e., ( ) or(2) ( )or(3) (2)

90%	MATCHING BLOCK 190/248	W
a d a a d a d a d a d ? ? ? ? ? ? ? ?		

etc.
n

60%	MATCHING BLOCK 194/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)
th Term of an A.P. The n th term of the A.P. , , 2 , 3 ,...... a a d a d			

a d ? ? ?
is given by (1) n T a n d ? ? ? ...(1)
Note Unit 7: Other Useful Mathematics Devices 204 This is also called the last term and is denoted by l. Sum to

75%	MATCHING BLOCK 192/248	W
n Terms of an A.P. The sum to n terms of A.P. , , 2 , 3 ,........, (1)		

31%	MATCHING BLOCK 193/248	W	

a a d a d a d a n d ? ? ? ? ? is ( ) (2) .... [(1)] a a d a d a n d ? ? ? ? ? ? ? ? which is given by ? ? 2 (1) 2 n n S a n d ? ? ? ... (2) Since (1) , l a n d ? ? ? we can write the formula for S n as ? ? (1) 2 n n S a a n d ? ? ? ? ? ? 2 n n S a l ? ? ? ? ? ? ? ? 2 (1) 2 2 n n n S a n d a l ? ? ? ? ? ? where (1) n l T a n d ? ? ? ? .

Example:
Find
the 10th term of

45%	MATCHING BLOCK 195/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

the A.P. 3, 1,1,3,5,...... ? ? Solution: The A.P. is 3, 1,1,3,5,...... ? ? 3and 2, 10 a d n ? ? ? ? ? term (– 1) ? ? th n a n d 10th term 3 (10 1)2 ? ? ? ? 3 18 15 ? ? ? ? Example: If the 5th term of an A.P. is 10 and 8th term is 16, find the first term and the common difference Solution: 5th term

is 10 4 10 a d ? ? ? ...(1) 8th term is 16 7 16 a d ? ? ? ...(2) Let us solve these equations. (2) (1) 3 6 2 d d ? ? ? ? ? Substituting 2 d ? in (1), we get
Business Mathematics Note 205 4(2) 10 a ? ? 10 8 2 a a ? ? ? ? ? Therefore, first term is 2 and the common difference is 2.
Example: If

41%	MATCHING BLOCK 196/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

the 4th term of an A.P. is 7 and the 7th term is 13, find the 12th term. Solution: 4 term 5 3 7 th a d ? ? ? ? ...(1) 7 term 13 6 13 th a d ? ? ? ? ...(2) Let us solve these equations. (2) (1) 3 6 d ? ? ? 2 d? ? Substituting 2 d ? in (1), we get 3(2) 7 7 6 1 a a ? ? ? ? ? ? 1, 2

a d ? ? 12th term 11 1 11(2) 1 22 23 a d ? ? ? ? ? ? ? ? 12th term is 23. Example:

57%	MATCHING BLOCK 197/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

Find the common difference, nth term and 15th term of the A.P. 3, 5, 13,...... ? ? Solution: 3, 5, 13,...... ? ? is an A.P. 3, 5 3 8 a d ? ? ? ? ? ? ? commondifference 8 ? ? ? thterm (1) 3 (1)(8) n a n d n ? ? ? ? ? ? ? 3 8 8 n ? ? ?

thterm 11 8 n n ? ? ? 15thterm 11 8(15) 11 120 109 ? ? ? ? ? ? ?
Note Unit 7: Other Useful Mathematics Devices 206 Example: Find the 17

38%	MATCHING BLOCK 208/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

th term of the series 3, 6, 9, 12,...... Solution: 3, 3, 17 a d n ? ? ? 17thterm 3 (17 1)3 3 16(3) 51 ? ? ? ? ? ? ? Example: Which term of the A.P. 1,3,7,...... ? is 79 ? Solution: 1,3,7,...... ? is the given A.P. 1, 4 thterm (1) 79 1 (1)4 79 1 4 4 79 4 5 79 4 84 21 ? a d n a n d n n n n n ? 21st term of the

given A.P. is 79. Example: Determine A.P. whose nth term is 5 4 n ? . Solution: nth term 5 4 n ? ? 1stterm 5(1) 4 9 2ndterm 5(2) 4 14 3rdterm 5(3) 4 19 ? ? ? ? ? ? ? ? ? ? ? A.P. is 9,14,19,..... Example: How many numbers are there between 12 and 108 which are divisible by 5 ? Solution: The first and the las numbers divisible by 5 between 12 and 108 are 15 and 105. Business Mathematics Note 207 15, 105 (1) 105 15 (1)5 (1)5 105 15 90 90 1 18 5 18 1 19 a l l a n d n n n n ? Hence there are 19 terms between 12 and 108 which are divisible by 5. Example: Determine the A.P. whose sum to n terms
is

54%	MATCHING BLOCK 198/248	W

n 2 . Solution: 2 n S n? 2 1 1 2 2 2 2 (1) (1) 2 1 n n n n S n T S S n n n n n ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2 3 2 1 2(1) 1 1 2(2) 1 3 2(3) 1 5 etc., n T n

T T T ? ? ? ? ? ? ? ? ? ? ? ? ? ? A.P. is 1, 3, 5, ... Example: Determine the A.P. whose nth term is 3 5 n ? . Find the 15th term and sum to 15 terms. Solution: 3 5 (given) n T n ? ? 1 2 3 3(1) 5 2 3(2) 5 1 3(3) 5 4 etc., ? ? ? ? ? ? ? ? ? ? ? ? T T T ? A.P. is 2,1,4,..... ? Here 2, 3 a d ? ? ? . thterm (1) 15thterm 2 (15 1)3 2 42 40 ? ? ? ? ? ? ? ? ? ? ? ? n a n d
Note Unit 7: Other Useful Mathematics Devices 208 ? ? ? ? 15 2 (1) 2 15 2(2) (15 1)3 2 n n S a n d S ? ? ? ? ? ? ? ? ? ? 15 4 42 2 ? ? ? 15 38 15 19 285 2 ? ? ? ? ? ? ? Sum to 15 terms is 285. Example: Find the sum to n terms of the A.P. 1,2,5,..... ? and hence find the sum to 10 terms. Solution: The A.P. is 1,2,5,..... ? 1, 5 2 2 (1) 3 a d ? ? ? ? ? ? ? ? ? Sumto terms

70%	MATCHING BLOCK 199/248	W

n ? ? ? ? ? ? ? ? 2 (1) 2 2(1) (1)3 2 2 3 3 2 (3 5) 2 n n S a n d n n n n n n ? ? ? ? ? ? ? ? ? ? ? ? put 10 n ? 10 10(30 5) 2 10 25 5 25 125 2

S ? ? ? ? ? ? ? ? Example: How many terms of the A.P. 42,36,30,..... amount to 150 ? Solution: It is given that 150 n S ? . We have to find n.
Business Mathematics Note 209 ? ? ? ? ? ? 150 2 (1) 150 2 Here 42, 6 2(42) (1)(6) 150 2 84 6 6 150 2

68%	MATCHING BLOCK 200/248	W

n S n a n d a d n n n n ? 90 6 150 2 n n ? ? ? 2 2 2 2 2(45 3) 150 2 (45 3) 150 45 3 150 15 50 (dividingby3) 15 50 0 10 5 50 0 (10) 5(10) 0 (10)(5) 0 10, 5 n n n n n n n n n n n n n n n n n n n n n ?

either 10 terms or 5 terms of the A.P. amount to 150. Verification: 42 36 30 24 18 12 6 0 (6) (12) ? ? ? ? ? ? ? ? ? ? ? Sum of 10 terms =150 Sum of 5 terms =150 Example: If 5, 52 and 470 ? ? ? ? n a l S , find n and d. Solution: ? ? (1) and 2 (1) 2 n

54%	MATCHING BLOCK 201/248	W

n l a n d S a n d ? ? ? ? ? ? ? ? 52 5 (1) and 470 2(5) (1) 2 n n d n d ? ? ? ? ? ? ? ? ? ? ? 57 (1) and 470 10 (1) 2 n n d n d ? ? ? ? ? ? ? Note

Unit 7: Other Useful Mathematics Devices 210 ? ? ? ? 470 10 (1) 2 470 10 57 2 470 (47) 2 2 470 2 10 20 47 n n d n n n ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Now, 57 (1) 57 (20 1) 57 19 n d d d ? ? ? ? ? ? ? 57 3 19 d ? ? ? Therefore 20and 3. ? ? n d
Example:

40%	MATCHING BLOCK 202/248	W

The sum of 3 numbers in A.P. is −24 and then product is 288. Find the numbers. Solution: Let , , a d a a d ? ? be the 3 numbers in A.P. Sum 24 3 24 8 a d a a d a a ? ? ? ? ? ? ? ? ? ? ? ? ? ? Product ( ) ( ) 288 a d a a d ? ? ? ? substituting 8 a ? ? , we get 2 2 2 (8)(8)(8) 288 (64) 36 64 36 100 10 d d d d d d ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? the numbers are 8 10, 8, 8 10

or 8 10, 8, 8 10 ? ? ? ? ? ? ? ? ? ? i.e., 18, 8,2or2, 8, 18 ? ? ? ? Example: Find the sum of the first 20 positive integers.
Business Mathematics Note 211 Solution: The first 20 positive integers are 1, 2, 3, 4,.........20. Here 1, 1, 20 a d l ? ? ? ? ? ? ? 20 2 20 1 20 2 10(21) 210 n n S a l S ? ? ? ? ? ? ? ? ? the sum of first 20 positive integers is 210. Example: The sum of the first eleven terms of an A.P. is 1 82 2 . If the 15th term is 12, find the A.P. Solution: ? ? 2 (1) , (1) 2 n
n n

57%	MATCHING BLOCK 203/248	W

S a n d T a n d ? ? ? ? ? ? ? ? ? 11 1 82 2 11 165 2 (11 1) 2 2 11(2 10) 165 S a d a d ? ? ? ? ? ? ? ? ? ? 2 10 15 ? ? a d ...(1) 15 12 ? T ? (15 1) 12 ? ? ? a d ? 14 12 ? ? a d ...(2) ? 2 28 24 ? ? a d (multiplying by (2)) 2 10 15 2 28 24 ? ? ? ? a d a

d
Subtracting, we get 18 9 1 2 d d ? ? ? Substituting 1 2 d ?
in (1), we get
Note Unit 7: Other Useful Mathematics Devices 212 1 2 10 15 2 i.e., 2 15 5 2 10 5 a a a a ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? the A.P. is 1 1 5,5 ,6,6 ,....... 2 2 Example: The ratio of 7th term to 3rd term of an

40%	MATCHING BLOCK 204/248	W

A.P. is 12:5. Find the ratio of the 13th term to the 4th term. Solution: Let the A.P. be , , 2 ,..... a a d a d ? ? 7thterm 6 12 3rdterm 2 5 ? ? ? ? a d a d 5 30 12 24 a d a d ? ? ? ? 7 6 a d ? ? ...(1) 13thterm 12 4thterm 3 7(12) (multiplybothnumeratoranddenominatorby7) 7(3) 7 84 = 7 21 6 84 [using (1)] 6 21 90 10 27 3 ? ? ? ? ? ? ? ? ? ? ? ? ? ? a d a d a d a d a d a d d d d d d d Therefore the 13

th term : 4
th term is 10:3 Example: The sum of the first four terms of an

35%	MATCHING BLOCK 205/248	W

A.P. is 16 and the sum of their squares is 84. Find the numbers. Solution: Let 3 , , , 3 a d a d a d a d ? ? ? ? be the four terms of an A.P. um 3 3 16 4 16 4 S a d a d a d a d a a ? ? ? ? ? ? ? ? ? ? ? ? ? ? Business Mathematics Note 213 2 2 2 2 2 2 2 2 2 2 2 2 2 Sum of their squares (3) ( ) ( ) (3) (4 3) (4) (4) (4 3) 16 24 9 16 8 16 8 16 24 9 64 20 a d a d a d a d d d d d d d

d d

27%	MATCHING BLOCK 206/248	W

d d d d d ? 2 2 2 64 20 84 20 84 64 20 1 1 d d d d ? ? ? ? ? ? ? ? ? ? ? ? Therefore, the numbers are 4 3,4 1,4 1,4 3 or4 3,4 1,4 1,4 3 i.e.,1,3,5,7or7,5,3,1 ? ? ? ? ? ? ? ? Example: If the 5th term of an A.P. exceeds the 2nd term by 12 and 15th term is 28, find the

A.P. Solution: 5thterm 4 ,2ndterm ? ? ? ?

21%	MATCHING BLOCK 207/248	W

a d a d (4) ( ) 12 3 12 4 a d a d d d ? ? ? ? ? ? ? ? ? 15thterm 14 28 14 4 28 28 56 28 ? ? ? ? ? ? ? ? ? ? ? a d a a a ? The A.P. is 28, 24, 16, 12, 8, 4,0,4,8,..... ? ? ? ? ? ? Example If the nth terms of the A.Ps 3, 10, 17, ... and 63, 65, 67,.... are equal, find the value of n. Solution: 3, 10, 17,....... 3, 7 a d ? ? thterm (1) 3 (1)7 7 4 ? ? ? ? ? ? ? ? ? n a n d n n 63, 65, 67......... 63, 2 a d ? ? thterm (1) 63 (1)2 2 61 7 4 2 61 5 65 13 n a n d n n n

n n n ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?

Note Unit 7: Other Useful Mathematics Devices 214 Example: The first year, a man saves $ 100. In each succeeding year, he saves $ 25 more than the year before. How much money would be accumulated at the end of 20 years? Solution: It is given that 100, 25, 20. a d n ? ? ? ? ? ? ? ? ? ? ? ? 2 1 2 20 2 100 20 1 25 2 10 200 475 10 675 6750 n n S a n d ? Therefore, he would have accumulated $ 6750 at the end of 20 years. Example A man is employed in a firm on a pay of $ 350 per month with an annual increment of $ 15. What will be his pay during 10th year? What are his total earnings during the 10 years? Solution: It is given that 350, 15, 10. a d n ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 1 350 10 1 15 350 135 485 2 1 2 10 2 350 10 1 15 2 5 700 135 5 835 4175 n n T a n d n S a n d ? Therefore, his pay during the 10th year will be $ 485 per month. His total earnings during 10 years is $ 4175x12=$ 50100. Example: A car purchased for $ 10,000 depreciates in value 10% every year. Find its value at the end of 5 years. Solution: At the end of year 1 =10000−1000=9000

Business Mathematics Note 215 At the end of year 2 =9000−900=8100 At the end of year 3 =8100−810=7290 etc., 9000,commonratio 1 1 0.1 0.9and 5 a r n ? ? ? ? ? ? ? ? ? the value of the car at the end of 5 years is given by 1n n T ar ? ? ? ? 5 1 5 4 4 4 4 9000(0.9) 9000 0.9 9 9000 10 9000 9 9 9 10000 10 59049 5904.9 10 T ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Therefore, the value of the car at the end of 5 years is $ 5904.90. Example A ball rebounds 2 3 the distance it falls. It is dropped from a height of 5 meters. How far does it travel before coming to rest? Solution: 2 5, 3 a r ? ? 1 5 5 3 15 15 2 3 2 1 1 3 a S r S ? ? ? ? ? ? ? ? ? ? ? ? ? Therefore, ball travels 15 meters before coming to rest. 7.6.2 Geometric progression A geometric progression is a sequence of numbers in which each term after the first term is obtained by multiplying its previous term by a constant quantity called common ratio. In general, a Geometric Progression is given by 2 3 , , , ,...... a ar ar ar Here a is the first term and 'r' is the common ratio. To find the common ratio of a G.P. divide any term from its previous term,

Note Unit 7: Other Useful Mathematics Devices 216 i.e., 2 3 2 or or ar ar ar r ar ar etc. nth Term of a G.P. The nth term of the G.P. 2 3 , , , ,....... a ar ar ar is given by 1n n T

58%	MATCHING BLOCK 209/248	W

ar ? ? ...(4) Sum to 'n' Terms of a G.P. The sum to 'n' terms of the G.P. 2 3 1 , , , ,....

76%	MATCHING BLOCK 210/248	W

n a ar ar ar ar ? is 2 3 1 ..... n a ar ar ar ar ? ? ? ? ? ?

which is given by (1) 1 n n a r S r ? ? ? where | | 1 r ? ....(5) If | | 1 r ? then (1) 1 n n a r S r ? ? ? ....(6) Sum to Infinity of a G.P. If | | 1 r ? then sum to infinity of the G.P. 2 3 , , , ,...... a ar ar ar is given by 1 a S r ? ? ? ....(7) Worked Examples on G.P. Example:

28%	MATCHING BLOCK 211/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

Find the 10th term of the G.P. 1, 2, 4, 8, ... Solution: The G.P. is 1, 2, 4, 8,.... 1 10 1 9 1, 2 thterm 10thterm 1(2) 2 512 ? ? ? ? ? ? ? ? ? n a r n ar Example: If the 5th term of a G.P. is 1 32 and 8th term is 1 256 , find the first term and the common ratio.

Solution: 5th term of a G.P. 5 1 4 ar ar ? ? ?

Business Mathematics Note 217 4 1 32 ar ? ? ...(1) 8th term 8 1 7 ? ? ? ar ar 7 1 256 ar ? ? ...(2) ? Dividing (2) by (1) we get 7 4 1 256 1 32 ar ar ? 3 32 1 256 8

71%	MATCHING BLOCK 212/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

r ? ? ? 1 2 r ? ? Substituting 1 2 r ? in (1), we get 4 1 1 2 32 1 1 32 1 2 16 a a ? ? ? ? ? ? ? ? ? ? ? first term is 1 2 and the common ratio

is 1 2 . Example If the 3rd term of a G.P. is 8 and the 6th term is 64, find the 10th term. Solution: 2 3rd term 8 ar ? ? ...(1) 5 6thterm 64 ? ? ar ...(2) Dividing (2) by (1), we get 5 2 3 64 8 8 2 ar ar r r ? ? ? ? ?

Note Unit 7: Other Useful Mathematics Devices 218 Substituting 2 r ? in (1), we get 2 9 9 10 (2) 8 4 8 2 10thterm 2(2) 2 1024 ? ? ? ? ? ? ? ? ? ? a a a ar Example Find the common ratio, nth term and 10th term of the G.P., 3, 9, 27, 81,...... ? ? Solution: 3, 9,27, 81,...... ? ? is the G.P. 9 27 81 3, 3 3 9 27 a r ? ? ? ? ? ? ? ? ? ? common ratio 3 ? ? ? 1 1 1 1 1 1 thterm 3(3) (1) 3 (1) 3 ? ? ? ? ? ? ? ? ? ? ? ? ? ? n n n n n n n ar 10 1 10 9 10 10 10thterm (1) 3 (1) 3 3 59049 ? ? ? ? ? ? ? ? ? ? 59049 ? ? Example Find the 6th term of the G.P. 5,15,45,...... Solution: The G.P. is 5,15,45,...... 1 6 1 5 15 45 5, 3 5 15 nthterm 6thterm 5(3) 5 3 5 243 1215 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? n a r ar Example Which term of the G.P. 3,12,48,..... is 768? Solution: 3,12,48,..... is the given G.P.

Business Mathematics Note 219 12 48 3, 4 3 12 a r ? ? ? ? ? 1 1 1 1 1 4 thterm 3(4) 768 768 4 3 4 256 4 4 1 4 5 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? n n n n n n ar n n ? 5th term is 768 Example Determine the G.P. whose nth term is given by 1 3 5 n? ? . Solution: 1 3 5 n n T ? ? ? 1 1 0 1 2 1 2 3 1 2 3 3 5 3 5 3 3 5 3 5 15 3 5 3 5 75 etc., T T T ? the G.P. is 3, 15, 75,........ Example If 5, 160and 315, n a l S ? ? ? ? ? ? find
n and r. Solution: 1 (1) , 1 n n n a r l ar S r ? ? ? ? ? 1 1 1 5(1) 160 5 , 315 1 1 32,63 1 1 63 1 (32) 1 63 1 63(1) 32 1 63 63 32 1 63 32 63 1 31 62 2 n n

79%	MATCHING BLOCK 213/248	SA	Maths Extended Essay.pdf (D31546002)

n n n r r r r r r r r r r r r r r r r r r ?

Note Unit 7: Other Useful Mathematics Devices 220 Substituting 2 r ? in 1 32, n r ? ? we get 1 5 2 2 1 5 6 6and 2 n n n n r ? ? ? ? ? ? ? ? ? ? Example The sum of three numbers in a G.P. is 63 4 and their product is 27, find the numbers. Solution: Let , , a a ar r be the 3 numbers in G.P. 63 4 a a ar r ? ? ? ? ...(1) 27 a a ar r ? ? ? ...(2) 3 (2) 27 3 a a ? ? ? ? Substituting 3 a ? in (1), we get 3 63 3 3 4

61%	MATCHING BLOCK 214/248	SA	Maths Extended Essay.pdf (D31546002)

r r ? ? ? Multiply by 4r 2 2 2 2 2 12 12 12 63 12 12 63 12 0 12 51 12 0 4 17 4 0 4 16 4 0 4 (4) 1(4) 0 (4)(4 1) 0 1 4or 4 r r r r r r r r r r r r r r r r r r ?

Therefore, the numbers are

Business Mathematics Note 221 3 3 ,3,3 4or3 4,3, 4 4 3 3 i.e., ,3,12or12,3, 4 4 ? ? Example Find the sum of 1, 2, 4, 8,.... up to 10 terms. Solution: 1, 2, 4, 8,..... is the G.P. Here 2 4 8 1, 2 1 2 4 a r ? ? ? ? ? 10 10 (1) 1 1(2 1) 2 1 2 1 2 1 1024 1 1023 n n n n a r S r S ? ? ? ? ? ? ? ? ? ? ? ? ? ? Example:

78%	MATCHING BLOCK 215/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

The sum of three numbers in G.P. is 42 5 and their product is −8. Find the numbers.

Solution: Let , , a a ar r be the three numbers in G.P. 42 5 a a ar r ? ? ? ? ...(1) 8 a a ar r ? ? ? ? ...(2) 3 (2) 8 2 a a ? ? ? ? ? ? Substituting 2 a ? ? in (1), we get 2 42 2 2 5 r r ? ? ? ? Multiply by 5r

Note Unit 7: Other Useful Mathematics Devices 222 2 2 2 2 2 10 10 10 42 10 10 42 10 0 10 52 10 0 5 26 5 0 5 25 5 0 5 (5) 1(5) 0 (5)(5 1) 0 1 5, 5

77%	MATCHING BLOCK 216/248	SA	Maths Extended Essay.pdf (D31546002)

r r r r r r r r r r r r r r r r r r r ?

the numbers are 2 2 , 2,(2)(5)or(2)(5), 2, 5 5 ? ? ? ? ? ? ? ? ? ? i.e., 2 2 , 2,10or10, 2, 5 5 ? ? Example: Find the three numbers in G.P. whose product is 64 and the sum of their squares is 84. Solution: Let , , a a ar r be the three numbers in G.P. 64 a a ar r ? ? ? ? ...(1) 2 2 2 2 2 84 a a a r r ? ? ? ...(2) 3 (1) 64 4 a a ? ? ? ? Substituting 4 a ? in (2), we get 2 2 16 16 16 84 r r ? ? ? Multiply by 2 r , we get 2 4 2 4 2 2 4 2 4 2 16 16 16 84 i.e., 16 16 84 16 0 i.e.,16 68 16 0 i.e.,4 17 4 0

69%	MATCHING BLOCK 217/248	SA	Maths Extended Essay.pdf (D31546002)

r r r r r r r r r ? ? ? ? ? ? ? ? ? ? ? ? ? 4 2 2 2 2 2 2 2 2 2 i.e.,4 16 4 0 4 (4) 1(4) 0 4 0,4 1 0 1 4, 4 1 2, 2 r r r r r r r r r r r r ?

if 1 2 or , 2 r ? the numbers are
Business Mathematics Note 223 4 4 ,4,4(2)or4(2),4, i.e., 2,4,8 or 8,4,2 2 2 and if 1 2 or , 2 r ? ? ? the numbers are 4 4 , 4,4(2) or 4(2), 4, , . ., 2, 4, 8 or 8, 4 2 2 2 i e ? ? ? ? ? ? ? ? ? Example How many terms of the series 3, 6, 12, 24,...... must be taken to make the sum 381? Solution: 3, 6, 12, 24,..... is the given A.P. Here 3, 2 a r ? ? (1) 1 3(2 1) 381 2 1 381 3 2 3 n n n n a r S r ? ? ? ? ? ? ? ? ? Dividing by 3, we get 7 127 2 1 2 127 1 128 2 2 7 n n n n ? ? ? ? ? ? ? ? ? ? ? 7 terms must be taken. Example: Find the common ratio, nth term, sum to n terms, 12th term and sum to 12 terms of the G.P. 1 1 1 1 , , , ,..... 3 6 12 24 Solution: The given G.P. is 1 1 1 1 , , , ,..... 3 6 12 24 Here 1 1/6 1 , 3 1/3 2 a r ? ? ? ? the common ratio is 1 2 nth term 1n ar ? ?
Note Unit 7: Other Useful Mathematics Devices 224 1 1 1 1 1 i.e., 3 2 3 2

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n n n T ? ? ? ? ? ? ? ? ? ? ? ...(1) Sum to n terms (1) 1 n a r r ? ? ? 1 1 1 3 2 1 1 2 n ? ? ? ? ? ? ? ? ? 1 2 1 2 3 2 n n ? ? ? ? ? ? ? ? ? 1 1 2 1 i.e., 3 2 ? ? ? ? ? ? ? ? ? ? n n n

S ...(2) Substituting 12 n ? in (1), we get 12 12 1 11 1 1 1 1 3 2048 6144 3 2 3 2 T ? ? ? ? ? ? ? ? ? Substituting 12 n ? in (2), we get ? ? 12 12 12 1 11 4096 1 1 2 1 1 3 3 2 2 1 4095 1365 3 2048 2048 S ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Example The first term of a G.P. is 10 and the fourth term is 640. Find the common ratio and the sum of the first four terms. Solution: It is given that 4 10and 640 a T ? ? 4 3 3 3 640 640 10 640 64 4 T ar r r r ? ? ? ? ? ? ? ? ? ? common ratio is 4. 4 4 4 (1) 1 10(4 1) 10 (256 1) 10 255 850 4 1 3 3 a r S r ? ? ? ? ? ? ? ? ? ? ? ?
Business Mathematics Note 225 Example: Find the sum to n terms of the series 3 33 333 .... ? ? ? Solution: 3 33 333 ...... n S ? ? ? ? 3(1 11 111 .....) ? ? ? ? Dividing and multiplying by 9, we get ? ? ? ? ? ? ? ? ? ? ? ? 3 9 99 999 ..... 9 1 10 1 100 1 1000 1 .... 3 1 10 100 1000 ...... 1 1 1 ..... 3 n S ? Now, 10 100 1000..... ? ? to n terms is a G.P. with first term = 10 and common ratio = 10. i.e., 10, 10 ? ? a r ? ? n 10 (10 - 1) their sum 10.1 1 10(10 1) 3 10 1 1 10(10 1) 3 9
n n n S n n ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Example
Find the
value
of

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x if 1, 2, 1 x x x ? ? ? are in G.P. Solution: 1, 2, 1 x x x ? ? ? are in G.P. 2 1 1 2 x x x x ? ? ? ? ? ? By cross multiplication, we get 2 (2) (1)(1) x x x ? ? ? ? ? 2 2 i.e., 4 4 1 4 5 4 x x x x x ? ? ? ? ? ? ? ? ?

Note Unit 7: Other Useful Mathematics Devices 226 Example: Find the sum to infinity of the G.P. 1 1 1 1, , , ,...... 2 4 8 Solution: 1 1 1 1, , , ,...... 2 4 8 is the given G.P. Here 1 1, 2 a r ? ? 1 1 1 2 1 1 1 2 2 a S r ? ? ? ? ? ? ? Example: Three numbers are in G.P.

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The sum of the first two terms is 15 and the sum of the

second and the third is 30. Find the numbers. Solution: Let 2 , , a ar ar be the three numbers in G.P. 15 a ar ? ? ? ...(1) 2 30 a ar ? ? ...(2) (2) ( ) 30 (15) 30 2 ? ? ? ? ? ? ? ? r a ar r r (using (1)) Substituting 2 r ? in (1), we get 2 15 3 15 5 a a a a ? ? ? ? ? ? ? the numbers are 2 5,5 2,5 2 ? ? i.e., 5,10,20 Example: If a, b, c, d, e are in G.P. Prove that ae bd ? . Solution: a, b, c, d, e are in G.P.
Business Mathematics Note 227 b c d e a b c d b e a d ? ? ? ? ? ? Cross multiplying, we get ae bd ? Example: If three numbers are in G.P., prove that their logarithms are in A.P. Solution: Let a, b, c be three numbers in G.P. 2 (commonratio) (bycrossmultiplication) b c a b b ac ? ? ? ? Taking logarithms on both sides, we get 2

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log log( ) 2log log log log log log log log log log log

b
ac b a c b b a c b a c b ? ? ? ? ? ? ? ? ? ? ? ? These are common differences which are equal. ? log ,log ,log a b c are in A.P. 7.6.3 Arithmetic – Geometric Series A series whose terms are obtained by multiplying together the corresponding terms of A.P. and G.P. is called an arithmetic -geometric series. The standard form of an arithmetic - geometric series is ? ? ? ? ? ? ? ? ? ? ? 2 3 n 1 a a d r a 2d r a 3d r .... a n 1 d r .... ? ? ? ? ? ? ? ? ? ? ? E.g. Consider the series 2 3 2 6 10 14 .... x x x ? ? ? ? (i) The terms 2 3 2 ,6 ,10 14 .... x x x x ? of the above series are obtained by multiplying corresponding terms of the A.P. 2, 6, 10, 14, ... and the G.P. 2 3 1, , , , x x x .... The series (i) is an arithmetic – geometric series.
Note Unit 7: Other Useful Mathematics Devices 228 The sum of n terms of the arithmetic - Geometric Series Consider the standard arithmetic - geometric Series: ? ? ? ? ? ? ? ? ? ? ? 2 3 1 2 3 .... 1 ....
n a a d r a d r a d r a n d r ? ? ? ? ? ? ? ? ? ? ? ? (1)

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Let S n be the sum of n terms of the

given series then ? 2 n 1 n rS ar a d
r .... a n 2 d r a n 1 d
r n ....(2) Subtracting (2) from (1) , we get ? 2 1 2 2 1 1 2 1 .... 1 1 .... 1 1 1 1 1 1 1 1 1

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n n n n n n n n n n r S a dr dr dr a n d r a dr r r r a n d r dr r a a n d r r dr r a n d r a S r r r ?? ? ? n 1 n n Cor : If r 1Then r ,r

and nr all tend to zero as n a dr lim S . n 2 n 1 r 1 r
Thus, when |r|> 1 the sum of the infinite series ? ? ? ? ? ? 2 3 2 3 .... a a d r a d r a d r ? ? ? ? ? ? ? ? is given by ? ? 2 1 1 a dr S r r ? ? ? ? 7.6.4 Sum of First N Natural Numbers Let S be the sum of first n natural numbers 1, 2, 3, ...n then S= 1+ 2+ 3+ ..... upto n terms. This is an A.P. with first term 1 and common difference 1. ?

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n S 2a n 1 d 2 n 2.1 n 1 1 2 n n 1 S 2 n n 1 is n 2 7.6.5 The Sum of The Squares of First N Natural Numbers

Let

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the sum of the squares of n natural numbers be S.

Then
Business Mathematics Note 229 S=1 2 + 2 2 + 3 2 + 4 2 +.......... + n 2 Now since ? 3 3 2 3 3 2 3 3 2 3 3 2 3 3

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n n 1 3n 3n 1 Putting n 1,2,3,............, n 1 ,n ,we have 1 0 3.1 3.1 1 2 1 3.2 3.2 1 3 2 3.3 3.3 1 .... n 1 n 2 3 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 3 3 2 . n 1 3 n 1 1 n n 1 3n 3n 1 Adding these, we have ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? ? ? 3 2 2 2 2 3 3 1 2 3 .... 3 1 2 3 .... 1 3 3. 2 3 1 or 3 2 3 1 1 1 2 3 1 1 2 n n n n n n S n n n S n n n n n n n n n n ?? ? ? 2 1 1 2 1 2 1 6 n n n n n n S ? ? ? ? ? ? ?

Self Assessment Fill in the blanks: 19. (1,2, 3,4) is a .... of no. 20. A sequence of nos. is a set of nos. arranged in .... order. 21. An arithmetic progression is a sequence of numbers in which each term after the first term is obtained by adding a constant quantity to its .... term. 22. The constant quantity is called .... 23. 5,10,12,15,18,20 are .... in sequence 24 G.P Stands for .... 25. A geometric progression is a sequence of numbers in which .... 26. Common ratio is represented by ....
Note Unit 7: Other Useful Mathematics Devices 230 27. If three numbers a, b and c are in G.P log ,log ,log a b c are in .... Task Design an activity to check whether the given sequence is an AP or not by paper cutting and pasting. 7.7 Inequalities An inequality is a statement how the relative size or order of two objects, or about whether they are the same or not. Examples a>b a is less than b. a<b a is greater than b. a=b a is equal to b. If we add or subtract the same number to each side of an inequality then we don not change the truth of the inequality. Example Solve the inequality for a a+b<c We subtract b from both sides: a+b-b<c-b a<c-b If we multiply or divide by a negative number than we must reverse the order of the inequality If c is positive (and not zero) and a > b, then ac > bc and a/c > b/c If c is negative (and not zero) and a > b, then ac < bc and a/c < b/c Solution of linear equation by graph is known. Now we shall proceed to solve linear equations. The general form of inequations of two variables are ax + by ≥ 0, ax + by < c, ax – by > 0, or ax + by ≤ 0. Whose a, b, c are real numbers. Note An ordered pair (x1, y1) will be a solution of the inequation ax + by ≥ c if ax1 + by1 ≥ c holds. The set of all such solutions is known as solution set. Now plotting of such ordered pairs (by usual process in plain or graph paper) is called the graph of the given inequation. 7.7.1 Method of Drawing Let the equation be ax + by ≥ c (or ax + by < c) : (i) Replace the sign ≥ (or <) by equality (i.e, take ax + by = c) Business Mathematics Note 231 (ii) Draw the graph of ax + by = c, which will be a straight line. (iii) For the sign ≥ (or ?), the points on the line are included, and a thick line should be drawn. (iv) The line drawn divides the XOY plant in two regions (or points). Now to identify which region satisfies the inequation, plot any point. If this point satisfies the inequation, then the region containing the plotted point will be the desired region. If, however, that point does not satisfy the inequation, the other region will be the desired result. Now shade the relevant region. Example In the XOY plane draw the graph of x^{3} – 3. In the inequation x^{3} – 3, there is no y. at first draw the graph of x = – 3 which is a straight line parallel to y axis passing through the point x = – 3. Figure 7.1: Graph of x 3 – 3 Put the coordinates (0, 0) in x ? – 3, we find 0 ? ?– 3 which is true. So the region containing (0, 0) is the required region. The points on the line are included and the line should be thick. Example: Sketch the graphs of the linear equations x – 2y + 11 = 0 and 2x – 3y + 18 = 0 Indicate in the graph : (i) The solution set of the system of equations, and (ii) The solution set of the system of inequations
Note Unit 7: Other Useful Mathematics Devices 232 The two graphs of the equations x – 2y + 11 = 0 and 2x – 3y + 18 = 0 intersect at P (–3, 4). So the solution is (–3, 4). Now putting the Co-ordinates of origin (0, 0) in inequations we find both are true. Hence both the graphs are on the origin side. Figure 7.2: Graph of intersecting Lines All the points are on the lines; so thick lines are used. The cross area indicates the solution set of the inequations. 7.8 Concept of 'Function' A function is a technical term used to symbolise relationship between variables. A function can be expressed by formula, graph or by equation. Therefore, A function f from a set X (domain) to a set Y, a subset of X ? Y, is a rule that associates a unique element of set Y (target) to each element in X. The unique element y in Y corresponding to an element x in X is denoted as f(x) and is called the image of x. Alternatively, we write a functional relationship by writing f: X ??Y. We sometimes also use the words transformation or map or mapping instead of function. Diagrammatically, a functional relationship can be represented as shown in Figure 7.3.
Business Mathematics Note 233 Figure 7.3: Functional Relationship Note One to One Mapping If different elements of the set X have different images in the set Y, the mapping is termed as one to one mapping. On to Mapping If every element of the set Y be an image of the set X and no element of Y remains unused, the mapping is termed as on to mapping. The set {f(x) : x?X} is called the range of the function. It is the set of all possible values of f(x) in Y. When we write a function as y = f(x), x is termed as the independent variable or argument or input of the function and y is termed as the dependent variable or output. A function can be described either (i) by a set of ordered pairs, or (ii) by one or more algebraic formulae or (iii) in words. 1. Using a set of ordered pairs: We can write a function as the set of ordered pairs ? ? ? ? ? ? ? ? n n y x y x y x S , ...,,,, , 2 2 1 1 ? , which denotes a function from the set ? ? n x x x X ...., ,, 2 1 ? to the set ? ? n y y y Y .....,,, 2 1 ? . Here first element of each pair belongs to domain and the second element belongs to the range of the function. Alternatively the set of ordered pairs, can also be presented in the form of a table. 2. Using One or More Formulae: A function can also be described by using one or more formulae. For example, ? ? , 2 c bx ax xf y ? ? ? ? where x ? R and y ? R are its domain and range respectively. Similarly we can write a function by using more than one algebraic formulae, e.g. 3 2 ? ? x y when x > 5 = 100 3 5 2 ? ? x x when 16 5 ? ? x = 4 when 16?x 3. Using words: A function can also be expressed in words. Let X be the set of students in a college and Y the set of colonies in a city. We have, of course, to assume here that every student has a unique name. Figure 7.4: Functional Relationship Fig. 3.8

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y ? o x ? x y x 1 X 2 1 1 , y x 1 2 , y x x y X Y

f

Note Unit 7: Other Useful Mathematics Devices 234 From the definition of a function, we should note that a function is also a relation but a relation may or may not be a function. The relation given in example (i) in is also a function. As another example, consider the subset given by {(x, y) : y = x 2 }. We note that corresponding to each value of x we can associate only one value of y. Thus y = x 2 is a function. Another important point to be noted is that although the definition of a function requires that there exists only one value of y corresponding to a given value of x, the converse of this need not be true i.e. it is possible to associate more than one value of x to a single value of y. This type of situation is shown in Fig. 3.7. Some Examples of Functions from Economics (i) Demand function, written as x = D(p) where p, the price, is an independent variable and x, the quantity demanded, is a dependent variable. (ii) Cost function, written as C = F(x) where x, the level of output, is an independent variable and C, the cost of production, is a dependent variable. (iii) Total revenue function, written as R = f(x) where x, the level of output, is an independent variable and R, the total revenue, is a dependent variable. (iv) Consumption function C = f(Y), where Y, the level of national income, is an independent variable and C, the level of national consumption, is a dependent variable. 7.8.1 Classification of Functions Functions can be classified into different categories according to the nature of their definition or of symbolic expressions. To facilitate this, we first define a few technical terms relating to functions. 1. Increasing or Decreasing Function: Let y = f(x) be a function defined in an interval and x 1 , x 2 be two points of the interval such that

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x 1 > x 2 . If f(x 2) < f(x 1) when x 1 > x 2 , then f(x) is increasing. If f(x 2) > f(x 1) when x 1 > x 2 , then f(x)

is decreasing. If, however, the strict inequality holds in the above statements, then f(x) is strictly increasing (or decreasing) function. 2. Monotonic Function: A function y = f(x) is said to be monotonic if y is either increasing or decreasing over its domain, as x increases. If the function is increasing (decreasing) over its domain, it is called monotonically increasing (decreasing) function. 3. Implicit and Explicit Function When a relationship between x and y is written as y = f(x), it is said to be an explicit function. If the same relation is written as F(x, y) = 0, it is said to be an implicit function. Production possibility function or the transformation function is often expressed as an implicit function.

Business Mathematics Note 235 4. Inverse Function If a function y = f(x) is such that for each element of the range we can associate a unique element of the domain (i.e. one to one function), then the inverse of the function, denoted as x = g(y), is obtained by solving y = f(x) for x in terms of y. The functions f(x) and g(y) are said to be inverse of each other and can be written as either g[f(x)] = x or f[g(y)] = y. We note here that an implicit function F(x, y) = 0, can be expressed as two explicit functions that are inverse of each other. 5. Single-Valued and Multi-Valued Functions If corresponding to each value of x the function y = f(x) assumes a single value, it is termed as single-valued function of x. If corresponding to each value of x the function y = f(x) assumes more than one value, it is termed as a multi-valued function of x. We note that multi-valued functions have been defined as relations in §3.2. 6. Symmetry of a Function Symmetry of a function is often helpful in sketching its graph. Following types of symmetry are often useful: (i) Symmetry about y-axis A function ? ?x f y ? is said to be symmetric about y-axis if ? ?x f ? = ? ?x f for all x in its domain. For example, the function 2 x y ? is symmetric about y-axis. Such a function is also known as even function. Similarly, if g(y) = g(?y), then the function x = g(y) is said to be symmetric about x-axis. (ii) Symmetry about origin A function ? ?x f y ? is said to be symmetric about origin if ? ? ? ? x f ? ?x f? , for all values of x in its domain. For example, the function 3 x y ? is symmetric about origin. Such a function is also known as odd function. (iii) Symmetry about the line y = x (45° line) Two functions are said to be symmetrical about the line y = x (45° line), if the interchange of x and y in one function gives the other function. This type of symmetry implies that y as an explicit function of x is exactly of the same form as x as an explicit function of y. The equation of rectangular hyperbola xy = ? is symmetric in x and y. The two inverse functions of xy = ? are x y ? ? and y x ? ? , which are said to be of identical form. The Graph of such functions are symmetrical about the line y = x.

Note Unit 7: Other Useful Mathematics Devices 236 Notes: (i) Two points with coordinates),(ba and (b, a) are said to be symmetrical about the line y = x. (ii) If there are two functions y = f(x) and y = g(x) which are inverse of each other, then we can write b = f(a) and a = g(b), for all values of a and b in the domain of f and g respectively, the graphs of these functions are mirror images of each other with respect to the line y = x. Figure 7.4: Graph of Two Functions To illustrate this we consider y = f(x) = 2x + 5 and y = g(x) = ? ?5 2 1 ? x . Note that (1, 7) is a point on the graph of y = 2x + 5 and (7, 1) is a point on the graph of y = ? ?5 2 1 ? x . The graphs of these functions are shown in Figure 7.5. (iv) The point of intersection of the two functions that are symmetric about the 45° line occurs at this line. 7. Composite Function If y is a function of u and u is a function of x, then y is said to be composite function of x. For example, if y = f(u) and u = g(x), then y = f[g(x)] is a composite function of x. A composite function can also be written as y = (fog)(x), where fog is read as f of g. 7.9 Polynomial A function of the form n n xa xa xa a y ? ? ? ? ? ? 2 2 1 0 , where n is a positive integer and 0? n a , is called a polynomial function of degree n. (i) If n = 0, we have 0 a y ? , a constant function. (ii) If n = 1, we have xa a y 1 0 ? ? , a linear function. (iii) If n = 2, we have 2 2 1 0 xa xa a y ? ? ? , a quadratic or parabolic function. (iv) If n = 3, we have 3 3 2 2 1 0 xa xa xa a y ? ? ? ? , a cubic function etc. Thus, A polynomial is a mathematical expression constructed with constants and variables using the four operations:
Business Mathematics Note 237 Polynomial Example Degree Constant 1 0 Linear 2x+1 1 Quadratic 3x 2 +2x+1 2 Cubic 4x 3 +3x 2 +2x+1 3 Quartic 5x 4 +4x 3 +3x 2 +2 x+1 4 In other words, we have been calculating with various polynomials all along. When two polynomials are divided it is called a rational expression . The diagrammatic representation of the polynomial functions is given as follows: x y x ? x? o x o y ? y ? (a) (b) Fig. 3.10 y 3 3 2 2 1 0 xa xa xa a y ? ? ? ? 3 ax y ? 0 a ? ? ? ? ? Did u know? In general, a polynomial of nth degree is supposed to have at most (n ?1) turning points or wiggles in its curve. 7.9.1 Shifting of Graphs 1. Vertical Shifts: If a positive number k is added to the right hand side of the function y = f(x) i.e. y = f(x) + k or y – k = f(x), the graph of the function f(x) + k is given by the vertically upward shift of the graph of f(x) by k units. The shift will be vertically downwards if k is negative. 2. Horizontal Shifts: If a positive number h is subtracted from x i.e. y = f(x – h), the graph of f(x – h) is given by the horizontal shift towards the right hand side of the graph of f(x) by h units. The shift will be towards the left hand side if h is negative. 7.10 Sigma (?) Notation The sum of any series is represented by using the greek letter sigma (???? before

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the n th term of the series. E.g. ? 2 2 2 2 2 2 3 3 3 3 3 2 n 1 2 3 4 .... n n 1 2 3 4 .... n n 2n 1 1.3 2.5 3.7 .... n(2n 1) n 1 2 3 4 .... n

Note Unit 7: Other Useful Mathematics Devices 238 Note: Denote the series where (???? placed before a term signifies the sum of all terms of which that term is the general type 7.11 Interpolation In the mathematical field of numerical analysis, interpolation is a method of constructing

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new data points within the range of a discrete set of known data points.

A different Example which is closely related to interpolation is the approximation of a complicated function by a simple function. Suppose the formula for some given function is known, but too complex to evaluate efficiently. A few known data points from the original function can be used to create an interpolation based on a simpler function. Of course, when a simple function is used to estimate data points from the original, interpolation errors are usually present; however, depending on the Example domain and the interpolation method used, the gain in simplicity may be of greater value than the resultant loss in accuracy. Exampl: For example, suppose we have a table like this, which gives some values of an unknown function f. Plot of the data points as given in the table. x f(x) 0 0 1 0 . 8415 2 0 . 9093 3 0 . 1411 4 −0 . 7568 5 −0 . 9589 6 −0 . 2794 Interpolation provides a means of estimating the function at intermediate points, such as x = 2.5. There are many different interpolation methods Piecewise constant interpolation: The simplest interpolation method is to locate the nearest data value, and assign the same value. In simple Examples, this method is unlikely to be used, as linear interpolation (see below) is almost as easy, but in higher-

Business Mathematics Note 239 dimensional multivariate interpolation, this could be a favourable choice for its speed and simplicity. Linear interpolation: One of the simplest methods is linear interpolation (sometimes known as lerp). Consider the above example of estimating f(2.5). Since 2.5 is midway between 2 and 3, it is reasonable to take f(2.5) midway between f(2) = 0.9093 and f(3) = 0.1411, which yields 0.5252. Generally, linear interpolation takes two data points, say (x a ,y a) and (x b ,y b), and the interpolant is given by: Linear interpolation is quick and easy, but it is not very precise. Another disadvantage is that the interpolant is not differentiable at the point x k . Polynomial interpolation: Polynomial interpolation is a generalization of linear interpolation. Note that the linear interpolant is a linear function. We now replace this interpolant with a polynomial of higher degree. Consider again the Example given above. The following sixth degree polynomial goes through all the seven points: Substituting x = 2.5, we find that f(2.5) = 0.5965. Generally, if we have n data points, there is exactly one polynomial of degree at most n–1 going through all the data points. The interpolation error is proportional to the distance between the data points to the power n. Furthermore, the interpolant is a polynomial and thus infinitely differentiable. So, we see that polynomial interpolation overcomes most of the Examples of linear interpolation. However, polynomial interpolation also has some disadvantages. Calculating the interpolating polynomial is computationally expensive (see computational complexity) compared to linear interpolation. Furthermore, polynomial interpolation may exhibit oscillatory artifacts, especially at the end points. Spline interpolation: Remember that linear interpolation uses a linear function for each of intervals [x k ,x k+1]. Spline interpolation uses low-degree polynomials in each of the intervals, and chooses the polynomial pieces such that they fit smoothly together. The resulting function is called a spline. For instance, the natural cubic spline is piecewise cubic and twice continuously differentiable. Furthermore, its second derivative is zero at the end points. Case Study: American History Illustrated Imagine you are the staff assistant to the publisher of American History Illustrated. Imagine further that you reported yesterday to your boss, the publisher, that the renewal rate for subscriptions increased from 0.512 in January to 0.641 in February. The renewal rate is computed as the number of subscriptions renewed in a given month divided by the total number of subscriptions that expired in that month. It gives the fraction renewed of subscriptions that could have been renewed. Imagine, finally, that your boss was pleased to hear this and wanted to know
Note Unit 7: Other Useful Mathematics Devices 240 which kinds of subscriptions were contributing most to the increase. He asked you to prepare a further summary by breaking the subscriptions down into several categories: gift subscriptions, previously renewed subscriptions, direct mail subscriptions, subscriptions from a subscription agency, and those from a catalog agent. This follows a standard industry classification scheme. You have obtained the data, as follows. For example, the total renewal rate for January was 23,545/45,955=.512, and the renewal rate for February for previously renewed subscriptions was 3907/5,140=0.760. Question Provide the summary the boss requested, and interpret it for him. Source: http://www.mhhe.com/business/opsci/bstat/bryantcases/samplecase8.mhtml Self Assessment State whether the following statements are true or false: 28. Interpolation is a method of constructing

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new data points within the range of a discrete set of known data points 29.

Depending on the Example domain and the interpolation method used, the gain in simplicity may be of greater value than the resultant loss in accuracy. 30. Polynomial interpolation is a generalization of non- linear interpolation 7.12 Summary ? A mathematical instruments and devices are tools or devices used in the study or practice of mathematics. ? An arithmetic progression (A.P.) is given by

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a, a+d, a+2d............... a is the first term and d is the common difference. ?

The nth term of the A.P. T n = a+(n – 1)d ? Sum to n terms of an A.P. S n = [2 (1)] ( ) 2 2 n n a n d a l ? ? ? ? , where l is the last term. ? A Geometric Progression is given by a, ar, ar 2 , ar 3 ,........
Business Mathematics Note 241 Where a is first term and r is common ratio. ? n th

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term of a G.P. T n = ar n-1 ? Sum to n terms of a G.P. S n = (1) 1 n a r r ? ?

Sum to infinity of a G.P. (if r >1) 1 a S r ? ? ? 7.13 Keywords Arithmetic Progression: A sequence of numbers in which each term after the first term is obtained by adding a constant quantity to its previous term. Common Difference: Difference between two successive terms in a A.P. Common Ratio: The constant multiplying factor (quantity). Geometric Progression: A sequence of numbers in which each term after the first term is obtained by multiplying its previous term by a constant quantity. Function: A function is a technical term used to symbolize relationship between variables. Domain: Set of all numbers (values) can be occupied by independent variable. Range: Set of all numbers (values) can be occupied by dependent variable. 7.14 Review Questions 1. Find the sum of first 15 terms of the following series: 10, 15, 20, 25,................. 2. The fourth term of an A.P. is 14 and the eight term is 26. Find the sum of first 10 terms. 3. Find the nth term, sum to n terms of the following arithmetic progressions. Also find them for the given values of n. (i) 1, 4, 7, ... and n = 5 (ii) 3, 5, 13 ? ? ,..... and 10 n ? (iii) 1, 6, 11,..... and 12 n ? (iv) 3, 1,1,.... ? ? and 8 n ? (v) 3,0, 3,..... ? and 9 n ? (vi) 1, 5, 9,..... and 11 n ?

Note Unit 7: Other Useful Mathematics Devices 242 (vii) 1 1 1, , ,.... 3 3 ? and 6 n ? (viii) 1 1, ,0,..... 2 ? ? and 15 n ? (ix) 2, 5, 8,.... and 20 n ? (x) 1 5, 2, 2 5...... ? ? and 14 n ? 4. If 4 3 n T n ? ? , find the first term, second term, third term and hence find the arithmetic progression and the common difference. 5. If the nth term is 2 1 n ? , find the A.P. and hence the common difference. 6. If the sum to

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n terms of an A.P. is given by 2 5 4 n n ? , find the nth term and the A.P. 7.

If

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the sum to n terms of an A.P. is given by 2 2n n? , find the nth term and the A.P. 8. If the sum to n terms is 2 2

n n ? , find the nth term and the A.P. 9.

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Find three numbers in A.P. whose sum is 9 and product is 15. 10.

100%	MATCHING BLOCK 237/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

Find three numbers in A.P. whose sum is 15 and product is 80. 11.

100%	MATCHING BLOCK 238/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

Find three numbers in A.P. whose sum is 12 and

the product is 28. 12. Find the

100%	MATCHING BLOCK 239/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

three numbers in A.P. whose sum is 18 and product is 162. 13.

100%	MATCHING BLOCK 240/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

Find three numbers in A.P. whose sum is 15 and product is 105. 14.

Find four numbers in A.P. whose sum is 16 and the product is 105. 15. Find four numbers in A.P. whose sum is 20 and the product is 384. 16.

100%	MATCHING BLOCK 241/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

Find three numbers in A.P. whose sum is 15 and

sum of their squares is 93. 17.

100%	MATCHING BLOCK 242/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

Find three numbers in A.P. whose sum is 3 and

sum of their squares is 35. 18. Which term of the A.P. (i) 3, 5, 7...... is 79? (ii) 0.5, 0.75,1,.... is 5.5? 19. Which term of the A.P. (i) 4, 1 2 5 ,6 ,.... 3 3 is 104? (ii) 9, 15, 21,.... is 183? 20. If the 7th term of A.P. is 21 and the 10th term 35, find the common difference, first term, nth term and the 4th term. 21. If the sum of 25 terms of an A.P. is 412 and the 1st term is 7, find the common difference. 22. Find how many terms of the A.P. 2, 7, 12,.... amount to 632? 23. Find how many terms of the A.P. must be taken so that the sum may be −480? 24 Find the sum of the odd numbers between 0 and 50. 25 Find the sum of the even numbers between 0 and 50. 26 Find whether −300 a term of the A.P. 10, 7, 4, ...
Business Mathematics Note 243 27 If

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the 10th term of an A.P. is 23, and the 32nd term is 67, find

the 20th term. 28. A ball rebounds 2 3 the distance it falls. If it is dropped from a height of 5 meters, how far does it travel before coming to rest. 29. The first term of a G.P. is 8 and the common ratio is 3. Find the sum of first 10 terms. 30. How many numbers are there between 20 and 100 which are divisible by 5? 31. Water in a water tank becomes half its previous volume in 1 hour. In how many hours will it become 1 512 th of the original volume? 32. Find the nth term, sum to n terms of the following geometric progressions. Also find them for the given values of n. (i) 1, 4, 16,....... and 5 n ? (ii) 2, 4, 8,..... and 10 n ? (iii) 1 1 1, , ,.... 2 4 and 12 n ? (iv) 1,3, 9,.... ? ? and 8 n ? (v) 5, 10, 20,..... and 9 n ? (vi) 3, 6, 12,...... and 11 n ? (vii) 3, 9, 27,..... and 6 n ? (viii) 2, 4,8,..... ? and 15 n ? (ix) 1 1 1 , , ,..... 3 9 27 and 12 n ? (x) 2 4 8 , , ,..... 3 3 3 and 6 n ? 33. If the nth term is 1 2 n? , find the first term, second term, third term and hence find the geometric progression and the common ratio. 34. If the nth term is 1 1 3 n? , find the G.P. and hence find the common ratio. 35. If the fourth term and seventh term

53%	MATCHING BLOCK 244/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

of a G.P. are 1 and 1 8 respectively, find the ninth term. 36. The sum of three numbers in G.P. is 26 and their product is 216. Find the numbers. 37.

Find the value of k if 3, 1 k ? , 12 are in G.P. 38. Find k if 1, 2, 3 k k k ? ? ? are in G.P.
Note Unit 7: Other Useful Mathematics Devices 244 39.

87%	MATCHING BLOCK 245/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

Find three numbers in G.P. whose sum is 28 and product is 512. 40.

87%	MATCHING BLOCK 246/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

Find three numbers in G.P. whose sum is 31 and product is 125. 41.

Find three numbers in G.P. whose product is 13 and sum of their squares

85%	MATCHING BLOCK 248/248	SA	Fundamental of Mathematics- Block-I, II.pdf (D144415340)

is 91. 42. The first term and the last term of a G.P. are respectively 3 and 768 and the sum is 1533. Find the common ratio and the number of terms. 43. Find the

sum to n terms and the sum to infinity of the following G.Ps. (i) 1 1 1, , ,..... 2 4 (ii) 1 3, 1, ,..... 3 ? (iii) 1 1 1, , ,..... 5 25 44. Find the sum to n terms of the following: (i) 2 22 222 ..... ? ? ? (ii) 7 77 777 ..... ? ? ? (iii) 0.2 0.22 0.222 .... ? ? ? (iv) 0.5 0.55 0.555 ... ? ? ? 45. How many terms of the series 2, 4, 8,.... must be taken to make the sum 510? 46. Which term of the G.P. −2, 8, −32, ... is 2048? 47. Three numbers are in G.P.

55%	MATCHING BLOCK 247/248	W

The sum of the first two terms is 15 and the sum of the second and third terms is 30. Find the

numbers. 48. A person borrows $ 5115 to be repaid in 10 monthly instalments. If each instalment is double the value of the previous instalment, find the value of first and last instalments. 49. A labourer is paid a salary of $ 100 per month. His salary is raised by $ 10 per month every year. He deposits 25% of his salary in a bank. How much has he saved in the bank after working for 5 years? 50. A man is employed in a firm on a pay of $ 350 per month with an annual increment of $ 15. What will be his salary during 10th year? What are his total earnings during 10 years? 51. The first year, a man saves $ 100. In each succeeding year, he saves $ 25 more than that in the year before. How much has he accumulated at the end of 20 years? 52. A ball dropped from a height of 10 feet rebounds and reaches half the height every time. Find the total distance covered by the ball before it comes to rest. 53. A car purchased for $ 10,000 depreciates in value 10% every year. Find its value at the end of 5 years. 54. Find the nth term and the sum of n terms of the series:
Business Mathematics Note 245 1 + 2 + 5 + 12 + 25 + 46 + ... 55. Find the sum of the series to n terms: 3 + 7 + 14 + 24 + 37 + ... Answers: Self Assessment 1. Rounding 2. Second 3. Third 4. whole 5. previous 6. do not 7. False 8. True 9. True 10. True 11. True 12. Precision 13. approximations 14. No 15. Significant figures 16. Accurate 17. Critical 18. Cannot 19. Set 20. Particular 21. Previous 22. common difference 23. not 24. geometric progression 25. each term after the first term is obtained by multiplying its previous term by a constant quantity 26. r 27. A.P 28. True 29. True 30. False 7.15 Further Readings Books Kenneth Rosen, Discrete Mathematics and Its Applications, 7/e, 2012, McGraw-Hill. Bathul, Shahnaz, Mathematical Foundations of Computer Science, PHI Learning. Lambourne, Robert, Basic Mathematics for the Physical Sciences,2008, John Wiley & Sons. Bari, Ruth A.; Frank Harary. Graphs and Combinatorics, Springer. F. Ernest Jerome, Connect for Jerome, Business Mathematics in Canada, 7e, Canadian Edition. S Rajagopalan and R Sattanathan, Business Mathematics, 2 edition, 2009, Tata McGraw Hill Education. Garrett H.E. (1956), Elementary Statistics, Longmans, Green & Co., New York.
Note Unit 7: Other Useful Mathematics Devices 246 Guilford J.P. (1965), Fundamental Statistics in Psychology and Education, Mc Graw Hill Book Company, New York. Hannagan T.J. (1982), Mastering Statistics, The Macmillan Press Ltd., Surrey. Lindgren B.W (1975), Basic Ideas of Statistics, Macmillan Publishing Co. Inc., New York. Selvaraj R., Loganathan C., Quantitative Methods in Management. Sharma J.K., Business Statistics, Pearson Education Asia Walker H.M. and J. Lev, (1965), Elementary Statistical Methods, Oxford & IBH Publishing Co., Calcutta. Wine R.L. (1976), Beginning Statistics, Winthrop Publishers Inc., Massachusetts. Online links en.wikipedia.org/wiki/Arithmetic_progression en.wikipedia.org/wiki/Geometric_prograession www.mathwords.com/a/arithmetic_sequence.htm home.windstream.net/okrebs/page133.html www.purplemath.com/modules/series3.htm www.mathcentre.ac.uk/resources/uploaded/mc-ty-apgp-2009-1.pdf
Business Mathematics Note 247 Unit 8: Frequency Distribution CONTENTS Objectives Introduction 8.1 Observations 8.2 Frequency 8.3 Statistical Series 8.3.1 Time Series 8.3.2 Qualitative or Quantitative Series 8.4 Frequency Distribution and its Construction 8.4.1 Construction of a Frequency Distribution 8.4. Construction of a Continuous Frequency Distribution 8.5 Cumulative and Relative Frequency 8.5.1 Cumulative Frequency Distribution 8.5.2 Relative or Percentage Frequency Distribution 8.6 Diagrammatic and Graphic Presentation 8.6.1 One-Dimensional Diagrams 8.6.2 Two-Dimensional Diagrams 8.6.3 Three Dimensional Diagrams 8.7 Graphs of a Frequency Distribution 8.8 Summary 8.9 Keywords 8.10 Review Questions 8.11 Further Readings Objectives After studying this unit, you will be able to: 1. Focus on observations and frequency 2. Discuss statistical series 3. Describe frequency distributions and its Construction 4. Explain cumulative and relative frequency 5. Understand diagrammatic representation, and Frequency curve

Note Unit 8: Frequency Distribution 248 Introduction Frequency distribution is a series when a number of observations with similar or closely related values are put in separate bunches or groups, each group being in order of magnitude in a series. It is simply a table in which the data are grouped into classes and the number of cases which fall in each class are recorded. It shows the frequency of occurrence of different values of a single Phenomenon. In this unit, we will discuss Focus on observations and frequency .We will also focus on statistical series, frequency distributions and its Construction. Further we will focus on cumulative and relative frequency. Finally, we will focus on diagrammatic representation, and Frequency curve. 8.1 Observations A set of data derived/occurrence or happenings from an object (experimental unit)is known as observation. Each object is measured according to various aspects, such as temperature, concentration of some constituents, frequency of occurrence of some phenomenon, etc. Each of these aspects is denoted as a variable or feature. By assembling all available data on all objects we can build a matrix - a table where the columns represent the variables and the rows represent the measured observations. For example, we can take the list of physical characteristics of 10 persons. The objects of the matrix are the persons, the variables are the measured properties, such as the weight or the color of the eyes. Table 8.1: Physical Characteristics of 10 Persons Age [years] Sex Weight [kg] Eye Color Body Temperature [°C] individuum 1 42 female 52.9 brown 36.9 individuum 2 37 male 87.0 green 36.3 individuum 3 29 male 82.1 blue 36.4 individuum 4 61 female 62.5 blue 36.7 individuum 5 77 female 55.5 gray 36.6 individuum 6 33 male 95.2 green 36.5 individuum 7 32 female 81.8 brown 37.0 individuum 8 45 male 78.9 brown 36.3 individuum 9 18 male 83.4 green 36.6 individuum 10 19 male 84.7 gray 36.1 Depending on the nature of the measured item and the measurement process the variables may be further classified into discrete (nominal and ordinal scales) and continuous variables (interval and ratio scales). Did u know? The eye color is a discrete variable, the weight or the body temperature is a continuous variable.

Business Mathematics Note 249 8.2 Frequency If the value of a variable, e.g., height, weight, etc. (continuous), number of students in a class, readings of a taxi-meter (discrete) etc., occurs twice or more in a given series of observations, then the number of occurrence of the value is termed as the “frequency” of that value. Therefore, frequency of a value of a variable is the number of times it occurs in a given series of observations. Note A tallysheet may be used to calculate the frequencies from the raw data (primary data not arranged in the Tabular form). A tally-mark (/) is put against the value when it occurs in the raw data. The following example shows how raw-data can be represented by a tally-sheer : Example: Raw data Marks in Mathematics of 50 students.(selected from among the candidates in ICAI Examination) Represent the given table by using tall sheet/marks. Table 8.2: Raw Data 37 47 32 26 21 41 38 41 50 45 52 46 37 45 31 40 44 48 46 16 30 40 36 32 47 37 47 50 40 45 51 52 38 26 41 33 38 39 37 32 40 38 50 38 48 41 36 41 41 52 Table 8.3: Note Unit 8: Frequency Distribution 250 Such a representation of the data is known as the Frequency Distribution. The number of classes should neither be too large nor too small. It should not exceed 20 but should not be less than 5, normally, depending on the number of observations in the raw data. Self Assessment Fill in the blanks: 1. A set of data derived/occurrence or happenings from an object (experimental unit)is known as .... 2. By assembling all available data on all objects we can build a matrix - a table where the columns represent the variables and the rows represent the .... 3. Depending on the nature of the measured item and the measurement process the variables may be further classified into discrete (nominal and ordinal scales) and continuous variables (........................scales). 4. If the value of a variable, e.g., height, weight, etc. (continuous), number of students in a class,readings of a taxi-meter (discrete) etc., occurs twice or more in a given series of observations, then the number of occurrence of the value is termed as the .... of that value. 5. Frequency of a value of a .... is the number of times it occurs in a given series.observations. 6. A tally-mark (/) is put against the value when it occurs in the .... data. 8.3 Statistical Series The classified data when arranged in some logical order, e.g., according to the size, according to the time of occurrence or according to some other measurable or non- measurable characteristics, is known as a Statistical Series. H. Secrist defined a statistical series as, "A series, as used statistically, may be defined as things or attributes of things arranged according to some logical order.” Another definition given by L. R. Connor as, "If the two variable quantities can be arranged side by side so that the measurable differences in the one correspond to the measurable differences in the other, the result is said to form a statistical series." Note A statistical series can be one of the following four types: (i) Spatial Series, (ii) Conditional Series, (iii) Time Series, and (iv) Qualitative or Quantitative Series The series formed by the geographical or spatial classification is termed as spatial series. Similarly, a series formed by the conditional classification is known as the conditional series. The examples of such series are already given under their respective classification category.

Business Mathematics Note 251 8.3.1 Time Series A time series is the result of chronological classification of data. In this case, various figures are arranged with reference to the time of their occurrence. For example, the data on exports of India in various years is a time series is given below: Table 8.4: Data on Exports of India in Various Years Year 2006 2007 2008 2009 2010 2011 2012 2013 2014 Exports (in $ cr.) 6591 7242 8302 8810 9981 10427 11490 15741 20295 8.3.2 Qualitative or Quantitative Series This type of series is obtained when the classification of data is done on the basis of qualitative or quantitative characteristics. Accordingly, we can have two types of series, namely, qualitative and quantitative series. 1. Qualitative Series: In case of qualitative series, the number of items in each group are shown against that group. These groups are either expressed in ascending order or in descending order of the number of items in each group. The example of such a series is given below. Table 8.5: Distribution of Students of a College according to Sex Sex Males Females Total No. of Student 1700 500 2200 2. Quantitative Series: A quantitative series is obtained when the collected data are classified either according to magnitude or according to choronology (time series) or according to alphabetical ordering of statistical units or accordingly to some other criterion. A quantitative series can be of two types: Individual series/simple series and Frequency Distribution 3. Individual series/simple series: In an individual series, the names of the individuals are written against their corresponding values. For example, the list of employees of a firm and their respective salary in a particular month. Frequency distribution is discuss in detail in further discussion. 8.4 Frequency Distribution and its Construction Frequency Distribution is a table in which the frequencies and the associated values of the variable are written side by side, is known as a frequency distribution. According to Croxton and Cowden, "Frequency distribution is a statistical table which shows the set of all distinct values of the variable arranged in order of magnitude, either individually or in a group with their corresponding frequencies side-by-side."Thus, the way of tabulating a pool of data of a variable and their respective frequencies side by side is called a 'frequency distribution' of those data. A frequency distribution can be discrete or continuous depending upon whether the variable is discrete or continuous. Note Unit 8: Frequency Distribution 252 8.4.1 Construction of a Frequency Distribution A frequency distribution is constructed for three main reasons: 1. To facilitate the analysis of data. 2. To estimate frequencies of the unknown population distribution from the distribution of sample data and 3. To facilitate the computation of various statistical measures Construction of a Discrete Frequency Distribution A discrete frequency distribution may be ungrouped or grouped. In an ungrouped frequency distribution, various values of the variable are shown along with their corresponding frequencies. If this distribution fails to reveal any pattern, grouping of various observations become necessary. The resulting distribution is known as grouped frequency distribution of a discrete variable. Did u know? A grouped frequency distribution is also constructed when the possible values that a variable can take are large. (a) Ungrouped Frequency Distribution of a Discrete variable Suppose that a survey of 150 houses was conducted and number of rooms in each house was recorded as shown below: Table 8.6: Survey of 150 Houses 5 4 4 6 3 2 2 6 6 2 6 3 3 4 5 6 3 2 2 5 3 1 4 5 1 5 1 4 3 2 5 1 5 3 2 2 4 2 2 4 4 6 3 2 4 2 3 2 4 6 3 3 2 6 4 1 4 4 5 2 4 1 4 2 1 5 1 3 3 2 5 6 1 3 1 5 3 4 3 1 1 4 1 1 2 2 1 5 2 3 6 3 5 2 2 3 3 3 3 4 5 1 6 2 1 2 1 1 6 5 2 1 1 5 6 4 2 2 3 3 3 4 3 2 1 5 2 3 1 1 4 6 4 6 2 2 4 5 6 3 6 4 1 2 4 2 2 3 4 5 Counting of frequency using Tally Marks The method of tally marks is used to count the number of observations or the frequency of each value of the variable. Each possible value of the variable is written in a column. For every observation, a tally mark denoted by is noted against its corresponding value. Five observations are denoted as , i.e., the fifth tally mark crosses the earlier four marks and so on. The method of tally marks is used below

Business Mathematics Note 253 to determine the frequencies of various values of the variable for the data given above. Table 8.7: Frequencies of Various Values of the Variable In the above frequency distribution, the number of rooms 'X' is a discrete variable which can take integral values from 1 to 6. This distribution is also known as ungrouped frequency distribution. It should be noted here that, in case of ungrouped frequency distribution, the identity of various observations is not lost, Did u know? It is possible to get back the original observations from the given frequency distribution. (b) Grouped Frequency Distribution of a Discrete Variable Consider the data on marks obtained by 50 students in statistics. The variable 'X' denoting marks obtained is a discrete variable, let the ungrouped frequency distribution of this data be as given in the following table. Table 8.8: Ungrouped Frequency Distribution of Data Marks Frequency Marks Frequency Marks Frequency 33 1 35 2 39 1 41 2 42 1 45 1 48 2 50 1 52 1 53 1 54 1 55 2 57 1 59 1 60 2 61 1 64 1 65 3 66 2 67 1 69 2 71 1 73 2 74 2 76 1 77 2 78 1 80 1 81 1 84 1 85 2 88 1 89 1 91 1 94 2 98 1 This frequency distribution does not reveal any pattern of behaviour of the variable. In order to bring the behaviour of the variable into focus, it becomes necessary to convert this into a grouped frequency distribution. Instead of above, if the individual marks are grouped like marks between and including 30 and 39, 40 and 49, etc. and the respective frequencies are written against them, we get a grouped frequency distribution as shown below:

Note Unit 8: Frequency Distribution 254 Table 8.9: Frequency Distribution 30 - 39 40 - 49 50 - 59 60 - 69 70 - 79 80 - 89 90 - 99 4 6 8 12 9 7 4 50 Marks between and including Frequency Total The above frequency distribution is more revealing than the earlier one. It is easy to understand the behaviour of marks on the basis of this distribution. It should, however, be pointed out here that the identity of observations is lost after grouping. For example, on the basis of the above distribution we can only say that 4 students have obtained marks between and including 30 - 39, etc. Thus, it is not possible to get back the original observations from a grouped frequency distribution. 8.4. Construction of a Continuous Frequency Distribution As opposed to a discrete variable, a continuous variable can take any value in an interval. Measurements like height, age, income, time, etc., are some examples of a continuous variable. As mentioned earlier, when data are collected regarding these variables, it will show discreteness, which depends upon the degree of precision of the measuring instrument. Therefore, in such a situation, even if the recorded data appear to be discrete, it should be treated as continuous. Since a continuous variable can take any value in a given interval, therefore, the frequency distribution of a continuous variable is always a grouped frequency distribution. To construct a grouped frequency distribution, the whole interval of the continuous variable, given by the difference of its largest and the smallest possible values, is divided into various mutually exclusive and exhaustive sub-intervals. These sub- intervals are termed as class intervals. Then, the frequency of each class interval is determined by counting the number of observations falling under it. The construction of such a distribution is explained below: The figures, given below, are the 90 measurements of diameter (in mm.) of a wire. 1.86, 1.58, 1.13, 1.46, 1.53, 1.65, 1.49, 1.03, 1.10, 1.36, 1.37, 1.46, 1.44, 1.46, 1.95, 1.67, 1.59, 1.35, 1.44, 1.40, 1.50, 1.41, 1.19, 1.16, 1.27, 1.21, 1.82, 1.55, 1.52, 1.42, 1.17, 1.62, 1.42, 1.22, 1.56, 1.78, 1.98, 1.31, 1.29, 1.69, 1.32, 1.68, 1.36, 1.55, 1.54, 1.67, 1.81, 1.47, 1.30, 1.33, 1.38, 1.34, 1.40, 1.37, 1.27, 1.04, 1.87, 1.45, 1.47, 1.35, 1.24, 1.48, 1.41, 1.39, 1.38, 1.47, 1.73, 1.20, 1.77, 1.25, 1.62, 1.43, 1.51, 1.60, 1.15, 1.26, 1.76, 1.66, 1.12, 1.70, 1.57, 1.75, 1.28, 1.56, 1.42, 1.09, 1.07, 1.57, 1.92, 1.48. The following decisions are required to be taken in the construction of any frequency distribution of a continuous variable. 1. Number of Class Intervals: Though there is no hard and fast rule regarding the number of classes to be formed, yet their number should be neither very large nor very small. If there are too many classes, the frequency distribution appears to be too fragmented to reveal the pattern of behaviour of characteristics. Fewer classes

Business Mathematics Note 255 imply that the width of the class intervals will be broad and accordingly it would include a large number of observations. As will be obvious later that in any statistical analysis, the value of a class is represented by its mid-value and hence, a class interval with broader width will be representative of a large number of observations. Thus, the magnitude of loss of information due to grouping will be large when there are small number of classes. On the other hand, if the number of observations is small or the distribution of observations is irregular, i.e., not uniform, having more number of classes might result in zero or very small frequencies of some classes, thus, revealing no pattern of behaviour. Therefore, the number of classes depends upon the nature and the number of observations. If the number of observations is large or the distribution of observations is regular, one may have more number of classes. In practice, the minimum number of classes should not be less than 5 or 6 and in any case there should not be more than 20 classes. Note: The approximate number of classes can also be determined by Struge's formula : $n = 1 + 3.322 \times \log 10 N$, where n (rounded to the next whole number) denotes the number of classes and N denotes the total number of observations. Based on this formula, we have $n = 1 + 3.322 \times 2.000 = 7.644$ or 8, when N = 100 $n = 1 + 3.322 \times 2.699 = 9.966$ or 10, when N = 500 $n = 1 + 3.322 \times 4.000 = 14.288$ or 15, when N = 10,000 $n = 1 + 3.322 \times 4.699 = 16.610$ or 17, when N = 50,000 From the above calculations we may note that even the formation of 20 class intervals is very rarely needed. For the given data on the measurement of diameter, there are 90 observations. The number of classes by the Sturge's formula are $n = 1 + 3.322 \times \log 10\ 90 = 7.492$ or 8 2. Width of a Class Interval: After determining the number of class intervals, one has to determine their width. The problem of determining the width of a class interval is closely related to the number of class intervals. As far as possible, all the class intervals should be of equal width. However, there can be situations where it may not be possible to have equal width of all the classes. Suppose that there is a frequency distribution, having all classes of equal width, in which the pattern of behaviour of the observations is not regular, i.e., there are nil or very few observations in some classes while there is concentration of observations in other classes. In such a situation, one may be compelled to have unequal class intervals in order that the frequency distribution becomes regular. The approximate size of a class interval can be decided by the use of the following formula: Class Interval = Largest observation - Smallest observation Number of class intervals

Note Unit 8: Frequency Distribution 256 or using notations, In the example, given above, L = 1.98 and S = 1.03 and n = 8. ? Approximate size of a class interval = 1.98 - 1.03 8 =0.1188 or 0.12 (approx.) Before taking a final decision on the width of various class intervals, it is worthwhile to consider the following points: (a) Normally a class interval should be a multiple of 5, because it is easy to grasp numbers like 5, 10, 15, ..., etc. (b) It should be convenient to find the mid-value of a class interval. (c) Most of the observations in a class should be uniformly distributed or concentrated around its mid-value. (d) As far as possible, all the classes should be of equal width. A frequency distribution of equal class width is convenient to be represented diagrammatically and easy to analyse. On the basis of above considerations, it will be more appropriate to have classes, each, with interval of 0.10 rather than 0.12. Further, the number of classes should also be revised in the light of this decision. n = L - S Class Interval = 1.98 - 1.03 0.10 = 0.95 0.10 =9.5 or 10 (rounded to the next whole number) 3. Designation of Class Limits: The class limits are the smallest and the largest observation in a class. These are respectively known as the lower limit and the upper limit of a class. For a frequency distribution, it is necessary to designate these class limits very unambiguously, because the mid-value of a class is obtained by using these limits. As will be obvious later, this mid-value will be used in all the computations about a frequency distribution and the accuracy of these computations will depend upon the proper specification of class limits. Note The class limits should be designated keeping the following points in mind: (a) It is not necessary to have lower limit of the first class exactly equal to the smallest observation of the data. In fact it can be less than or equal to the smallest observation. Similarly, the upper limit of the last class can be equal to or greater than the largest observation of the data. (b) It is convenient to have lower limit of a class either equal to zero or some multiple of 5. (c) The chosen class limits should be such that the observations in a class tend to concentrate around its mid-value. This will be true if the observations are uniformly distributed in a class. The designation of class limits for various class intervals can be done in two ways : (i) Exclusive Method and (ii) Inclusive Method.

Business Mathematics Note 257 (i) Exclusive Method: In this method the upper limit of a class is taken to be equal to the lower limit of the following class. To keep various class intervals as mutually exclusive, the observations with magnitude greater than or equal to lower limit but less than the upper limit of a class are included in it. For example, if the lower limit of a class is 10 and its upper limit is 20, then this class, written as 10 - 20, includes all the observations which are greater than or equal to 10 but less than 20. The observations with magnitude 20 will be included in the next class. (ii) Inclusive Method: Here all observations with magnitude greater than or equal to the lower limit and less than or equal to the upper limit of a class are included in it. The two types of class intervals, discussed above, are constructed for the data on the measurements of diameter of a wire as shown below: Table 8.10: Frequency Distribution Class Intervals Frequency 20-29 30-39 40-49 50-59 Total 8 15 10 7 40 Mid-Value of a Class In exclusive types of class intervals, the mid-value of a class is defined as the arithmetic mean (to be discussed later) of its lower and upper limits. However, in the case of inclusive types of class intervals, there is a gap between the upper limit of a class and the lower limit of the following class which is eliminated by determining the class boundaries. Here, the mid-value of a class is defined as the arithmetic mean of its lower and upper boundaries. To find class boundaries, we note that the given data on the measurements of diameter of a wire is expressed in terms of millimeters, approximated upto two places after decimal. This implies that a value greater than or equal to 1.095 but less than 1.10 is approximated as 1.10 and, thus, included in the class interval 1.10 - 1.19. Similarly, an observation less than 1.095 but greater than 1.09 is approximated as 1.09 and is included in the interval 1.00 - 1.09. Keeping the precision of measurements in mind, various class boundaries, for the inclusive class intervals, given above, can be obtained by subtracting 0.005 from the lower limit and adding 0.005 to the upper limit of each class. These boundaries are given in the third column of the above table. Construction of a grouped frequency distribution for the data on the measurements of diameter of a wire. Taking class intervals as 1.00 - 1.10, 1.10 - 1.20, etc. and counting their respective frequencies, by the method of tally marks, we get the required frequency distribution as given below: Note Unit 8: Frequency Distribution 258 Table 8.11: Grouped Frequency Distribution Example Given below are the weights (in pounds) of 70 students. (i) Construct a frequency distribution when class intervals are inclusive, taking the lowest class as 60 - 69. Also construct class boundaries. (ii) Construct a frequency distribution when class intervals are exclusive, taking the lowest class as 60 - 70. 61, 80, 91, 113, 100, 106, 109, 73, 88, 92, 101, 106, 107, 97, 93, 96, 102, 114, 87, 62, 74, 107, 109, 91, 72, 89, 94, 98, 112, 103, 101, 77, 92, 73, 67, 76, 84, 90, 118, 107, 108, 82, 78, 84, 77, 95, 111, 115, 104, 69, 106, 105, 63, 76, 85, 88, 96, 90, 95, 99, 83, 98, 88, 72, 75, 86, 82, 86, 93, 92. Solution. (i) Construction of frequency distribution using inclusive class intervals. Table 8.12: Frequency Distribution To determine the class boundaries, we note that measured weights are approximated to the nearest pound. Therefore, a measurement less than 69.5 is approximated as 69 and included in the class interval 60 - 69. Similarly, a measurement greater than or equal to 69.5 is approximated as 70 and is included in the class interval 70 - 79. Thus, the class boundaries are obtained by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of various classes. These boundaries are shown in the last column of the above table. (ii) The frequency distribution of exclusive type of class intervals can be directly written from the above table as shown below:

Business Mathematics Note 259 Table 8.13: Frequency Distribution Class Inervals Frequency Total 60 - 70 70 - 80 80 - 90 90 - 100 100 - 110 110 - 120 5 11 14 18 16 6 70 Example: Determine the class boundaries for the following distribution of ages of 40 workers of a factory, where quoted age is the age completed on last birthday. Table 8.14: Frequency Distribution Class Intervals Frequency 20 - 29 30 - 39 40 - 49 50 - 59 Total 8 15 10 7 40 Solution: The determination of class boundaries depends upon the nature of approximation. Since the quoted age is the age completed on last birthday, therefore, a number greater than 29 but less than 30 is approximated as 29. Therefore, the boundaries of this class will be 20 - 30. Similarly, the boundaries of other classes will be 30 - 40, 40 - 50 and 50 - 60, respectively. Self Assessment State whether the following statements are true or false: 7. The classified data when arranged in some logical order, e.g., according to the size, according to the time of occurrence or according to some other measurable or non-measurable characteristics, is known as a Statistical Series. 8. A statistical series can be one of the following four types: (i) Spatial Series, (ii) Conditional Series, (iii) Time Series, and (iv) Qualitative or Quantitative Series 9. The series formed by the geographical or spatial classification is termed as spatial series. 10. A series formed by the conditional classification is known as the conditional series. 11. A time series is the result of chronological classification of data. 12. In case of qualitative series, the number of items in each group are shown against that group. 13. A quantitative series is obtained when the collected data are classified either according to magnitude or according to choronology (time series) or according to alphabetical ordering of statistical units or accordingly to some other criterion. 14. In an individual series, the names of the individuals are written against their corresponding values.

Note Unit 8: Frequency Distribution 260 15. A table in which the frequencies and the associated values of the variable are written side by side, is known as a frequency distribution. 16. A frequency distribution can be discrete or continuous depending upon whether the variable is discrete or continuous. 17. A discrete frequency distribution may be ungrouped or grouped. 8.5 Cumulative and relative frequency 8.5.1 Cumulative Frequency Distribution In order to answer the questions like; the measurements on diameter that are less than 1.70 or the number of measurements that are greater than 1.30, etc., a cumulative frequency distribution is constructed. A cumulative frequency distribution can be of two types: (i) Less than type cumulative frequency distribution (ii) More than type cumulative frequency distribution These frequency distributions, for the data on the measurements of diameter of a wire, are shown in Table I and Table II respectively. Table 8.15: Less than type CFD Table 8.I6: More than type CFD Diameters Cumulative Frequency less than 1.10 less than 1.20 less than 1.30 less than 1.40 less than 1.50 less than 1.60 less than 1.70 less than 1.80 less than 1.90 less than 2.00 4 11 21 35 55 68 77 83 87 90 Diameters Cumulative Frequency l More than 1.00 More than 1.10 More than 1.20 More than 1.30 More than 1.40 More than 1.50 More than 1.60 More than 1.70 More than 1.80 More than 1.90 90 86 79 69 55 35 22 13 7 3 8.5.2 Relative or Percentage Frequency Distribution If instead of frequencies of various classes their relative or percentage frequencies are written, we get a relative or percentage frequency distribution. Relative frequency of a class Percentage frequency of a class = Relative frequency × 100 These frequencies are shown in the following table. Table 8.17: Relative Frequency Distribution = Frequency of the class Total Frequency

Business Mathematics Note 261 8.6 Diagrammatic and Graphic Presentation In the earlier study, we have already learnt that the techniques of classification and tabulation that help in summarizing the collected data and presenting them in a systematic manner. However, these forms of presentation do not always prove to be interesting to the common man. One of the most convincing and appealing ways in which statistical results may be presented is through diagrams and graphs. Just one diagram is enough to represent a given data more effectively than thousand words. Moreover even a layman who has nothing to do with numbers can also understands diagrams. Evidence of this can be found in newspapers, magazines, journals, advertisement, etc. An attempt is made in this chapter to illustrate some of the major types of diagrams and graphs frequently used in presenting statistical data. An important function of statistics is the presentation of complex mass of data in a simple way so that it becomes easier to understand. Classification and tabulation are the techniques that help in presenting the data in an intelligible form. But with increase in volume of data, it becomes more and more inconvenient to understand even after its classification and tabulation. Thus, to understand various trends of the data at a glance and to facilitate the comparison of various situations, the data are presented in the form of diagrams and graphs. Diagrams A diagram is a visual form for presentation of statistical data, highlighting their basic facts and relationship. If we draw diagrams on the basis of the data collected they will easily be understood and appreciated by all. It is readily intelligible and save a considerable amount of time and energy. Significance of Diagrams and Graphs Diagrams and graphs are extremely useful because of the following reasons. 1. They are attractive and impressive. 2. They make data simple and intelligible. 3. They make comparison possible 4. They save time and labour. 5. They have universal utility. 6. They give more information. 7. They have a great memorizing effect. General Rules for Making Diagrams A diagrammatic presentation is a simple and effective method of presenting the information contained in statistical data. The construction of a diagram is an art, which can be acquired only through practice. However, the following rules should be observed, in their construction, to make them more effective and useful. (i) Appropriate title and footnote: Every diagram must have a suitable title written at its top. The title should be able to convey the subject matter in brief and unambiguous manner. The details about the title, if necessary, should be provided below the diagram in the form of a footnote.

Note Unit 8: Frequency Distribution 262 (ii) Attractive presentation: A diagram should be constructed in such a way that it has an immediate impact on the viewer. It should be neatly drawn and an appropriate proportion should be maintained between its length and breadth. The size of the diagram should neither be too big nor too small. Different aspects of the problem may be emphasised by using various shades or colours. (iii) Accuracy: Diagrams should be drawn accurately by using proper scales of measurements. Accuracy should not be compromised to attractiveness. (iv) Selection of an appropriate diagram: There are various types of geometrical figures and pictures which can be used to present statistical data. The selection of an appropriate diagram should be carefully done keeping in view the nature of data and objective of investigation. (v) Index: When a diagram depicts various characteristics distinguished by various shades and colours, an index explaining these should be given for clear identification and understanding. (vi) Source-Note: As in case of tabular presentation, the source of data must also be indicated if the data have been acquired from some secondary source. (vii) Simplicity: As far as possible, the constructed diagram should be simple so that even a layman can understand it without any difficulty. Self Assessment 18. Diagrams are attractive and .... 19. Diagrams .... data 20. Diagrams are useful in making .... 21. Diagrams are liable to be misused for presenting .... picture of the problem. 22. A .... is a simple and effective method of presenting the information contained in statistical data. 23. The construction of a diagram is an .... , which can be acquired only through practice. 24. The choice of a suitable diagram depends upon the .... of the given data. Types of Diagrams There are a large number of diagrams which can be used for presentation of data. The selection of a particular diagram depends upon the nature of data, objective of presentation and the ability and experience of the person doing this task. For convenience, various diagrams can be grouped under the following categories: 1. One-Dimensional Diagrams 2. Two-Dimensional Diagrams 3. Three-Dimensional Diagrams

Business Mathematics Note 263 4. Pictograms 5. Cartograms 8.6.1 One-Dimensional Diagrams One-dimensional diagrams are also known as bar diagrams. In case of one- dimensional diagrams, the magnitude of the characteristics is shown by the length or height of the bar. The width of a bar is chosen arbitrarily so that the constructed diagram looks more elegant and attractive. It also depends upon the number of bars to be accommodated in the diagrams. If large number of items are to be included in the diagram, lines may also be used instead of bars. Different types of bar diagrams are: 1. Line Diagram: In case of a line diagram, different values are represented by the length of the lines, drawn vertically or horizontally. The gap between the successive lines is kept uniform. The comparison of values of various items is done by the length of these lines. Although the comparison is easy, the diagram is not very attractive. This diagram is used when the number of items is relatively large. Example: The income of 12 workers on a particular day was recorded as given below. Represent the data by a line diagram. S. No. of workers : 1 2 3 4 5 6 7 8 9 10 11 12 Income (in $) : 25 35 30 45 50 55 40 50 60 55 40 35 Solution. Line Diagram of income of 12 workers I n c o m e (i n $) Figure 8.1 2. Simple Bar Diagram: In case of a simple bar diagram, the vertical or horizontal bars, with height

Note Unit 8: Frequency Distribution 264 Simple Bar Diagram: In case of a simple bar diagram, the vertical or horizontal bars, with height proportional to the value of the item, are constructed. The width of a bar is chosen arbitrarily and is kept constant for every bar. Different bars are drawn so that the gap between the successive bars is same. Bar diagrams are particularly suitable for presenting individual series, such as time and spatial series. Example Represent the following data by a suitable diagram. Table 8.18: CFA Enrolments Data in Various Years Years : C. F. A Enrolments : 7300 9400 12100 14600 16700 2006 2007 2008 2009 2010 Solution. 2006 2007 2008 2009 2010 Figure 8.2: Simple Bar Diagram 3. Multiple Bar Diagram: This type of diagram, also known as compound bar diagram, is used when comparisons are to be shown between two or more sets of data. A set of bars for a period or a related phenomena are drawn side by side without gaps while various sets of bars are separated by some arbitrarily chosen constant gap. Different bars are distinguished by different shades or colours. In order that various bars are comparable, it is necessary to draw them on the same scale. Example The following table gives the figures of Indo-US trade during 2007 to 2010. The figures of Indian exports and imports are in $ billion. Table 8.19: Figures of Indo-US trade during 2007-2010 Year : Export : 2.529 2.952 3.314 3.191 Import : 1.460 2.484 2.463 2.486 2007 2008 2009 2010 Present the above data by a suitable diagram.
Business Mathematics Note 265 Solution: 2007 2008 2009 2010 Source: U.S. Department of Commerce Figure 8.3: Multiple Bar Diagram of INDO-US Trade (2007-2010) 4. Sub-divided or Component Bar Diagram: In case of a sub-divided bar diagram, the bar corresponding to each phenomenon is divided into various components. The portion of the bar occupied by each component denotes its share in the total. For example, the bar corresponding to the number of students in a course can be sub-divided into boys and girls. The subdivisions of different bars should always be done in the same order and they should be distinguished from each other by using different colours or shades. Sub-divided bar diagram is useful when it is desired to represent the comparative values of different components of a phenomenon. The main limitation of this diagram is that since various components are not drawn on a common base, they are difficult to compare. This diagram is used only if there are few components of a phenomenon. Figure 8.4: Multiple Bar Diagram of the Faculty-wise distribution of students 5. Percentage Sub-Divided Bar Diagram: A sub-divided diagram is used to show absolute magnitudes of various components. These magnitudes can be changed into relative by converting them as a percentage of the total. The length of each bar is taken as 100 and length of each component is denoted by its percentage value. As before the different components are distinguished from each other by different type of shades or colours. Example: The following table gives the debt position of Central Government as the amount ($ Crores) outstanding at the end of March 31 of each year. Represent the data by a percentage sub-divided bar diagram.
Note Unit 8: Frequency Distribution 266 Table 8.20: Debt Position of Central Govt Years InternalDebt External Debt Other Liabilities Total : : : : : 2007 08 58537 71039 86313 98646 16637 18153 20299 23223 38267 48422 59934 73692 113441 137614 166546 195561 - 2008-09 2009-10 2010 -11 Solution: The amounts are given in absolute terms, therefore, these are to be converted into percentages in order to show them on a percentage sub-divided bar diagram. Table 8.21: Percentage Value table Years Internal Debt External Debt OtherLiabilities Total : : : : : 2007 08 51 52 52 50 15 13 12 12 34 35 36 38 100 100 100 100 - 2008-09 2009-10 2010 -11 2007-08 2008-09 2009-10 2010-11 (Years) Figure 8.5: Percentage Sub-divided Bar Diagram 6. Deviation Bar Diagram: This diagram represents net quantities like profit and loss, positive and negative balance of trade, surplus and deficit, etc. Positive quantities are shown above X-axis and negative quantities are shown below it. Example: Represent the following data by a suitable diagram. Table 8.22: Net profit/Loss of Airlines . . . . (–) (–) (–) . . . 2003 04 2004 05 2005 06 2006 07 163 1 204 3 206 0 252 2 2007 08 2008 09 2009 10 15 2 64 6 211 0 - - - - - - - Year Profit/Loss(in $ Crores) Year Profit/Loss(in $ Crores)
Business Mathematics Note 267 03-04 04-05 05-06 06-07 07-08 08-09 09-10 Figure 8.6: Deviation Bar Diagram 7. Duo-Directional Bar Diagram: This diagram is used to show an aggregate of two components. One of the components is shown above X-axis and the other below it. Both the components added together give total value. Example: Represent the following data by a Duo-directional bar diagram. Table 8.23: Manufacturing Income and Expenses Years Total Income of a Manufacturer (in $ '000) Manufacturing Expenses (in $ '000) Net Income (in $ '000) 2005 25 10 15 2006 28 15 13 2007 30 15 15 2008 27 16 11 2009 29 17 12 2010 28 18 10 2005 2006 2007 2008 2009 2010 Net Income Manufacturing ($ '000) Exps. ($ '000) Figure 8.7: Duo-Directional Bar Diagram

Note Unit 8: Frequency Distribution 268 8. Sliding Bar Diagram: Sliding bar diagrams are similar to duo-directional bar diagrams. Whereas absolute values are shown by duo-directional bar diagrams, the percentage is shown using sliding bar diagrams. The length of each sliding bar is same, which represents 100%. The bars can be drawn horizontally or vertically. Example: Represent the following data by a sliding bar diagram. Table 8.24: Result of a college in various courses 70 80 90 85 95 30 20 10 15 5 Pass (Percentage) 1. B.A. (Pass) 2. B.A. (Hons.) 3. B.Com. 4. B.Sc. (Gen.) 5. B.Sc. (Hons.) Fail (Percentage) Figure 8.8: Sliding Bar Diagram 9. Pyramid Diagram: This diagram is used to represent the distribution of population according to sex, age, occupation, education, etc. The bars are drawn adjacently one above the other so as to look like a pyramid, as shown in the diagram. Example: Given below are the figures of enrolment in a college during 2005-2010. Represent the data with the help of a suitable diagram. Table 8.25: Figures of Enrolment in a College Years : 2005 2006 2007 2008 2009 2010 Male Students : 800 850 1120 1300 1360 1600 Female Students : 400 450 480 500 540 600 Total : 1200 1300 1600 1800 1900 2200

Business Mathematics Note 269 2005 2006 2007 2008 2009 2010 Figure 8.9: Sex-wise Distribution of Students of a College 10. Broken-Scale Bar Diagram: When there are one or more figures of unusually high magnitude while the majority of the figures are of low magnitude, the diagrammatic representation is done by using a broken scale as shown in the following example. Example: Represent the following data by a suitable diagram: Table 8.26: Profit of a Firm Firms A B C D E Profit in : (') : 000 10 12 15 80 90 $ $ Figure 8.10: Bar Diagram Representing Profits of five Firms Task "Diagrams do not add anything to the meaning of statistics but when drawn and studied intelligently they bring to view the salient characteristics of groups and series". Justify it. Did u Know? A sub-divided or percentage sub-divided bar diagram is also known as a stacked bar diagram.

Note Unit 8: Frequency Distribution 270 Self Assessment 25. One-dimensional diagrams are also known as bar diagrams. 26. In case of one-dimensional diagrams, the magnitude of the characteristics is shown by the length or height of the bar. 27. Sub-divided bar diagram is useful when it is desired to represent the comparative values of different components of a phenomenon. 28. Duo-Directional Bar Diagram is used to show an aggregate of two components. 29. Pyramid Diagram is used to represent the distribution of population according to sex, age, occupation, education, etc. 8.6.2 Two-Dimensional Diagrams One dimensional diagrams are used to represent the values of items with the help of length or height of different bars. In case of a two-dimensional diagram, the value of an item is represented by an area. Such diagrams are also known as 'surface' or 'area diagrams'. Popular forms of two-dimensional diagrams are: 1. Rectangular Diagrams 2. Square Diagrams 3. Circular or Pie Diagrams. 1. Rectangular Diagrams: Rectangular diagrams are used to compare values of different items in two or more situations. These can further be sub-divided into (a) Absolute Diagram and (b) Percentage Diagram. (a) Absolute Diagram: In this diagram the areas corresponding to each item is in proportion to its absolute value. This is illustrated with the help of following example. Example: The detailed cost of production and total revenue of a firm in a particular month are given below. Represent the data by a suitable diagram so that cost and profit per unit of each firm can be compared. Table 8.27: Cost of Production and Total Revenue of a Firm Firm Firm i Raw MaterialCost ii Labour Cost iii OtherOverheadExpenses iv MiscellaneousExpenses a TotalCost b Total Revenue c Profit d No of units Producedand sold X Y ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) . , , , , , , , , , , , , , , 8 000 5 000 2 500 1 000 16 500 20 000 3 500 1 000 5 000 2 500 1 500 500 9 500 12 500 3 000 500

Business Mathematics Note 271 Solution: The figures of total revenue and total cost of each firm will be represented by areas of their respective rectangles where the width of each rectangle will denote the corresponding output of firm. Total length of rectangle for firm X = Total Revenue output = 20,000 1,000 = 20 units Length of the corresponding total cost = 16,500 1000 = 16.5 units ? ? Length for per unit Profit = 20 ? 16.5 = 3.5 units. Similar type of calculations are to be done for firm Y. Figure 8.11: Rectangular Diagram for Comparing Cost and Profit per Unit of the two Firms (b) Percentage Diagram: Here the area corresponding to each item represents its percentage of total. Writing the data of previous example (related to cost of production and total revenue of a firm) as % of total revenue we get Table 8.28: Percentage Table Firm Firm i Raw MaterialCost ii Labour Cost iii OtherOverheadExpenses iv MiscellaneousExpenses v Profit Total X Y ( ) ( ) ( ) ( ) ( ) % % . % % . % % % % % % 40 25 12 5 5 17 5 40 20 12 4 24 100 100 Note that the width of rectangles here will be in the ratio 1000 : 500 as before. 2. Square Diagrams: When the values of items vary widely, the use of rectangular diagrams become very inconvenient. For example, if in the above example firm X produces 10,000 units and firm Y produces 500 units, then the ratio of the width of the two rectangles will be 20 : 1. If rectangular diagrams are drawn in this case, then one of the rectangles would be having an unusually large width than the other and it may be very difficult to draw these rectangles on the same page. A

Note Unit 8: Frequency Distribution 272 more convenient way, in such a case, is to draw squares. For drawing square diagrams the square root of the value of an item is obtained and by suitable choice of scale, the square is drawn with side equal to this value. Figure 8.12 Example Represent the following data by a suitable diagram. Year India s Exports in Crores : ' ( ) : 1972 1976 1980 1984 1988 1823 4970 6591 9981 20295 $ Solution The length of the sides of different squares are calculated as shown below : Year India s Exports in Crores Square Root : ' ( ) : : . . . . . 1972 1976 1980 1984 1988 1823 4970 6591 9981 20295 42 7 70 5 81 2 99 9 142 5 $ Scale in $ Figure 8.13: Square Diagram 3. Circular or Pie Diagrams: In the above example, one can also draw circles in place of squares. The radius of the circle is given by , where A denotes area of the circle whose value is given by the value of an item.

Business Mathematics Note 273 ( ) ÷ π Year : 1972 1976 1980 1984 1988 ' India s Exports X : 1823 4970 6591 9981 20295 $ (in Crores) Radius X :24.1 39.8 45.8 56.4 80.4 ( ) Figure 8.14: Pie diagram 1994 1998 2002 2006 2010 Scale: $ 80 = 0.5 inch Figure 8.15: Pie Diagram In order to show proportions of various components, a circle can also be partitioned into sections in a similar manner as in component bar diagrams. Since there are 360 o at the centre of a circle, these are divided in proportions to the magnitude of values of different items. The diagram, thus obtained is known as Angular Sector Diagram or more popularly as Pie Diagram. The construction of a pie diagram is explained by the following example: Example: Show the following data of expenditure of an average working class family by a suitable diagram. Table 8.29: Expenditure of an Average Working Class Family Item of Expenditure Percent of Total Expenditure (i) Food (ii) Clothing (iii) Housing (iv) Fuel and Lighting (v) Miscellaneous 65 10 12 5 8 The angles of different sectors are calculated as shown below: Figure 8.16: Pie Chart

Note Unit 8: Frequency Distribution 274 Self Assessment Multiple Choice Questions: 30. In order to show .... of various components, a circle can also be partitioned into sections in a similar manner as in component bar diagrams. (a) Ratios (b) Proportions (c) Properties (d) Amount 31. Since there are .... at the centre of a circle, these are divided in proportions to the magnitude of values of different items. The diagram, thus obtained is known as Pie Diagram. (a) 90º (b) 180º (c) 270º (d) 360º 32. Angular Sector Diagram is more popularly known as .... (a) Pie diagram (b) Bar diagram (c) Histogram (d) Pyramid Diagram 8.6.3 Three Dimensional Diagrams With the help of three dimensional diagrams, the values of various items are represented by the volume of cube, sphere, cylinder, etc. These diagrams are normally used when the variations in the magnitudes of observations are very large. Example: The production figures (in thousand tons) of three companies A, B and C in a particular year are 100, 630 and 1,750 respectively. Represent the data by a three dimensional diagram. Solution: Since the three dimensional diagram is not specified, we shall use cubes for the representation of data. The procedure for the representation is as shown below: Table 8.30: Production Figure of Three Companies Production Cube root of Side of Company (in '000 tons) the output the cube A 100 4.641 1.00 cm B 630 8.572 1.85 cm C 1750 12.050 2.60 cm * Scale: 1 cm = 4641 units Figure 8.18: Three Dimensional Diagram Representing production of Three Companies in a particular year

Business Mathematics Note 275 Pictogram When numerical figures are represented by pictures, we get pictogram. Although such diagrams are very attractive and easier to understand, they are difficult to be drawn by everybody. The following pictogram represents the number of students in a college during the four academic sessions. Table 8.31: Number of Students in a College during the Four Academic Sessions Academic Sessions : No. of Students : 1625 2000 2250 3000 2010-11 2011-12 2012-13 2013-14 2010-11 2011-12 2012-13 2013-14 Figure 8.19: No. of Students in a College Cartogram or Map Diagram Cartograms are used to represent data relating to a particular country or to a geographical area. Such a diagram can be used to represent various types of characteristics like density of population, yield of a crop, amount of rainfall, etc. Figure 8.20: Choice of A Suitable Diagram Diagrammatic presentation of data can be done in various ways. The choice of a suitable diagram is a practical problem and should be done in the light of the following considerations: (i) The nature of data (ii) Purpose of the diagram (iii) The calibre of the persons to whom the information is to be communicated.

Note Unit 8: Frequency Distribution 276 The choice of a suitable diagram depends upon the nature of the given data. It may be recalled that two or three-dimensional diagrams are more appropriate if there are large variations in the magnitudes of observations. Many a times, the purpose of drawing a diagram may also give a clue to its choice. For example, if it is desired to indicate the comparison of values relating to different situations, bar diagrams will be most suitable. Further, if one wishes to indicate various components of a characteristics, sub-divided bar diagrams can be used. The relative importance of various components can be shown by using percentage sub-divided bar diagram. When the number of components become very large, i.e., more than three or four, circular diagrams are preferred because bar diagrams look more crowded. If the statistical data consists of a series of observations with different components for each observation, percentage sub-divided bar diagrams are more suitable than the circular diagrams. Caution! Before the choice of a suitable diagram, it is very necessary to know the level of education of the person for whom the diagram is to be drawn. For persons with little knowledge of statistics, the pictograms or cartograms may be more suitable. Further, if the data are related to different geographical areas, the cartograms may be the most appropriate choice. Utility and Advantages of Diagrammatic Presentation Data presented in the form of diagrams are useful as well as advantageous in many ways, as is obvious from the following : (i) Diagrams are attractive and impressive: Data presented in the form of diagrams are able to attract the attention of even a common man. It may be difficult for a common man to understand and remember the data presented in the form of figures but diagrams create a lasting impression upon his mind. Due to their attractive and impressive character, the diagrams are very frequently used by various newspapers and magazines for the explanation of certain phenomena. Diagrams are also useful in modern advertising campaign. (ii) Diagrams simplify data: Diagrams are used to represent a huge mass of complex data in simplified and intelligible form which is easy to understand. (iii) Diagrams give more information: In addition to the depiction of the characteristics of data, the diagrams may bring out other hidden facts and relations which are not possible to know from the classified and tabulated data. (iv) Diagrams save time and labour: A lot of time is required to study the trend and significance of voluminous data. The same data, when presented in the form of diagrams, can be understood in practically no time. (v) Diagrams are useful in making comparisons: Many a times the objective of the investigation is to compare two or more situations either with respect to time or places. The task of comparison can be very conventionally done by the use of diagrams. (vi) Diagrams have universal applicability: Diagrams are used in almost in every field of study like economics, business, administration, social institutions and other fields.

Business Mathematics Note 277 Limitations Inspite of the above advantages of diagrams, their usefulness is somewhat limited. One has to be very careful while drawing conclusions from diagrams. Their main limitations are : (i) Diagrams give only a vague idea of the problem which may be useful for a common man but not for an expert who wishes to have an exact idea of the problem. (ii) Diagrams can at best be a supplement to the tabular presentation but not an alternative to it. (iii) The information given by the diagrams vis-a-vis classification and tabulation is limited. (iv) The level of precision of values indicated by diagrams is very low. (v) Diagrams are helpful only when comparisons are desired. They don't lead to any further analysis of data. (vi) Diagrams can portray only limited number of characteristics. Larger the number of characteristics the more difficult it is to understand them using diagrams. (vii) Diagrams are liable to be misused for presenting an illusory picture of the problem. (viii) Diagrams don't give a meaningful look when different measurements have wide variations. (ix) Diagrams drawn on a false base lines should be analysed very carefully. Self Assessment State whether the following statements are true or false: 33. When numerical figures are represented by pictures, we get pictogram. 34 . Cartograms are used to represent data relating to a particular country or to a geographical area. Graphic Presentation Graphic presentation is another way of pictorial presentation of the data. Graphs are commonly used for presentation of time series and frequency distributions. In situations where the diagrams as well as the graphs can be used, the later is preferred because of its advantages over the former. Graphic presentation of data, like diagrammatic presentation, also provides a quick and easier way of understanding various trends of data and to facilitate the process of comparison of two more situations. In addition to this, it can also be used as a tool of analysis. Graphic methods are sometimes used in place of mathematical computations to save time and labour, e.g., free hand curves may be fitted in place of mathematical curve to determine trend values. Advantages of Graphic Presentation A properly constructed graph may provide more information than the tabular or diagrammatically presented data. Graphic presentation may indicate the nature of

Note Unit 8: Frequency Distribution 278 trend present and the manner in which it is likely to change in future. Various advantages of a graphic presentation are : (i) Graphs provide an attractive and lasting effect. (ii) Graphs are easy to understand. (iii) Graphs provide easy comparison of two or more phenomena. (iv) Graphs provide a method of locating certain positional averages like median, mode, quartiles, etc. It can also be used to study correlation between two variables. (v) Graphs also facilitate the process of interpolation, extrapolation and forecasting. (vi) It saves time and energy of the statistician as well as of the observer. (vii) It can indicate the nature and the direction of trend of the data. Construction of a Graph A point in a plane can be located with reference to two mutually perpendicular lines. The horizontal line is called the X- axis and the vertical line as the Y- axis. Their point of intersection is termed as origin. The position of a point in a plane is located by its distance from the two axes. If a point P is 4 units away from Y- axis and 3 units away from X- axis, its location will be as shown in the Figure 4.4. Figure 8.21: Construction of a Graph It should be noted here that the distance of the point from Y- axis is measured along X- axis and its distance from X- axis is measured along Y- axis. To measure 4 units from Y- axis, one moves 4 units along X- axis and erects a perpendicular at this point. Similarly, to measure 3 units from X- axis, one moves 3 units along Y- axis and erects a perpendicular. The point of intersection of these two perpendiculars will be the required point. The position of a point is denoted by a pair of numbers, e.g., (4,3) for the point P, that are respectively termed as abscissa and ordinate of the point. Jointly they are termed as the coordinates of a point. The coordinates of a point, in general form, are written as (x, y). The four parts of the plane are called quadrants, as shown in the above figure. It may be noted that both x and y are positive in the first, x is negative and y is positive in the second, x and y are both negative in the third and x is positive and y is negative in the fourth quadrant.

Business Mathematics Note 279 Different points can be plotted for a different pairs of values, e.g., for data on demand of a commodity at different prices, we can locate a point for each pair of quantity and price combination. These points are then joined by a curve or a line to get the required graph. General Rules for the Graphic Presentation (i) Every graph must have a suitable title written at its top. This title should indicate the facts presented by the graph in a comprehensive and unambiguous manner. (ii) By convention, the independent variable is normally measured along X- axis and the dependent variable on Y- axis. The scale on Y- axis must always start from zero. If the fluctuations are small as compared to the size of the variable, there is no need to show the entire vertical axis from origin. This can be done by showing a gap in the vertical axis and drawing a horizontal line from it. This line is often termed as a false base line. (iii) The choice of a scale of measurement should be such that the whole data can be accommodated in the available space and all of its important fluctuations are clearly depicted. (iv) Proportional changes in the values of the variables can be shown by drawing a ratio or logarithmic scale. (v) A graph must not be overcrowded with curves. (vi) When more than one curve is to be shown on the same graph, it is necessary to distinguish them by drawing curves of different pattern or colour. Some common patterns are: (vii) An index should always be given to show the scales and the interpretations of different curves. (viii) The source of data should be mentioned as a footnote. Figure 8.22: Interpretations of Different Curves Difference between a Diagram and a Graph A brief distinction between a diagram and a graph is given below.

Note Unit 8: Frequency Distribution 280 Table 8.32: Difference between a Diagram and a Graph Diagram Graph 1. Can be drawn on an ordinary paper. 1. Can be drawn on a graph paper. 2. Easy to grasp. 2. Needs some effort to grasp. 3. Not capable of analytical treatment. 3. Capable of analytical treatment. 4. Can be used only for comparisons. 4. Can be used to represent a mathematical relation. 5. Data are represented by bars, rectangles, pictures, etc. 5. Data are represented by lines and curves. A graphic presentation is used to represent two types of statistical data : (i) Time Series Data and (ii) Frequency Distribution. 8.7 Graphs of a Frequency Distribution A frequency distribution can also be represented by means of a graph. The most common forms of graphs of a frequency distribution are: 1. Histogram: A histogram is a graph of a frequency distribution in which the class intervals are plotted on X- axis and their respective frequencies on Y-axis. On each class, a rectangle is erected with its height proportional to the frequency density of the class. (a) Construction of a Histogram when Class Intervals are equal: In this case the height of each rectangle is taken to be equal to the frequency of the corresponding class. The construction of such a histogram is illustrated by the following example. Example The frequency distribution of marks obtained by 60 students of a class in a college is given below: Marks : 30-34 35-39 40-44 45-49 50-54 55-59 60-64 No. of Students : 3 5 12 18 14 6 2 Draw a histogram for the distribution. Solution: Since the upper limit of a class is not equal to the lower limit of its following class, the class boundaries will have to be determined. The distribution after adjustment will be as given below. Marks No. of Students 29.5-34.5 3 34.5-39.5 5 39.5-44.5 12 44.5-49.5 18 49.5-54.5 14 54.5-59.5 6 59.5-64.5 2

Business Mathematics Note 281 Figure 8.23: Histogram (b) Construction of a Histogram when Class Intervals are not equal: When different classes of a frequency distribution are not equal, the frequency density (frequency ¸ width) of each class is computed. The product of frequency density and the width of the class having shortest interval is taken as the height of the corresponding rectangle. Example: Represent the following frequency distribution by a histogram. Class Intervals : 0-10 10-15 15-30 30-40 40-60 60-80 80-90 90-100 Frequency : 8 10 36 44 52 20 16 10 Solution: The height of the rectangle = Frequency Density ? Shortest Class Width Figure 8.33: Frequency Distribution = ? Figure 8.24: Histogram

Note Unit 8: Frequency Distribution 282 Caution! If the mid points of various classes are given in place of class intervals then these must first be converted into classes. 2. Frequency Polygon: A frequency polygon is another method of representing a frequency distribution on a graph. Frequency polygons are more suitable than histograms whenever two or more frequency distributions are to be compared. A frequency polygon is drawn by joining the mid-points of the upper widths of adjacent rectangles, of the histogram of the data, with straight lines. Two hypothetical class intervals, one in the beginning and the other in the end, are created. The ends of the polygon are extended upto base line by joining them with the mid-points of hypothetical classes. This step is necessary for making area under the polygon to be approximately equal to the area under the histogram. Frequency polygon can also be constructed without making rectangles. The points of frequency polygon are obtained by plotting mid-points of classes against the heights of various rectangles, which will be equal to the frequencies if all the classes are of equal width. Example The daily profits (in rupees) of 100 shops are distributed as follows : Figure 8.34: Frequency Distribution Table No. of Shops : 12 18 27 20 17 6 Construct a frequency polygon of the above distribution. Solution: Figure 8.25: Frequency Polygon Example. Represent the following data by a frequency polygon. Figure 8.35: Frequency Distribution Table Frequency : 10 16 18 15 12 5 2 Business Mathematics Note 283 Solution: Here the frequency polygon is drawn by plotting mid-points of class intervals against their respective frequencies. Figure 8.26: Frequency Polygon 3. Frequency curve: When the vertices of a frequency polygon are joined by a smooth curve, the resulting figure is known as a frequency curve. As the number of observations increases, there is need of having more and more classes to accommodate them and hence the width of each class will become smaller and smaller. In such a situation the variable under consideration tend to become continuous and the frequency polygon of the data tends to acquire the shape of a frequency curve. Thus, a frequency curve may be regarded as a limiting form of frequency polygon as the number of observations become large. The construction of a frequency curve should be done very carefully by avoiding, as far as possible, the sharp and sudden turns. Smoothing should be done so that the area under the curve is approximately equal to the area under the histogram. A frequency curve can be used for estimating the rate of increase or decrease of the frequency at a given point. It can also be used to determine the frequency of a value (or of values in an interval) of the variable. This method of determining frequencies is popularly known as interpolation method. 4. Cumulative Frequency Curve or Ogive The curve obtained by representing a cumulative frequency distribution on a graph is known as cumulative frequency curve or ogive. Since a cumulative frequency distribution can be of 'less than' or 'greater than' type, accordingly, there are two types of ogive, 'less than ogive' and 'more than ogive'. An ogive is used to determine certain positional averages like median, quartiles, deciles, percentiles, etc. We can also determine the percentage of cases lying between certain limits. Various frequency distributions can be compared on the basis of their ogives. Example Draw 'less than' and 'more than' ogives for the following distribution of monthly salary of 250 families of a certain locality. Figure 8.36: Distribution of Monthly Salary of 250 Families Income Intervals 0−500 500−1000 1000−1500 1500−2000 2000−2500 2500−3000 3000−3500 3500−4000 No. of Families 50 80 40 25 25 15 10 5

Note Unit 8: Frequency Distribution 284 Solution: First we construct 'less than' and 'more than' type cumulative frequency distributions. Figure 8.37: Less than and More than Type Income less than Cumulative Frequency Income more than Cumulative Frequency 500 50 0 250 1000 130 500 200 1500 170 1000 120 2000 195 1500 80 2500 220 2000 55 3000 235 2500 30 3500 245 3000 15 4000 250 3500 5 Ogive Figure 8.27: Ogive We note that the two ogives intersect at the median. Task Look at the Graph (Figure 8.28) and answer the following questions: 1. How many turtles are 1 year old? 2. How many turtles are 2 years old? 3. How many turtles are 3 years old? 4. How many turtles are 4 years old? 5. How many turtles are 5 years old? 6. How many turtles lived in the pond? 7. What percentage of turtles in the pond are 2 years old? 8. What fraction of turtles in the pond are 4 years old? 9. Turtles of which age make up the largest fraction of the turtles in the pond?

Business Mathematics Note 285 Figure 8.28: Frequency Distribution Case Study: ASSOCHAM Associated Chamber of Commerce and Industry (ASSOCHAM) is very much concerned about the employment of youths and their pay rolls in small engineering industries, with special reference to automobile parts manufacturing, transport for hire, taxis, dealers of new and old vehicles, petrol stations and automobile repair garages. The chamber has employed you to collect the data regarding employment and pay role as on 31st March, 2014 and present it suitably through diagram so that it can be include in the final memorandum to be submitted to Minister for Industries. The data that you have collected is as follows: Industry Employment on 31-3-2014 Avg. Earnings per employee per year ($) 1. Automobile parts manufacturers 4,34,856 56,540 2. Transport for hire 15,26,897 26,348 3. Taxis 11,32,560 42,685 4. Dealers of new and used vehicles 1,09,805 13,684 5. Retail filling stations 22,25,960 15,008 6. Automobile repair garages 12,35,200 12,048 Question: Present the data using a suitable diagram(s) so as to bring out the finer points.

Note Unit 8: Frequency Distribution 286 Self Assessment Fill in the blanks 35. .... are commonly used for presentation of time series and frequency distributions 36. .... methods are sometimes used in place of mathematical computations to save time and labour. 37. Graphs provide .... comparison of two or more phenomena. 38. Graphs also facilitate the process of interpolation, extrapolation and .... 39. Every graph must have a suitable title written at its .... 40. A graph ....be overcrowded with curves 8.8 Summary ? The diagrammatic presentation of data provides a quick and an easier way to understand the broad nature and trends of the given data. ? Diagrams are capable of being understood easily even by a common man. In addition to this, they facilitate the process of comparison of data in two or more situations. ? While using diagrams, their limitations must always be kept in mind. ? Diagrams give only a vague idea of the problem and therefore, cannot be used as a substitute for classification and tabulation. ? The diagrams can portray only a limited number of characteristics and are no longer useful when the number of characteristics become large. ? The main limitation of the diagrams being that these cannot be used as a tool of analysis. ? Various types of diagrams can be divided into five broad categories, viz. one- dimensional, two-dimensional, three-dimensional, pictograms and cartograms. ? Some important one-dimensional diagrams are line diagram, bar diagram, multiple bar diagram, component bar diagram, etc. ? Rectangular, square and circular diagrams are examples of two-dimensional diagrams. ? Cubes sphere and cylinder, etc., are three-dimensional diagrams. ? The diagrams can also be constructed by using relevant pictures or maps. 8.9 Keywords Frequency distribution: A statistical series, in which data are arranged according to magnitude of one or more characteristics, is known as a frequency distribution. Bar diagrams: One-dimensional diagrams are also known as bar diagrams. Broken-Scale Bar Diagram: When there are one or more figures of unusually high magnitude while the majority of the figures are of low magnitude, the diagrammatic representation is done by using a broken scale.

Business Mathematics Note 287 Cartograms: Cartograms are used to represent data relating to a particular country or to a geographical area. Such a diagram can be used to represent various types of characteristics like density of population, yield of a crop, amount of rainfall, etc. Deviation Bar Diagram: This diagram represents net quantities like profit and loss, positive and negative balance of trade, surplus and deficit, etc. Positive quantities are shown above X- axis and negative quantities are shown below it. Duo-Directional Bar Diagram: This diagram is used to show an aggregate of two components. One of the components is shown above X-axis and the other below it. Both the components added together give total value. Line Diagram: In case of a line diagram, different values are represented by the length of the lines, drawn vertically or horizontally. Multiple Bar Diagram: This type of diagram, also known as compound bar diagram, is used when comparisons are to be shown between two or more sets of data. A set of bars for a period or a related phenomena are drawn side by side without gaps. One-dimensional diagrams: In case of one-dimensional diagrams, the magnitude of the characteristics is shown by the length or height of the bar. The width of a bar is chosen arbitrarily so that the constructed diagram looks more elegant and attractive. Percentage Sub-Divided Bar Diagram: A sub-divided diagram is used to show absolute magnitudes of various components. These magnitudes can be changed into relative by converting them as a percentage of the total. Pictogram: When numerical figures are represented by pictures, we get pictogram. Pie Diagram: In order to show proportions of various components, a circle can also be partitioned into sections in a similar manner as in component bar diagrams. Since there are 360º at the centre of a circle, these are divided in proportions to the magnitude of values of different items. The diagram, thus obtained is known as Angular Sector Diagram or more popularly as Pie Diagram. Pyramid Diagram: This diagram is used to represent the distribution of population according to sex, age, occupation, education, etc. The bars are drawn adjacently one above the other so as to look like a pyramid. Simple Bar Diagram: In case of a simple bar diagram, the vertical or horizontal bars, with height proportional to the value of the item, are constructed. The width of a bar is chosen arbitrarily and is kept constant for every bar. Sliding Bar Diagram: Sliding bar diagrams are similar to duo-directional bar diagrams. Whereas absolute values are shown by duo-directional bar diagrams, the percentage is shown using sliding bar diagrams. The length of each sliding bar is same, which represents 100%. The bars can be drawn horizontally or vertically. Sub-divided or Component Bar Diagram: In case of a sub-divided bar diagram, the bar corresponding to each phenomenon is divided into various components. The portion of the bar occupied by each component denotes its share in the total. 8.10 Review Questions 1. Describe the merits and limitations of the diagrammatic presentation of data. 2. What are different types of diagram which are used in statistics to show salient characteristics of groups and series? Illustrate your answer with examples. 3. What are the advantages of presentation of data through diagram? Give brief description of various types of diagram. Note Unit 8: Frequency Distribution 288 4. Explain clearly the necessity and importance of diagrams in statistics. What precautions should be taken in drawing a good diagram? 5. Describe, with suitable examples, the following type of diagrams: (a) Bar Diagram (b) Multiple Bar Diagram (c) Pie Diagram (d) Pictogram 6. Describe, in brief, different types of two-dimensional diagrams. 7. Discuss the usefulness of diagrammatic representation of facts and explain how would you construct circular diagrams? 8. Represent the following data by a line diagram: S.No. : 1 2 3 4 5 6 7 8 9 10 Weekly Income($) : 240 270 315 318 330 345 354 360 375 390 9. Represent the following data by multiple bar diagram: $ 10. The following table gives the support price of Rabi-crops during 2009-2010 and 2010-2011. Represent the given data by a suitable diagram. 2009-10 2010-11 $ 11. The following table shows the yield of wheat in five countries during 2010. Represent the data by a suitable diagram. Country : China EEC USA India Canada Yield of Wheat (in Kg./ hect.) : 3194 5118 2656 2125 2272

Business Mathematics Note 289 12. Represent the increase or decrease of production of the year 2010 through bar diagram: (a) Increase in production of cotton textile industry = 60% (b) Increase in the production of iron and steel industry = 50% (c) Decrease in sugar production = 40% (d) Decrease in cement production = 30% 13. Show the following data by means of a pie diagram: Area-wise exports from India during 2010-11. West Europe : 31.38% Asia : 31.04% America : 16.49% East Europe : 18.24% Africa and others : 2.85% Answers: Self Assessment 1. observation. 2. measured observations 3. interval and ratio 4. frequency 5. variable 6. raw 7. True 8. True 9. True 10. True 11. True 12. True 13. True 14. True 15. True 16. True 17. True 18. impressive 19. simplify 20. comparisons 21. an illusory 22. diagrammatic presentation 23. art 24. nature 25. True 26. True 27. True 28. True 29. True 30. (b) 31. (d) 32. (a) 33. True 34. True 35. Graphs 36. Graphic 37. easy 38. forecasting 39. top 40. must not

Note Unit 8: Frequency Distribution 290 8.11 Further Readings Books Kenneth Rosen, Discrete Mathematics and Its Applications, 7/e, 2012, McGraw-Hill. Bathul, Shahnaz, Mathematical Foundations of Computer Science, PHI Learning. Lambourne, Robert, Basic Mathematics for the Physical Sciences,2008, John Wiley & Sons. Bari, Ruth A.; Frank Harary. Graphs and Combinatorics, Springer. F. Ernest Jerome, Connect for Jerome, Business Mathematics in Canada, 7e, Canadian Edition. S Rajagopalan and R Sattanathan, Business Mathematics, 2 edition, 2009, Tata McGraw Hill Education. Garrett H.E. (1956), Elementary Statistics, Longmans, Green & Co., New York. Guilford J.P. (1965), Fundamental Statistics in Psychology and Education, Mc Graw Hill Book Company, New York. Hannagan T.J. (1982), Mastering Statistics, The Macmillan Press Ltd., Surrey. Lindgren B.W (1975), Basic Ideas of Statistics, Macmillan Publishing Co. Inc., New York. Selvaraj R., Loganathan C., Quantitative Methods in Management. Sharma J.K., Business Statistics, Pearson Education Asia Walker H.M. and J. Lev, (1965), Elementary Statistical Methods, Oxford & IBH Publishing Co., Calcutta. Wine R.L. (1976), Beginning Statistics, Winthrop Publishers Inc., Massachusetts. Online links http://www.staff.vu.edu.au/mcaonline/units/statistics/presentation.html http://www.slideshare.net/edithosb/graphic-presentation-of-data http://nos.org/318courseE/L- 6%20PRESENTATION%20OF%20STATISTICAL%20DATA.pdf http://www.stars.rdg.ac.uk/data.html http://www.mathsisfun.com/data/bar-graphs.html http://www.mathsisfun.com/data/pie-charts.html http://www.mathsisfun.com/data/pictographs.html http://www.mathsisfun.com/data/histograms.html

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n n n n r ? ? ? ? (1)(2).....(1) n r p n n n n r ? ? ? ? ? ? In factorial notation, (1)(2).....(1) n r p n n n n r ? ? ? ? ? can be written as (1)(2).....(1)( )! ( )! n r n n n n r n r p n r ? ? ? ? ? ? ? ! ( )! n r n p n r ? ? ? Did u Know? Particular Cases If 0 ! ! 0,then 1 (0)! ! n n n r p n n ? ? ? ? ? If ! ! ,then ! ( )! 0! n n n n r n p n n n ? ? ? ? ? If 1 ! (1)! 1,then (1)! (1)! n n n n r p n n

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permutations of n objects taking all at a time is given by = n(n – 1)(n – 2) ...... 3.2.1 = n! 2. Permutations of n objects taking r at a time:

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r ≤ n. Thus, = n(n – 1)(n – 2) ...... [n – (r – 1)] = n(n – 1)(n – 2) ...... (n – r + 1). On multiplication and division of the R.H.S. by (n – r)!, we get ( )( ) ( )( ) ( ) n r n n 1 n 2 .... n r 1 n r ! n! P n r ! (n r)! - - - + - = = - - 3.

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n n p p ? find n. Solution: 4 2 12 n n p p ? (1)(2)(3) 12 (1)(2)(3) n n n n n n n n ? ? ? ? ? ? ? 2 2 5 6 12 0 5 6 0 (6)(1) 0 6 0, 1 0 6, 1 n n n n n n n n n n ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? But n cannot be –1 6 n? ? Example: Find the number of permutations of the letters of the

n-1) 2 3 – 1 3 3.1 3 + 3.1 + 1 .... (n) Adding these n n + 1) 3 – 1 3 3(1 2 + 2 2 + ... + n 2) + 3(1 + 2 + 3 + ... + S 2 + n(n+1) + n 3S 2 (n + 1) 3 – 1 3 – 3 (n + 1) – n = n 3 + 3n 2 + 3n – 3 (n + 1) – n = {2n 2 + 6 n + 6 – 3n – 3 – 2} = {2n 2 + 3n + 1} = (n + 1) (2n + 1), S 2 = (Let S 3 be the sum of the cubes of the

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n n n n ? ? ? ? ? ? Solution: (2)! (2)(2 1)(2 2)(2 3)(2 4)......4 3 2 1 n n n n n n ? ? ? ? ? ? ? ? (2)! (2)(2 2)(2 4)......4 2 1 3....(2 1) 2 2(1) 2(2).....2 2 2 1 3...(2 1) (2 2 2....2) (1)(2)...1 1 3...(2 1) 2 (!)[1 3 5...(2 1)] n n n n n n n n n n n n n n n n n ?

n – 2) 2 + 3(n – 2) + 1 .... (3) - - - - - - - - - - - 3 3 – 2 3 3.2 3 + 3.2 + 1 .... (n) Adding n identities (n + 1) 3 – 1 3 3(1 2 + 2 2 + ... + n 2) + 3(1 + 2 + 3 + ... + n) + 2 3S 2 + n(n+1) + n 3S 2 (n + 1) 3 – 1 3 – 3 (n + 1) – n = n 3 + 3n 2 + 3n – 3 (n + 1) – n = {2 n 2 + 6 n – 3 – 2} = {2n 2 + 3n + 1} = (n + 1) (2n + 1),

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8/248	SUBMITTED TEXT	90 WORDS	35%	MATCHING TEXT	90 WORDS

The number of permutations of 4 things taken 3 at a time is 4 3 4! 4! 4 3 2 1 24 (4 3)! p ? ? ? ? ? ? ? ? ? . The number of permutations of 4 things taken all at a time is 4 4 4! 4! 4 3 2 1 24 0! p ? ? ? ? ? ? ? ? Permutation of things when some of them are alike The number of permutations of n things when p things are alike of one kind, q things are alike of second kind, r things are alike of

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9/248	SUBMITTED TEXT	17 WORDS	89%	MATCHING TEXT	17 WORDS

Example 1. In how many ways can the letters of the word EDUCATION be arranged?

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10/248	SUBMITTED TEXT	15 WORDS	88%	MATCHING TEXT	15 WORDS

In how many ways can the letters of the word STATISTICS be arranged?

SA Objective Mathematics -II (BMGE- 02)- FINAL.pdf (D162012805)

11/248	SUBMITTED TEXT	18 WORDS	75%	MATCHING TEXT	18 WORDS

In how many ways 4 men and 3 women can be seated in a row such that

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12/248	SUBMITTED TEXT	40 WORDS	39%	MATCHING TEXT	40 WORDS

In how many ways 4 men and 4 women can be seated such that men and women occupy alternative places? Solution 1. 4 men can be seated in 4! ways and 3 women can be seated in 3! ways.

SA Objective Mathematics -II (BMGE- 02)- FINAL.pdf (D162012805)

13/248	SUBMITTED TEXT	21 WORDS	72%	MATCHING TEXT	21 WORDS

In how many ways 7 boys and 7 girls be seated at a round table so that no two girls

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14/248	SUBMITTED TEXT	14 WORDS	85%	MATCHING TEXT	14 WORDS

the number of permutations of n objects taking r at a time is n

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15/248	SUBMITTED TEXT	27 WORDS	44%	MATCHING TEXT	27 WORDS

the number of combinations of n objects taking r at a time, denoted by n r C , can be obtained by dividing by r!, i.e., . ( ) n n r

SA Objective Mathematics -II (BMGE- 02)- FINAL.pdf (D162012805)

16/248	SUBMITTED TEXT	103 WORDS	33%	MATCHING TEXT	103 WORDS

n r C Theorem 1 : ! n n r r P C r ? Theorem 2 : n n r n r C C ? ? Theorem 3 : 1 1 n n n r r r C C C ? ? ? ? Particular Cases 0 ! ! 1 (0)! 0! ! n n n C n n ? ? ? ? 1 ! ! (1)! (1)! 1! (1)! (1)! n n n n n C n n n n ? ? ? ? ? ? ? ? ! ! ! ! 1 ( )! ! 0! ! 0! ! ! n n n n n C n n n n n n ? ? ? ? ? ? Business Mathematics Note 51 Note: (a) Since n n r n r

SA Maths Extended Essay.pdf (D31546002)

17/248	SUBMITTED TEXT	31 WORDS	90%	MATCHING TEXT	31 WORDS

n C n 1 ´ n - n 1 C n 2 = n! n 1 !n 2 !n 3 ! . 2.

n C 0 = 1 n C 1 = n, n C 2 = n (n – 1), n C 3 = n (n – 1) (

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

18/248	SUBMITTED TEXT	23 WORDS	66%	MATCHING TEXT	23 WORDS

number of combinations of n distinct objects taking 1, 2, ... n respectively, at a time is n n n n 1 2

SA UNIT -1 -ECE18R316 -PTAND RP.ppt (D108941599)

19/248	SUBMITTED TEXT	39 WORDS	48%	MATCHING TEXT	39 WORDS

n C C ? Solution: 10 4 n n C C ? 10 4 ( ) 4 10 14 n n n n r n r C C C C n ? ? ? ? ? ? ? ? ? ? Example 3: From a group of 20 people,

SA CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

20/248	SUBMITTED TEXT	35 WORDS	71%	MATCHING TEXT	35 WORDS

can be chosen in 4 1 4 C ? ways. 2. goods can be chosen in 4 2 4 3 6 2 1 C ? ? ? ? ways. 3. goods can be chosen in 4 4 3 1 4 C C ? ? ways. 4.

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21/248	SUBMITTED TEXT	68 WORDS	27%	MATCHING TEXT	68 WORDS

In how many ways can 5 questions be selected? Solution: 5 questions can be selected in the following ways: (i) 2 and 3: This can be done in 4 4 2 3 C C ? ways 4 3 4 24 2 1 ? ? ? ? ? (ii) 3 and 2: This can be done in 4 4 3 2 C C ? ways 4 3 4 24 2 1 ? ? ? ? ? ?? 5

SA Objective Mathematics -II (BMGE- 02)- FINAL.pdf (D162012805)

22/248	SUBMITTED TEXT	27 WORDS	50%	MATCHING TEXT	27 WORDS

number of permutations of n distinct objects is n!, i.e., n r P = n! ? Permutation of n objects taking r at a time: ! ( )!

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23/248	SUBMITTED TEXT	30 WORDS	56%	MATCHING TEXT	30 WORDS

n p n n r c r r r n r ? ? ? ? n c n-r n c r ? ? The total number of combinations of n distinct objects at a time = 2

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24/248	SUBMITTED TEXT	15 WORDS	88%	MATCHING TEXT	15 WORDS

In how many ways can the letters of the word MATHEMATICS be arranged? 3.

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25/248	SUBMITTED TEXT	16 WORDS	100%	MATCHING TEXT	16 WORDS

In how many ways can 3 boys and 5 girls be arranged in a row,

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26/248	SUBMITTED TEXT	17 WORDS	66%	MATCHING TEXT	17 WORDS

be arranged in a row if balls of the same colour are to be together. 11.

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27/248	SUBMITTED TEXT	34 WORDS	50%	MATCHING TEXT	34 WORDS

n C C ? (ii) 30 5 n n C C ? (iii) 18 18 2 n n C C ? ? (iv) 2 3 2 : 44 : 3 n n C C ? (v) 6 4 3 2 n n C

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28/248	SUBMITTED TEXT	21 WORDS	71%	MATCHING TEXT	21 WORDS

the number of subsets of a set with n elements is 2 n . If the number of

The number of circular permutations of a set S with n elements is (n−1)!. Properties[edit] The number of

W http://en.wikipedia.org/wiki/Permutation

29/248	SUBMITTED TEXT	21 WORDS	50%	MATCHING TEXT	21 WORDS

For example, the set of all real numbers, the set of all firms in an industry, the set of all

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30/248	SUBMITTED TEXT	22 WORDS	83%	MATCHING TEXT	22 WORDS

Set Theory 72 3. A set is a collection of .... objects 4. The objects of a set are

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31/248	SUBMITTED TEXT	15 WORDS	78%	MATCHING TEXT	15 WORDS

Representation of a Set A set can be represented in the following two ways: (

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32/248	SUBMITTED TEXT	14 WORDS	84%	MATCHING TEXT	14 WORDS

Singleton set: A set containing only one element is called a singleton set.

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33/248	SUBMITTED TEXT	40 WORDS	42%	MATCHING TEXT	40 WORDS

A is also an element of the set B. We write it as A ? B, and read as 'A is a subset of B 'or' A is contained in B'. In symbols, if A &

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34/248	SUBMITTED TEXT	84 WORDS	17%	MATCHING TEXT	84 WORDS

B. Then A is a subset of B. Note: For every set A, A ? A, i.e., A is itself a subset of A. Also note that the empty set ? is always subset of every set. If A is not a subset of B. We denote A B. E.g.: If A = {a, b, c} and B = {a, b, c, e}. Then clearly A ? B. 4. Proper subset: If A is a subset of B

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35/248	SUBMITTED TEXT	51 WORDS	40%	MATCHING TEXT	51 WORDS

A, then A is called the proper subset of B. if A = {1, 2, 3} and B = {1, 2, 3, 4}. Then A is a proper subset of B, and is denoted by A ? B, read as ‘A is a proper subset of B’.

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36/248	SUBMITTED TEXT	48 WORDS	46%	MATCHING TEXT	48 WORDS

A ? B or B ? A. I. If A = {1, 2, 3}, B = {1, 2, 3, 4, 5}. Then A ? B so that A and B are comparable. II. If A = {a, b, c} and B = {a}.

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37/248	SUBMITTED TEXT	22 WORDS	92%	MATCHING TEXT	22 WORDS

Submitted text	Matching text
if every element of A is an element of B, and every element of B is also an element of A.	if every element of A is an element of B, and every element of B is an element of A;

W http://en.wikipedia.org/wiki/Set_(mathematics)

38/248	SUBMITTED TEXT	16 WORDS	96%	MATCHING TEXT	16 WORDS

Equal sets: Two sets are said to be equal if they contain the same elements,

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39/248	SUBMITTED TEXT	18 WORDS	78%	MATCHING TEXT	18 WORDS

a set with finite number of elements is a finite set. E.g.: I. The set of

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40/248	SUBMITTED TEXT	63 WORDS	47%	MATCHING TEXT	63 WORDS

all subsets of a given set A, is called the power set A. The power set A is denoted by P (A). E.g.: If A = {a, b, c}. Then its subsets are ?, {a}, {b}, {c}, {a, b},{a, c},{b, c},{a, b, c} P (A) = {?, {a}, {b}, {c},{a, b}, {a, c},{b, c},{a, b, c}} 12.

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41/248	SUBMITTED TEXT	17 WORDS	70%	MATCHING TEXT	17 WORDS

if every element of the set A is also an element of the set B 14.

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42/248	SUBMITTED TEXT	17 WORDS	78%	MATCHING TEXT	17 WORDS

B is called the union of A and B, denoted by A ∪ B. The

B, is called the Cartesian product of A and B and is denoted by A B. The

W http://www.cs.odu.edu/~toida/nerzic/level-a/set/set_operations.html

43/248	SUBMITTED TEXT	20 WORDS	83%	MATCHING TEXT	20 WORDS

The set of all subsets of a given set A, is called the .... set A

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44/248	SUBMITTED TEXT	30 WORDS	65%	MATCHING TEXT	30 WORDS

Disjoint Sets: Two sets A and B are said to be disjoint, if they have no element in common, i.e., A ? B = ?

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45/248	SUBMITTED TEXT	24 WORDS	58%	MATCHING TEXT	24 WORDS

sets A and B contains A ? B as a subset, i.e., (A ? B) ? A and ¶(A ? B) ? B.

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46/248	SUBMITTED TEXT	26 WORDS	41%	MATCHING TEXT	26 WORDS

The union of two sets A and B, written as B A? , is another set consisting of elements that belong to A or B (

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47/248	SUBMITTED TEXT	27 WORDS	56%	MATCHING TEXT	27 WORDS

The intersection of two sets A and B, written as B A? , is the set of all those elements that belong to A and B (

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48/248	SUBMITTED TEXT	24 WORDS	92%	MATCHING TEXT	24 WORDS

A cup B), is the set of all elements, which are either in A, or in B, or in both.

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49/248	SUBMITTED TEXT	34 WORDS	50%	MATCHING TEXT	34 WORDS

A cap B), is the set of all elements that belong to both A and B. Thus A ? B = {x : x ? A and x ? B}.

A ∩ B is the set of all things that are members of both A and B. If A ∩ B = ∅, then A and B

W http://en.wikipedia.org/wiki/Set_(mathematics)

50/248	SUBMITTED TEXT	97 WORDS	29%	MATCHING TEXT	97 WORDS

A and B are each a subset of A ? B. E.g. If A = {1, 2, 3, 4, 5} and B = {2, 4, 5, 6, 11, 12} Then A ? B = {1, 2, 3, 4, 5, 6, 11, 12}. Remark: i. When B ? A, then A ? B = A ii. A and B are both subsets of A ? B, i.e., A ? (A ? B) and B ? (A ? B) iii. A ? A = A

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51/248	SUBMITTED TEXT	19 WORDS	80%	MATCHING TEXT	19 WORDS

the number of elements in A and n 2 be the number of elements in B,

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52/248	SUBMITTED TEXT	17 WORDS	80%	MATCHING TEXT	17 WORDS

is called the symmetric difference of A and B, and is denoted by A ? B

is called the Cartesian product of A and B and is denoted by A B.

W http://www.cs.odu.edu/~toida/nerzic/level-a/set/set_operations.html

53/248	SUBMITTED TEXT	189 WORDS	14%	MATCHING TEXT	189 WORDS

A ? B is a subset of both A and B. Also A ? A = A. E.g.: If A = {1, 2, 3, 4, 5} Note Unit 3: Set Theory 80 B = (2, 4, 5, 6, 11, 12} Then A ? B = {2, 4, 5}. 3.8 Complement The complement of a set is defined as another set consisting of all elements of the universal set which are not elements of the original set. The complement of the set A for the universal set U is generally denoted by A?, and thus A? = {x : x ? A} or A? = {x | x ? U but x ? A} E.g.: i. If U = {1, 2, 3, 4, 5} and A = {1, 2, 4} Then A? = {3, 5} ii. If U is the set of all letters of English alphabet and A the set of vowels then A? is the set of

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54/248	SUBMITTED TEXT	63 WORDS	31%	MATCHING TEXT	63 WORDS

difference of two sets A and B denoted by A – B (read as A minus B), is the set of all elements of A which are not in B. Thus A – B = {x : x ? A, x ? B} Similarly B – A = {x : x ? B, x ? A}

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55/248	SUBMITTED TEXT	102 WORDS	47%	MATCHING TEXT	102 WORDS

of the sets A and B is the set of all elements of A and B, which are not common to both A and B. Thus A ? B = {x : x ? A and x ? B or x ? B and x ? A} E.g.: If A = {1, 2, 3}, B = {2, 3, 4, 5} Then A – B ={1, 2, 3} – {2, 3, 4, 5} = {1} and B – A = {2, 3, 4, 5} – {1, 2, 3} = {4, 5} ? ? A ? B = (

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56/248	SUBMITTED TEXT	28 WORDS	61%	MATCHING TEXT	28 WORDS

The ……………………… of two sets A and B, written as , is the set of all those elements that belong to A and B 22.

SA UNIT -1 -ECE18R316 -PTAND RP.ppt (D108941599)

57/248	SUBMITTED TEXT	23 WORDS	55%	MATCHING TEXT	23 WORDS

The ……………………… of a set is defined as another set consisting of all elements of the universal set which are not

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58/248	SUBMITTED TEXT	31 WORDS	42%	MATCHING TEXT	31 WORDS

of two sets A and B denoted by A – B (read as A minus B), is the set of all elements of A which are not in B. 3.10

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59/248	SUBMITTED TEXT	46 WORDS	42%	MATCHING TEXT	46 WORDS

A (ii) A ∩ B = B ∩ A 2. Associative Laws: For any three finite sets A, B and C; (i) (A U B) U C = A U (B U C) (ii) (A ∩ B) ∩ C = A ∩ (B ∩ C)

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60/248	SUBMITTED TEXT	50 WORDS	40%	MATCHING TEXT	50 WORDS

A = A (ii) A ∩ A = A 4. Distributive Laws: For any three finite sets A, B and C; (i) A U (B ∩ C) = (A U B) ∩ (A U C) (ii) A ∩ (B U C) = (A ∩ B) U (A ∩ C)

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61/248	SUBMITTED TEXT	74 WORDS	57%	MATCHING TEXT	74 WORDS

Morgan’s Laws: For any three finite sets A, B and C; (i) A – (B U C) = (A – B) ∩ (A – C) (ii) A - (B ∩ C) = (A – B) U (A – C) De Morgan’s Laws can also we written as: For any two finite sets A and B; (i) (A U B)’ = A' ∩ B' (ii) (A ∩ B)’ = A' U B'

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62/248	SUBMITTED TEXT	115 WORDS	33%	MATCHING TEXT	115 WORDS

A and B; (i) A – B = A ∩ B' (ii) B – A = B ∩ A' (iii) A – B = A ⇔ A ∩ B = ∅ (iv) (A – B) U B = A U B (v) (A – B) ∩ B = ∅ (vi) A ⊆ B ⇔ B' ⊆ A' (vii) (A – B) U (B – A) = (A U B) – (A ∩ B) 7. For any three finite sets A, B and C; (i) A – (B ∩ C) = (A – B) U (A – C) (ii) A – (B U C) = (A – B) ∩ (A –

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

63/248	SUBMITTED TEXT	43 WORDS	65%	MATCHING TEXT	43 WORDS

A A B A has dual ( ) . ? ? ? A A B A ( ) ? ? ? A A B A has dual ( ) . ? ? ? A A B A ? ? A A ?

a 11 b 1 a 13 b 2 a 22 a 23 a 21 b 2 a 23 b 3 a 32 a 33 a 31 b 3 a 33 and D 3 = a 11 a 12 b 1 a 21 a 22 a 31

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

64/248	SUBMITTED TEXT	31 WORDS	90%	MATCHING TEXT	31 WORDS

A ∩ (B - C) = (A ∩ B) - (A ∩ C) (iv) A ∩ (B △ C) = (A ∩ B) △ (A ∩ C) 3.11

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

65/248	SUBMITTED TEXT	98 WORDS	33%	MATCHING TEXT	98 WORDS

C C C A B A B ? ? ? Proof: , , . C C A U A B U B ? ? ? ? ( ) ( ) , ( ) . C x U A U B x U A U B x A B x A B ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ( ) , , . C C C C C x A B x A B x A B x A B x A B ? ? ? ? ? ? ? ? ? ? ? ? ?

SA CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

66/248	SUBMITTED TEXT	53 WORDS	41%	MATCHING TEXT	53 WORDS

the following laws: 1. Idempotent law: A ? A = A 2. Commutative law: A ? B = B ? A 3. Associative law: (A ? B) ? C = A ? (B ? C) 4. Identity law: (a) A ? ? = A, (b) A ? U =

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67/248	SUBMITTED TEXT	306 WORDS	15%	MATCHING TEXT	306 WORDS

A ∪ B ⊆ B ∪ A .... (1) Let y ∈ B ∪ A ⇒ y ∈ B or y ∈ A ⇒ y ∈ A or y ∈ B ⇒ y ∈ A ∪ B i.e., B ∪ A ⊆ A ∪ B .... (2) From (1) & (2), we have A ∪ B = B ∪ A. 3. To prove (A ∪ B) ∪ C = A∪ (B ∪ C) Let x∈ (A ∪ B) ∪ C ⇒ x ∈ (A ∪ B) or x ∈ C ⇒ (x ∈ A or x ∈ B) or x ∈ C ⇒ x ∈ A or (x ∈ B or x ∈ C) ⇒ x ∈ A or (x ∈ B ∪ C) i.e., x ∈ A ∪ (B ∪ C) .... (1) Let y ∈ A ∪ (B ∪ C) ⇒ y ∈ A or (y ∈ B ∪ C) ⇒ y ∈ A or (y ∈ B or y ∈ C) ⇒ (y ∈ A or y ∈ B) or (y ∈ C) ⇒ (y ∈ A ∪ B) or (y ∈ C) ⇒ y ∈ (A ∪ B) ∪ C i.e., A ∪ (B ∪ C) ⊆ (A ∪ B) ∪ C .... (2) From (1) & (2), we have (A ∪ B)∪ C = A ∪ (B ∪ C) 4. To prove A ∪ φ = A and A ∪

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68/248	SUBMITTED TEXT	93 WORDS	47%	MATCHING TEXT	93 WORDS

A ∩ B = B ∩ A 3. Associative law: (A ∩ B) ∩ C = A ∩ (B ∩ C) 4. Identity law: A ∩ U = A, A ∩ φ = φ 5. Distributive law: A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) Proof: 1. To prove A ∩ A = A Let x ∈ A ∩ A ⇒ x ∈ A

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

69/248	SUBMITTED TEXT	281 WORDS	18%	MATCHING TEXT	281 WORDS

A ? B ? B ? A .... (1) Let y ? B ? A ? y ? B and y ? A ? y ? A and y ? B ? y ? A ? B i.e. B ? A ? A ? B .... (2) From (1) and (2), we have A ? B = B ? A 3. To prove (A ? B) ? C = A ? (B ? C) Let x ? (A ? B) ? C ? x ? (A ? B) and x ? C ? (x ? A and x ? B) and x ? C ? x ? A and (x ? B and x ? C) ? x ? A and (x ? B ? C) ? x ? A ? (B ? C) i.e. (A ? B) ? C ? A ? (B ? C) .... (1) Let y ? A ? (B ? C) ? y ? A and y ? (B ? C) ? y ? A and (y ? B and y ? C) ? (y ? A and y ? B) and y ? C ? y ? (A ? B) ? C i.e. A ? (B ? C) ? (A ? B) ? C .... (2) From (1) and (2), we have (A ? B) ? C = A ? (B ? C) 4.

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70/248	SUBMITTED TEXT	56 WORDS	39%	MATCHING TEXT	56 WORDS

A ? (B ? C) = (A ? B) ? (A ? C) Let x ? A ? (B ? C) ?? x ? A or (x ? B ? C) ?? x ? A or (x ? B and x ? C) ?? (x ? A

SA chapter 2-Summation of Series.docx (D33334824)

71/248	SUBMITTED TEXT	42 WORDS	57%	MATCHING TEXT	42 WORDS

Submitted text	Matching text
A ? A? = U 2. A ? A? = ? 3. (A?) ? = A 4. (A ? B)? = A? ? B?. 5. (A ? B) ? = A? ? B?. Proof 1: To prove A ? A? =	a 12 a 13 D 2 = a 11 b 1 a 13 b 2 a 22 a 23 a 21 b 2 a 23 b 3 a 32 a 33 a 31 b 3 a 33 and D 3 = a 11 a 12

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72/248	SUBMITTED TEXT	386 WORDS	15%	MATCHING TEXT	386 WORDS

A ? (B ? C) ? (A ? B) ? (A ? C) .... (1) Again, let y ? (A ? B) ? (A ? C) ?? (y ? A ? B) and (y ? A ? C) ?? (y ? A or y ?B) and (y ? A or y ? C) ?? y ? A or (y ?B and y ? C) ?? y ? A or (y ? B ? C) ?? y ? A ? (B ? C) i.e., (A ? B) ? (A ? C) ? A ? (B ? C) .... (2) From (1) and (2), we have A ? (B ? C) = (A ? B) ? (A ? C). (ii) To prove A ? (B ? C) = (A ? B) ? (A ? C). Let x ? A ? (B ? C) ?? x ? A and (x ? B ? C) ?? x ? A and (x ? B or x ? C) ?? (x ? A and x ? B) or (x ? A and x ? C) ?? (x ? A ? B) or (x ? A ? C) Note Unit 3: Set Theory 88 ?? x ? (A ? B) ? (A ? C) i.e., A ? (B ? C) ? (A ? B) ? (A ? C) .... (1) Similarly, let y ? (A ? B) ? (A ? C) ?? y ? (A ? B) or y ? (A ? C) ?? (y ? A and y ? B) or (y ? A and y ? C) ?? y ? A and (y ?B or y ? C) ?? y ? A and (y ? B ? C) ?? y ? A ? (B ? C) i.e., (A ? B) ? (A ? C) ? A ? (B ? C) .... (2) From (1) and (2), we have A ? (B ? C) = (A ? B) ? (A ? C). Laws of

SA CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

73/248	SUBMITTED TEXT	109 WORDS	33%	MATCHING TEXT	109 WORDS

x ? (A?)? Then x ? (A?)? ? x ? A? ? x ? A i.e. (A?)? ? A .... (1) Again, let x ? A Then x ? A ? x ? A? ? x ? (A?)? i.e. A ? (A?)? .... (2) From (1) and (2), we have (A?)? = A. Proof 4: Let x ? (A ? B) ? ? x ? A ? B ? x ? A or x ? B. ? x ? A? or x ? B? ? x ? A? ?

SA CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

74/248	SUBMITTED TEXT	83 WORDS	23%	MATCHING TEXT	83 WORDS

A ? B) ?? A? ? B? .... (i) Again, Let y ? A? ? B? y ? A? ? B? ? y ? A? and y ? B?. ? y ? A or y ? B ? y ? (A ? B) ? y ? (A ? B) i.e. A? ? B?? (A ? B) ? .... (ii) From (i) and (ii), we have (A ? B) ?= A? ? B?.

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75/248	SUBMITTED TEXT	127 WORDS	18%	MATCHING TEXT	127 WORDS

x ? A ? B ? x ? A or x ? B. ? x ? A? and x ? B? Note Unit 3: Set Theory 90 ? x ? A? ? B? i.e. (A ? B) ?? A? ? B? .... (i) Similarly, we can show A? ? B?? (A ? B) ? .... (ii) From (i) and (ii), we have (A ? B) ?= A? ? B?. Law of Symmetric Difference If A and B are any two sets. Then A ? B = (A ? B) – (A ? B). Proof: Let x ? A ? B. Then x ?(A – B) ? (B – A) ?? (x ?

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76/248	SUBMITTED TEXT	131 WORDS	16%	MATCHING TEXT	131 WORDS

x ? A) ?? (x ? A and x ? B) or x ? B and [x ? A and x ? B] or x ? A ?? [x ? B or (x ? A and x ? B)] and [x ? A or (x ? A and x ? B)] ?? [(x ? B or x ? A) and (x ? B or x ? B)] and [(x ? A or x ? A) and (x ? A or x ? B)] ?? [x ? A ? B and x ? U] and [x ? U and x ? A ? B)] ?? x ? (A ? B) and x ? (A ? B) ?? x ? [(A ? B) – (A ? B)]

SA CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

77/248	SUBMITTED TEXT	336 WORDS	20%	MATCHING TEXT	336 WORDS

A ? B = (A ? B) – (A ? B). 3. A – (B ? C) = (A – B) ? (A – C). 4. A – (B ? C) = (A – B) ? (A – C). 3. To prove A – (B ? C) = (A – B) ?(A – C). Let x ? A – (B ? C) ?? x ? A and x ? (B ? C) ?? x ? A and (x ? B and x ? C) ?? (x ? A and x ? B) or (x ? A and x ? C) ?? x ? (A – B) ? x ? (A – C) ?? x ? (A – B) ? (A – C). i.e., A – (B ? C) ? (A – B) ? (A – C) .... (i) Similarly, we can show (A – B) ? (A – C) ? A – (B ? C) .... (ii) From (i) and (ii), we have A – (B ? C) = (A – B) ? (A – C) Business Mathematics Note 91 4. To prove A – (B ? C) = (A – B) ? (A – C) Let x ? A – (B ? C) ?? x ? A and x ? (B ? C) ?? x ? A and (x ? B and x ? C) ?? (x ? A and x ? B) or (x ? A and x ? C) ?? x ? (A – B) or x ? (A – C) ?? x ? (A – B) ? (A – C) i.e., A – (B ? C) ? (A – B) ? (A – C) .... (i) Similarly, we can show (A – B) ? (A – C) ? A – (B ? C) .... (

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78/248	SUBMITTED TEXT	125 WORDS	26%	MATCHING TEXT	125 WORDS

A, B and C (ii) (A ∩ B) ∩ C = A ∩ (B ∩ C) 26. For any finite set A; (ii) A ∩ A = A 27. For any three finite sets A, B and C; (ii) A ∩ (B U C) = (A ∩ B) U (A ∩ C) 28. For any three finite sets A, B and C; (i) A – (B U C) = (A – B) ∩ (A – C) 29. For any two finite sets A and B; (i) A – B = A ∩ B' 30. For any three finite sets A, B and C; A ∩ (B - C) = (A ∩ B) - (A ∩ C) 31. A

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79/248	SUBMITTED TEXT	48 WORDS	47%	MATCHING TEXT	48 WORDS

x ∈ (A ∩ B)' ⇒ x ∉ (A ∩ B) ⇒ x ∉ A or x ∉ B ⇒ x ∈ A' or x ∈ B' ⇒ x ∈ A' U B' ⇒ x ∈ N

SA CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

80/248	SUBMITTED TEXT	112 WORDS	20%	MATCHING TEXT	112 WORDS

X ∩ Y)' = X' U Y'. Solution: We know, U = {j, k, l, m, n} X = {j, k, m} Y = {k, m, n} (X ∩ Y) = {j, k, m} ∩ {k, m, n} = {k, m} Therefore, (X ∩ Y)' = {j, l, n} .... (i) Again, X = {j, k, m} so, X' = {l, n} and Y = {k, m, n} so, Y' = {j, l} X' ∪ Y' = {l, n} ∪ {j,

x and y, and [(x + y) n + 1] j , k = [(x + y) n] j − 1 , k + [(x + y) n] j , k − 1 , {\displaystyle [(x+y)^{n+1}]_{ j,k}= [(x+y)^{n}]_{j-1,k}+[(x+y)^{n}]_{j,k-1},} {\ displaystyle [(x+y)^{n+1}]_{ j,k}=[(x+y)^{n}]_{j-1,k}+[(x+y)^{ n}]_{j,

W http://en.wikipedia.org/wiki/Binomial_theorem

81/248	SUBMITTED TEXT	110 WORDS	32%	MATCHING TEXT	110 WORDS

x ? ? ?)\()\(C A B A ? (b) Let) (\ C B A x ? ? ?) (and C B x A x ? ? ? ?] or [and C x B x A x ? ? ? ?] and [or] and [C x A x B x A x ? ? ? ? ?)\()\() (\ )\()\(C A B A C B A C A B A x ? ? ? ? ? ?

SA CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

82/248	SUBMITTED TEXT	143 WORDS	31%	MATCHING TEXT	143 WORDS

P = {4, 5, 6} and Q = {5, 6, 8}. Show that (P ∪ Q)' = P' ∩ Q'. Solution: We know, U = {1, 2, 3, 4, 5, 6, 7, 8} P = {4, 5, 6} Q = {5, 6, 8} P ∪ Q = {4, 5, 6} ∪ {5, 6, 8} = {4, 5, 6, 8} Therefore, (P ∪ Q)' = {1, 2, 3, 7} .... (i) Now P = {4, 5, 6} so, P' = {1, 2, 3, 7, 8} and Q = {5, 6, 8} so, Q' = {1, 2, 3, 4, 7} P' ∩ Q' = {1, 2, 3, 7, 8} ∩ {1, 2, 3, 4, 7} Therefore, P' ∩ Q' = {1, 2, 3, 7} .... (ii) Combining (i)and (ii) we get; (P ∪ Q)' = P' ∩

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83/248	SUBMITTED TEXT	268 WORDS	22%	MATCHING TEXT	268 WORDS

A ? B. If x ? A ? x ? B .... (1) Also B ? A. If x ? B ? x ? A .... (2) From (1) and (2), x ? A ? x ? B. ? A = B. Proved Business Mathematics Note 95 Theorem: 4. If A ? B and B ? C. then A ? C. Proof: A ? B ? x ? A ? x ? B .... (1) Again B ? C. ? x ? B ? x ? C .... (2) From (1) and (2), it is clear x ? A ? x ? C ?? A ? C Proved Theorem: If a

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84/248	SUBMITTED TEXT	53 WORDS	37%	MATCHING TEXT	53 WORDS

sets. (a) A ∪ B (b) A ∩ B (c) A' (d) B - A (e) (A ∩ B)' (f) (A ∪ B)' Solution: ξ = {a, b, c, d, e, f, g, h, i, j} A = {a, b, c, d, f}

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

85/248	SUBMITTED TEXT	22 WORDS	95%	MATCHING TEXT	22 WORDS

elements which are in A or in B or in both} = {

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86/248	SUBMITTED TEXT	83 WORDS	31%	MATCHING TEXT	83 WORDS

A x ? ? ? ? B x A x ? ? and ? ' and ' B x A x ? ? ?)"(B A x ? ? ? ")"(B A B A ? ? ? Similarly it can be shown that)"()"(B A B A ? ? ? Hence ")' (B A B A ? ? ? Example If n (A)

SA CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

87/248	SUBMITTED TEXT	17 WORDS	73%	MATCHING TEXT	17 WORDS

n(A) + n(B) = n(A ? B) + n(A ? B) showing A, B, B A?

SA UNIT -1 -ECE18R316 -PTAND RP.ppt (D108941599)

88/248	SUBMITTED TEXT	29 WORDS	63%	MATCHING TEXT	29 WORDS

A ? B') ?(A ? B) Since (A ? B) and (A ? B) are disjoint sets, we have n(A) =) ( )' (B

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89/248	SUBMITTED TEXT	173 WORDS	47%	MATCHING TEXT	173 WORDS

x| x 2 – 2x + 1 = 0}, S 3 = {1, 2, 3}, S 4 = {x| x 3 – 6x 2 + 11x – 6 = 0}. Solution We have S 1 = {1, 2, 2, 3} = {1, 2, 3}. Since x 2 – 2x + 1 = 0 ? (x – 1) 2 = 0 or x = 1,1. ? S 2 = {x| x 2 – 2x + 1 = 0} ={1, 1} = {1}. Again S 3 = {1, 2, 3} Also x 3 – 6x 2 + 11x – 6 = 0 or (x – 1)(x 2 – 5x + 6) = 0 or (x – 1) (x – 2)(x – 3) = 0 ? x = {1, 2, 3}.

SA lesson lesson lesson.pdf (D134982954)

90/248	SUBMITTED TEXT	44 WORDS	29%	MATCHING TEXT	44 WORDS

A = {x: x is a letter in the word REAP}, B = {x: x is a letter in the word PAPER}, and C = {x: x is a letter in the word RAPE}, Solution A = {x: x is a

SA CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

91/248	SUBMITTED TEXT	220 WORDS	40%	MATCHING TEXT	220 WORDS

x 4 – 1 = 0 ⇒ (x 2 + 1) (x 2 – 1) = 0 ⇒ x = ± 1, ± i Since, x ∈⇒ x = ± i ∴ A = {- i, i} Again x 3 – 1 = 0 ⇒ (x – 1)(x 2 + x + 1) = 0 ⇒ x = 1 and x = (–1 ± √1–4) / 2 ⇒ x = 1 and x = (–1 ± √3 i) / 2 Since, x ∈⇒ x = (–1 ± i√3) / 2 ∴ B = {(-1 + i√3) / 2, (–1 – i√3) / 2 A – B ={–i, i} – {(–1 + i√3) / 2, (–1 – i√ Business Mathematics Note 101 = {–i, i}. Answer A ? B = {–i, i} ? {(–1 + i?3) / 2, (–1 – i?3) / = {–i, i, (–1 + i?3) / 2, (–1 - i?3) / 2 }. Answer Example If A = {x: x ? 5, x ?}, B = {x: x ? 5, x ?}, C = {x: 2 > x > 6,

x – 3 / 32 x 2 + 7 / 128 x 3 ... (1 – x) ¼ = 1 – ¼ x – 3 / 32 x 2 – 7 / 128 x 3 ... (1 + x) ¼ + (1 – x) ¼ = 1 + ¼ x – 3 / 32 x 2 + 7 / 128 x 3 ... + 1 – ¼ x – 3 / 32 x 2 – 7 / 128 x 3 ... = 2 – 3 / 16 x 2 = a – bx 2 ⇒ a = 2, b = 3 / 16 Putting x = 1 / 16 (1 + x) ¼ + (1 – x) ¼ = (1 + 1 / 16) ¼ + (1 – 1 / 16) ¼ = 17 ¼ + 15 ¼ = 17 ¼ + 15 ¼ = 17 ¼ + 15 ¼ 16 16 16 ¼ 16 ¼ 2 2 = ½ (17 ¼ + 15 ¼) (1 + x) ¼ + (1 – x) ¼ = ½ (17 ¼ + 15 ¼) = 2 – 3 / 16 x 2 If x = 1 / 16 = ½ (17 ¼ + 15 ¼) = 2 – 3 / 16 (1 / 16) 2 = ½ (17 ¼ + 15 ¼) = 8189 4096 17 ¼ + 15 ¼ = 8189 x 2 = 3.998535156 4096 ∴ 17 ¼ + 15 ¼ ≈ 3.9935

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

92/248	SUBMITTED TEXT	353 WORDS	30%	MATCHING TEXT	353 WORDS

A = B = C. ⇒ A = B = C. Example If A = {1, 2, 3, 4}, B = {2, 3, 5, 6} and C = {4, 5, 6, 7}. Then verify that i). A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C) ii). A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) Note Unit 3: Set Theory 100 Solution (i) B ∩ C = {2, 3, 5, 6} ∩ {4, 5, 6, 7} = {5, 6}. A ∪ (B ∩ C) = {1, 2, 3, 4} ∪ {5, 6} = {1, 2, 3, 4, 5, 6} .... (i) A ∪ B = {1, 2, 3, 4} ∪ {2, 3, 5, 6} = {1, 2, 3, 4, 5, 6} A ∪ C = {1, 2, 3, 4} ∪ {4, 5, 6, 7} = {1, 2, 3, 4, 5, 6, 7}. ∴ (A ∪ B) ∩ (A ∪ C) = {1, 2, 3, 4, 5, 6} ∩ (1, 2, 3, 4, 5, = {1,2,3,4,5,6} .... (ii) From (1) & (2), we have A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C). (ii) B ∪ C = {2, 3, 5, 6} ∪ {4, 5, 6, 7} = {2, 3, 4, 5, 6, 7}. A ∩ (B ∪ C) = {1, 2, 3, 4} ∩ {2, 3, 4, 5, 6, 7} = {2, 3, 4} .... (i) Again A ∩ B = {1, 2, 3, 4} ∩ {2, 3, 5, 6} = {2, 3} .... (ii) A ∩ C = {1, 2, 3, 4} ∩ {4, 5, 6, 7} = {4}. .... (iii) ∴ (A ∩ B) ∪ (A ∩ C) = {2, 3} = {2,3,4} .... (iv) From (1) & (4), we observe A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

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93/248	SUBMITTED TEXT	83 WORDS	66%	MATCHING TEXT	83 WORDS

B ? C) = {3, 4, 5} A – (B ? C) = {1, 2} (ii) A – B = ?. and A – C = {1, 2}. (A – B) ? {A – C} = {1, 2} = A – (B ? C). (A – B) ? {A – C} = {1, 2} = A – (B ? C). Hence A – (B ? C) = (A – B) ? (A – C).

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

94/248	SUBMITTED TEXT	22 WORDS	52%	MATCHING TEXT	22 WORDS

is the least number of elements in A ? B? Solution♥ The number of elements in the set A ? B

SA CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

95/248	SUBMITTED TEXT	35 WORDS	52%	MATCHING TEXT	35 WORDS

n(S - D) = 10 n(S) =24 Therefore, n(S - D) = n(S) - n(S ∩ D) ⇒ n(S ∩ D) = n(S) - n(S -

n | ¯ i = s n ¯ | i (1 + i) = s n + 1 | ¯ i − 1 {\displaystyle {\ddot {s}}_{{\overline {n|}}i}=s_{{\overline {n}}|i}(1+i)=s_{{\overline {n+1|}}i}-1} {\ddot {s}}_{{\overline {n|}i}}=s_{{\overline {n}|i}}(1+i)=s_{{\

W http://en.wikipedia.org/wiki/Annuity

96/248	SUBMITTED TEXT	49 WORDS	50%	MATCHING TEXT	49 WORDS

n(C) = 20 n(C ∩ F) = 15 n (C U F) = 40 n (F) = ? n(C ∪ F) = n(C) + n(F) - n(C ∩ F) 40 = 20 + n(

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97/248	SUBMITTED TEXT	60 WORDS	41%	MATCHING TEXT	60 WORDS

n(S ∪ C) = 80 n(S) = 35 n(C) = 60 Therefore, n(S ∪ C) = n(S) + n(C) - n(S ∩ C) 80 = 35 + 60 - n(S ∩ C) 80 = 95 - n(S ∩ C) Therefore, n(S∩C) = 95 – 80 = 15

SA CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

98/248	SUBMITTED TEXT	64 WORDS	78%	MATCHING TEXT	64 WORDS

if A and B are two non – empty intersecting sets, then ? n (A ? B) = n (A) + n (B) – n(A ? B) ? n (A ? B) = n (A – B) + n (A ? B) + n (B – A) ? n (A – B) + n (A ? B) = n (A)

SA UNIT -1 -ECE18R316 -PTAND RP.ppt (D108941599)

99/248	SUBMITTED TEXT	14 WORDS	84%	MATCHING TEXT	14 WORDS

Singleton set: A set containing only one element is called a singleton set.

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100/248	SUBMITTED TEXT	29 WORDS	51%	MATCHING TEXT	29 WORDS

is a subset of B and if there is at least one element in B which is not in A, then A is called the proper subset of

is a subset of and if there is at least one of Q which is not a member of P , then P is a proper subset of

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101/248	SUBMITTED TEXT	16 WORDS	70%	MATCHING TEXT	16 WORDS

if every element of the set A is also an element of the set B.

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102/248	SUBMITTED TEXT	22 WORDS	92%	MATCHING TEXT	22 WORDS

if every element of A is an element of B, and every element of B is also an element of A.

if every element of A is an element of B, and every element of B is an element of A;

W http://en.wikipedia.org/wiki/Set_(mathematics)

103/248	SUBMITTED TEXT	16 WORDS	96%	MATCHING TEXT	16 WORDS

Equal sets: Two sets are said to be equal if they contain the same elements,

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104/248	SUBMITTED TEXT	12 WORDS	95%	MATCHING TEXT	12 WORDS

a set with finite number of elements is a finite set.

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105/248	SUBMITTED TEXT	43 WORDS	54%	MATCHING TEXT	43 WORDS

then the set is called a set of sets. Power set: The set of all subsets of a given set A, is called the power set A. The power set A is denoted by P (A). Universal set: A set, which contains

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106/248	SUBMITTED TEXT	13 WORDS	95%	MATCHING TEXT	13 WORDS

Union of sets: The union of two sets A and B,

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107/248	SUBMITTED TEXT	32 WORDS	62%	MATCHING TEXT	32 WORDS

Intersection of two sets : The intersection of two sets A and B, written as B A? , is the set of all those elements that belong to A and B (

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108/248	SUBMITTED TEXT	27 WORDS	66%	MATCHING TEXT	27 WORDS

Complement of a set : The complement of a set is defined as another set consisting of all elements of the universal set which are not

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109/248	SUBMITTED TEXT	38 WORDS	38%	MATCHING TEXT	38 WORDS

set. Difference of two sets : The difference of two sets A and B denoted by A – B (read as A minus B), is the set of all elements of A which are not in B.

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110/248	SUBMITTED TEXT	111 WORDS	18%	MATCHING TEXT	111 WORDS

A &B. 5. If A = {a, b, c, d}, & B = {e, f, c, d}, find A ? B. 6. If A = A ? B, show B = A ? B. 7. If A & B are two sets, find the value of A ? (A ? B). 8. If A, B are subsets of a set S 1 and A′, B′ are the complements of A& B respectively. Prove that A ? B ? B′ ? A′. 9. Prove that for any two sets A & B, (A – B) ? (B – A) = (A ? B) – (A ?

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

111/248	SUBMITTED TEXT	13 WORDS	87%	MATCHING TEXT	13 WORDS

still, logarithmic and exponential equations and functions are very common in mathematics.

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

112/248	SUBMITTED TEXT	23 WORDS	88%	MATCHING TEXT	23 WORDS

a positive real number (except 1), n is any real number and a n = b, then n is called

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

113/248	SUBMITTED TEXT	13 WORDS	100%	MATCHING TEXT	13 WORDS

logarithm of b to the base a. It is written as log

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

114/248	SUBMITTED TEXT	112 WORDS	83%	MATCHING TEXT	112 WORDS

is called the logarithmic form. For example: ? 3 2 = 9 ⇔ log 3 9 = 2 ? 5 4 = 625 ⇔ log 5 625 = 4 ? 7 0 = 1 ⇔ log 7 1 = 0 ? 2 -3 = 1 / 8 ⇔ log 2 (1 / 8) = -3 ? 10 -2 = 0.01 ⇔ log 10 0.01 = -2 ? 2 6 = 64 ⇔

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

115/248	SUBMITTED TEXT	50 WORDS	36%	MATCHING TEXT	50 WORDS

x = log a M then a log a M = a Proof: x = log a M. Therefore, a x = M or, a log a M = M [Since, x = log a

x Log a a - + = - Solution: [ ]) 1( ) (2 2 2 2 2 a x a Log x a Log a a - + = -]) 1(2 2 2 a x Log a Log a a - + =]) 1(2 2 2 a x Log a - + =

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116/248	SUBMITTED TEXT	25 WORDS	62%	MATCHING TEXT	25 WORDS

If no base is given, the base is assumed to be 10. For example: log 21 means log 10 21. 5.

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

117/248	SUBMITTED TEXT	13 WORDS	83%	MATCHING TEXT	13 WORDS

the power to which the base must be raised in order to

SA Objective Mathematics -II (BMGE- 02)- FINAL.pdf (D162012805)

118/248	SUBMITTED TEXT	127 WORDS	36%	MATCHING TEXT	127 WORDS

Convert the following exponential form to logarithmic form: (i) 10 4 = 10000 Solution: 10 4 = 10000 ⇒ log 10 10000 = 4 (ii) 3 -5 = x Solution: 3 -5 = x ⇒ log 3 x = -5 (iii) (0.3) 3 = 0.027 Business Mathematics Note 113 Solution: (0.3) 3 = 0.027 ⇒ log 0.3 0.027 = 3 Convert the following logarithmic form to exponential form: (i) log 3 81 = 4 Solution: log 3 81 = 4 ⇒ 3 4 = 81,

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

119/248	SUBMITTED TEXT	371 WORDS	43%	MATCHING TEXT	371 WORDS

exponential form. (ii) log 8 32 = 5/3 Solution: log 8 32 = 5/3 ⇒ 8 5/3 = 32 (iii) log 10 0.1 = -1 Solution: log 10 0.1 = -1 ⇒ 10 -1 = 0.1. By converting to exponential form, find the values of following: (i) log 2 16 Solution: Let log 2 16 = x ⇒ 2 x = 16 ⇒ 2 x = 2 4 ⇒ x = 4, Therefore, log 2 16 = 4. (ii) log 3 (1/3) Solution: Let log 3 (1/3) = x ⇒ 3 x = 1/3 ⇒ 3 x = 3 -1 ⇒ x = -1, Therefore, log 3 (1/3) = -1. (iii) log 5 0.008 Note Unit 4: Logarithm 114 Solution: Let log 5 0.008 = x ⇒ 5 x = 0.008 ⇒ 5 x = 1/125 ⇒ 5 x = 5 -3 ⇒ x = -3, Therefore, log 5 0.008 = -3. Solve the following for x: (i) log x 243 = -5 Solution: log x 243 = -5 ⇒ x -5 = 243 ⇒ x -5 = 3 5 ⇒ x -5 = (1/3) -5 ⇒ x = 1/3. (ii) log √5 x = 4 Solution: log √5 x = 4 ⇒ x = (√5) 4 ⇒ x = (5 1/2) 4 ⇒ x = 5 2 ⇒ x = 25. (iii) log √x 8 = 6

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

120/248	SUBMITTED TEXT	49 WORDS	47%	MATCHING TEXT	49 WORDS

a a = log b a x log a b or, log b a x log a b = 1 [since, log a a = 1] or, log b a = 1/log

a a + =)(log (2) B Log A Log B A a a a - = log (3) A BLog A a B a =)(log

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

121/248	SUBMITTED TEXT	227 WORDS	55%	MATCHING TEXT	227 WORDS

x log 2 64 = 6 ⇔ 2 6 = 64 log 4 32 = 5/2 ⇔ 4 5/2 = 32 log 1/8 2 = -1/3 ⇔ (1/8) -1/3 = 2 log 3 81 = x ⇔ 3 x = 81 log 5 x = -2 ⇔ 5 -2 = x log x = 3 ⇔ 10 3 = x Solve for x: 1. log 5 x = 2 x = 5 2 = 25 2. log 81 x = ½ x = 81 1/2 ⇒ x= (9 2) 1/2 ⇒ x = 9 3. log 9 x = -1/2 x = 9 -1/2 ⇒ x = (3 2) -1/2 ⇒ x = 3 -1 ⇒ x= 1/3 Note Unit 4: Logarithm 116 4. log 7 x = 0 x = 7 0 ⇒ x = 1 4.2

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

122/248	SUBMITTED TEXT	92 WORDS	22%	MATCHING TEXT	92 WORDS

M (iv) log a (MN) = log a M + log a N (v) log a (M/N) = log a M - log a N (vi) log a M n = n log a M (vii) log a M = log a M x log a b (viii) log b a x log a b = 1 (

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

123/248	SUBMITTED TEXT	113 WORDS	50%	MATCHING TEXT	113 WORDS

log 2 log 2 log 2 16 = 1. Business Mathematics Note 119 Solution: L. H. S. = log 2 log 2 log 2 2 4 = log 2 log 2 4 log 2 2 = log 2 log 2 2 2 [since log 2 2 = 1] = log 2 2 log 2 2 = 1 · 1 = 1.

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

124/248	SUBMITTED TEXT	120 WORDS	48%	MATCHING TEXT	120 WORDS

x = 2 log 10 y, find x in terms of y. Solution: 3 + log 10 x = 2 log 10 y or, 3 log 10 10 + log 10 x= 1og 10 y 2 [since log 10 10 = 1] or. log 10 10 3 + log 10 x = log 10 y 2 or, log 10 (10 3 · x) = log 10 y 2 or, 10 3 x = y 2

SA Objective Mathematics -II (BMGE- 02)- FINAL.pdf (D162012805)

125/248	SUBMITTED TEXT	396 WORDS	40%	MATCHING TEXT	396 WORDS

log (25/24) + log 2. Solution: Since,7 log (10/9) + 3 log (81/80) — 2 log (25/24) = 7(log 10 – log 9)+ 3(1og 81 - log 80)- 2(1og 25 - 1og 24) = 7[log(2 · 5) - log3 2] + 3[1og3 4 - log(5 · 2 4)] - 2[log5 2 - log(3 · 2 3)] = 7[log 2 + log 5 – 2 log 3] + 3[4 log 3 - log 5 - 4 log 2] - 2[2 log 5 – log 3 – 3 log 2] = 7 log 2+ 7 log 5 - 14 log 3 + 12 log 3 – 3 log 5 – 12 log 2 – 4 log 5 + 2 log 3 + 6 log 2 = 13 log 2 – 12 log 2 + 7 log 5 – 7 log 5 – 14 log 3 + 14 log 3 = log 2 Therefore 7 log(10/9) +3 log (81/80) = 2 log (25/24) + log 2. Proved. 6. If log 10 2 = 0.30103, log 10 3 = 0.47712 and log 10 7 = 0.84510, find the values of (i) log 10 45 Note Unit 4: Logarithm 120 (ii) log 10 105. (i) log 10 45 Solution: log 10 45= log 10 (5 × 9) = log 10 5 + log 10 9 = log 10 (10/2) + log 10 3 2 = log 10 10 - log 10 2 + 2 log 10 3 = 1 - 0.30103 + 2 × 0.47712 = 1.65321. (ii) log 10 105 Solution: log 10 105 = log 10 (7 x 5 x 3) = log 10 7 + log 10 5 + log 10 3 = log 10 7 + log 10 10/2 + log 10 3 = log 10 7 + log 10 10 -

SA Objective Mathematics -II (BMGE- 02)- FINAL.pdf (D162012805)

126/248	SUBMITTED TEXT	70 WORDS	29%	MATCHING TEXT	70 WORDS

log b a × log c b × log d c = log d a. Solution: L. H. S. = log b a × log c b × log d C = log c a × log d c [since log b

SA Objective Mathematics -II (BMGE- 02)- FINAL.pdf (D162012805)

127/248	SUBMITTED TEXT	78 WORDS	19%	MATCHING TEXT	78 WORDS

x/(y - z) = log y/(z - x) = log z/(x – y) show that, x x y y z z = 1 Solution: Let, log x/(y - z) = log y/(z - x) = log z/(x – y) = k Therefore, log x = k(y - z) ⇔ x log x = kx(y - z) or, log x x = kx(y -

x + y) 0 = 1 , (x + y) 1 = x + y , (x + y) 2 = x 2 + 2 x y + y 2 , (x + y) 3 = x 3 + 3 x 2 y + 3 x y 2 + y 3 , (x + y) 4 = x 4 + 4 x 3 y + 6 x 2 y 2 + 4 x y 3 + y 4 , (x + y) 5 = x 5 + 5 x 4 y + 10

W http://en.wikipedia.org/wiki/Binomial_theorem

128/248	SUBMITTED TEXT	233 WORDS	29%	MATCHING TEXT	233 WORDS

log d a = xyz = log b a × log c b × log d c. (putting the value of x, y, z) 8. Show that, log 4 2 × log 2 3= log 4 5 × log 5 3. Solution: L. H. S. = log 4 2 × log 2 3 = log 4 3 Business Mathematics Note 121 = log 5 3 × log 4 5. Proved. 9. Show that, log 2 10 - log 8 125 = 1. Solution: We have, log 8 125 = log 8 5 3 = 3 log 8 5 = 3 · (1/log 5 8) = 3 · (1/log 5 2 3) = 3 · (1/3 log 5 2) = log 2 5 Therefore, L.H. S. = log 2 10 - log 8 125 = log 2 10 - log 2 5 = log 2 (10/5) = log 2 2 = 1.

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

129/248	SUBMITTED TEXT	173 WORDS	25%	MATCHING TEXT	173 WORDS

log a a log a 2 x ⇒ log a p = log a 2 x · log a a ⇒ log a p = log a 2 x [since, log a a = 1] ⇒ log a p = 1/(log x a 2) [since, log n m = 1/(log m n)] ⇒ log a p = 1/(2 log x a) ⇒ log a p = (1/2) log a x ⇒ log a p = log a x ½ ⇒ log a p = log a √x Therefore, p = √x or, a log a 2

Log b a a = we put a N = in (*) 1= = ⇒ a bLog Log a Log b a a b Log b Log a a 1 = ⇒ (**) Another form of (*) is a Log N Log N Log b b a = Example: Show that:) 1(2) (2 2 2 2 a x Log x x Log a a - + = - Solution: [ ]) 1( ) (2 2 2 2 2 a x a Log x a Log a a - + = -]) 1(2 2 2 a x Log a a a - + =]) 1(2 2 2 a x Log a - + =

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

130/248	SUBMITTED TEXT	96 WORDS	43%	MATCHING TEXT	96 WORDS

a 2 - x · b 5x = a x + 3 · b 3x Therefore, b 5x /b 3x = a x + 3 /a 2 - x or, b 5x - 3x = a x + 3 – 2 + x or, b 2x = a 2x + 1 or, b 2x =a 2x · a or, (b/a) 2x = a

SA CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

131/248	SUBMITTED TEXT 181 WORDS	34% MATCHING TEXT 181 WORDS

x + log a z = 2 log a y = 2/(log y a) [since log a y × log y a = 1] Proved. 15. Solve log x 2 · log x/16 2 = log x/64 2. Solution: Let, log 2 x = a ; then, log x 2 = 1/ (log 2 x) = 1/a and log x/16 2 = 1/ [log 2 (x/16)] = 1/(log 2 x — log 2 16) = 1/(log 2 x — log 2 24) = 1/(a – 4) [since, log 2 2 = 1] Similarly, log x/64 2 = 1/[log 2 (x/64)] = 1/(log 2 x – log 2 64) = 1/(a - log 2 2 6) = 1/(

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

132/248	SUBMITTED TEXT 46 WORDS	53% MATCHING TEXT 46 WORDS

The characteristic of the logarithm of a number greater than 1 is positive and is one less than the number of digits in the integral part of the number. Example: (ii) To find the characteristic of the logarithm of a number

the characteristic of the logarithm of a number that is greater than one is positive and is one unit lesser than the total number of digits present on the left side of the decimal point. • Mantissa part: It refers to the decimal part of the logarithm of a number.

W http://www.vitutor.com/alg/log/antilogarithm.html

133/248	SUBMITTED TEXT 25 WORDS	50% MATCHING TEXT 25 WORDS

The characteristic of the logarithm of a positive number less than 1 is negative and is numerically greater by 1 than the number of

the characteristic of the logarithm of a number that is greater than one is positive and is one unit lesser than the total number of

W http://www.vitutor.com/alg/log/antilogarithm.html

134/248	SUBMITTED TEXT 20 WORDS	76% MATCHING TEXT 20 WORDS

move horizontally to the right to the column headed by 0 of part (b) and read the number 7782

move horizontally to the right to the column headed by 6 of the top-most row and read the number 3516.

W http://www.math-only-math.com/antilogarithm.html

135/248	SUBMITTED TEXT 20 WORDS	76% MATCHING TEXT 20 WORDS

move horizontally to the right to the column headed by 0 of part (b) and read the number 6812

move horizontally to the right to the column headed by 6 of the top-most row and read the number 3516.

W http://www.math-only-math.com/antilogarithm.html

136/248	SUBMITTED TEXT 20 WORDS	76% MATCHING TEXT 20 WORDS

move horizontally to the right to the column headed by 2 of part (b) and read the number 5933

move horizontally to the right to the column headed by 6 of the top-most row and read the number 3516.

W http://www.math-only-math.com/antilogarithm.html

137/248	SUBMITTED TEXT	54 WORDS	69% MATCHING TEXT	54 WORDS

move horizontally to the right to the column headed by 3 of part (b) and read the number 7185 in part (c) of the table. Again we move along the same horizontal line further right to the column headed by 4 of mean difference and read the number 3 there. If this 3

move horizontally to the right to the column headed by 6 of the top-most row and read the number 3516. Again we move along the same horizontal line further right to the column headed by 3 of mean difference and read the number 2 there. This 2

W http://www.math-only-math.com/antilogarithm.html

138/248	SUBMITTED TEXT	293 WORDS	98% MATCHING TEXT	293 WORDS

If log M = x, then M is called the antilogarithm of x and is written as M = antilog x. For example, if log 39.2 = 1.5933, then antilog 1.5933 = 39.2. If the logarithmic value of a number be given then the number can be determined from the antilog-table. Antilog-table is similar to log-table; only difference is in the extreme left-hand column which ranges from .00 to .99. Example Find antilog 2.5463. Solution: Clearly, we are to find the number whose logarithm is 2.5463. For this consider the mantissa .5463. Now find .54 in the extreme left-hand column of the antilog-table (see four-figure antilog-table) and then move horizontally to the right to the column headed by 6 of the top-most row and read the number 3516. Again we move along the same horizontal line further right to the column headed by 3 of mean difference and read the number 2 there. This 2 is now added to the previous number 3516 to give 3518. Since the characteristic is 2, there should be three digits in the integral part of the required number. Therefore, antilog 2.5463 = 351.8. Example If log x = -2.0258, find x. Solution: In order to find the value of x using antilog-table, the decimal part (i.e., the mantissa) must be made positive. For this we proceed as follows: log x = -2.0258 = - 3 + 3 - 2.0258 = - 3 + .9742 =3.9742 Therefore, x = antilog 3.9742. Now, from antilog table we get the number corresponding to the mantissa .9742 as (9419 + 4) = 9423. Again the characteristic in log x is (- 3).

If log M = x, then M is called the antilogarithm of x and is written as M = antilog x. For example, if log 39.2 = 1.5933, then antilog 1.5933 = 39.2. If the logarithmic value of a number be given then the number can be determined from the antilog-table. Antilog-table is similar to log-table; only difference is in the extreme left-hand column which ranges from .00 to .99. Example on antilogarithm: 1. Find antilog 2.5463. Solution: Clearly, we are to find the number whose logarithm is 2.5463. For this consider the mantissa .5463. Now find .54 in the extreme left-hand column of the antilog-table (see four-figure antilog-table) and then move horizontally to the right to the column headed by 6 of the top-most row and read the number 3516. Again we move along the same horizontal line further right to the column headed by 3 of mean difference and read the number 2 there. This 2 is now added to the previous number 3516 to give 3518. Since the characteristic is 2, there should be three digits in the integral part of the required number. Therefore, antilog 2.5463 = 351.8. 2. If log x = -2.0258, find x. Solution: In order to find the value of x using antilog-table, the decimal part (i.e., the mantissa) must be made positive. For this we proceed as follows: log x = -2.0258 = - 3 + 3 - 2.0258 = - 3 + .9742 =3.9742 Therefore, x = antilog 3.9742. Now, from antilog table we get the number corresponding to the mantissa .9742 as (9419 + 4) = 9423. Again the characteristic in log x is (- 3).

W http://www.math-only-math.com/antilogarithm.html

139/248	SUBMITTED TEXT	26 WORDS	100% MATCHING TEXT	26 WORDS

Hence, there should be two zeroes between the decimal point and the first significant digit in the value of x. Therefore, x = .009423.

Hence, there should be two zeroes between the decimal point and the first significant digit in the value of x. Therefore, x = .009423. •

W http://www.math-only-math.com/antilogarithm.html

140/248	SUBMITTED TEXT	24 WORDS	97% MATCHING TEXT	24 WORDS

If log M = x, then M is called the .... of x and is written as M = antilog x. 20.

If log M = x, then M is called the antilogarithm of x and is written as M = antilog x.

W http://www.math-only-math.com/antilogarithm.html

141/248	SUBMITTED TEXT	12 WORDS	100% MATCHING TEXT	12 WORDS

is in the extreme left-hand column which ranges from 4.6

is in the extreme left-hand column which ranges from .00

W http://www.math-only-math.com/antilogarithm.html

142/248	SUBMITTED TEXT	107 WORDS	48% MATCHING TEXT	107 WORDS

a · 1/(a - 4) = 1/(a – 6) or, a 2 - 4a = a - 6 or, a 2 - 5a + 6 = 0 or, a 2 - 2a - 3a + 6 = 0 or, a(a - 2) - 3(a - 2) = 0 or, (a - 2)(a - 3) = 0 Therefore, either, a - 2 = 0 i.e., a = 2 or, a - 3 = 0 i.e., a = 3 When a = 2

SA CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

143/248	SUBMITTED TEXT	23 WORDS	88% MATCHING TEXT	23 WORDS

a positive real number (except 1), n is any real number and a n = b, then n is called

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

144/248	SUBMITTED TEXT	23 WORDS	100% MATCHING TEXT	23 WORDS

If log M = x, then M is called the antilogarithm of x and is written as M = antilog x. 4.7

If log M = x, then M is called the antilogarithm of x and is written as M = antilog x.

W http://www.math-only-math.com/antilogarithm.html

145/248	SUBMITTED TEXT	14 WORDS	100% MATCHING TEXT	14 WORDS

logarithm of b to the base a. It is written as log

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

146/248	SUBMITTED TEXT	38 WORDS	52%	MATCHING TEXT	38 WORDS

the logarithm of 2025 to the base 3 5 2. The logarithm of a number to the base 2 is k. What is its logarithm to the base 2 2 ? Note

SA CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

147/248	SUBMITTED TEXT	142 WORDS	25%	MATCHING TEXT	142 WORDS

x log c b x log a c = 1. 5. Find the value of log 2 [log 2 {log 3 (log 3 27 3)}]. 6. If log 2 x + log 4 x + log 16 x=21/4 then, find x 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. Business Mathematics Note 133 18. 19. 20 21. If log 3 = 0.4771, find the number of digits in 3 43 . 22. Given log 10 2 = 0.30103, find log 10 (1000/256) 23. Find the value of : (

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

148/248	SUBMITTED TEXT	12 WORDS	100%	MATCHING TEXT	12 WORDS
Binomial Theorem A binomial is a polynomial with two terms. Binomial			Binomial Theorem A binomial is a polynomial with two terms Binomial		

W http://www.mathsisfun.com/algebra/binomial-theorem.html

149/248	SUBMITTED TEXT	64 WORDS	33%	MATCHING TEXT	64 WORDS

Given log 8 = 0.931, log 9 = 0.9542 ; find the value of log 60 correct to 4 decimal 4 places. 27. Given log 2 = 0.30103, log 3 = 0.47712 ; find the value of : (i) log 4500 (ii) log 0.015 (iii) log 0.1875. 28. Using tables find the value of : (i) 19.66 / 9.701 (ii) (

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

150/248	SUBMITTED TEXT	18 WORDS	64%	MATCHING TEXT	18 WORDS
The binomial theorem describes the algebraic expansion of powers of a binomial, hence it is referred to			the binomial theorem (or binomial expansion) describes the algebraic expansion of powers of a binomial. According to the theorem, it is possible to		

W http://en.wikipedia.org/wiki/Binomial_theorem

151/248	SUBMITTED TEXT	26 WORDS	93% MATCHING TEXT	26 WORDS

According to the theorem, it is possible to expand the power (x + y) n into a sum involving terms of the form

According to the theorem, it is possible to expand the polynomial (x + y)n into a sum involving terms of the form

W http://en.wikipedia.org/wiki/Binomial_theorem

152/248	SUBMITTED TEXT	33 WORDS	100% MATCHING TEXT	33 WORDS

where the exponents b and c are nonnegative integers with b + c = n, and the coefficient a of each term is a specific positive integer depending on n and b.

where the exponents b and c are nonnegative integers with b + c = n, and the coefficient a of each term is a specific positive integer depending on n and b.

W http://en.wikipedia.org/wiki/Binomial_theorem

153/248	SUBMITTED TEXT	27 WORDS	80% MATCHING TEXT	27 WORDS

n C r a n−r x r . As the (r + 1) th term from the

n C r a n¡r x r is the (r + 1)-th term. 3 In the

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

154/248	SUBMITTED TEXT	39 WORDS	100% MATCHING TEXT	39 WORDS

can be interpreted as the number of ways to choose k elements from an n-element set. This is related to binomials for the following reason: if we write (x + y) n as a product

can be interpreted as the number of ways to choose k elements from an n-element set. This is related to binomials for the following reason: if we write (x + y)n as a product (

W http://en.wikipedia.org/wiki/Binomial_theorem

155/248	SUBMITTED TEXT	39 WORDS	100% MATCHING TEXT	39 WORDS

then, according to the distributive law, there will be one term in the expansion for each choice of either x or y from each of the binomials of the product. For example, there will only be one term

then, according to the distributive law, there will be one term in the expansion for each choice of either x or y from each of the binomials of the product. For example, there will only be one term

W http://en.wikipedia.org/wiki/Binomial_theorem

156/248	SUBMITTED TEXT	17 WORDS	100% MATCHING TEXT	17 WORDS

corresponding to choosing x from each binomial. However, there will be several terms of the form

corresponding to choosing x from each binomial. However, there will be several terms of the form

W http://en.wikipedia.org/wiki/Binomial_theorem

157/248	SUBMITTED TEXT 27 WORDS	100% MATCHING TEXT 27 WORDS
	y 2 , one for each way of choosing exactly two binomials to contribute a y. Therefore, after combining like terms, the coefficient of	y2, one for each way of choosing exactly two binomials to contribute a y. Therefore, after combining like terms, the coefficient of

W http://en.wikipedia.org/wiki/Binomial_theorem

158/248	SUBMITTED TEXT 23 WORDS	100% MATCHING TEXT 23 WORDS
	y 2 will be equal to the number of ways to choose exactly 2 elements from an n-element set. 5.2.3	y2 will be equal to the number of ways to choose exactly 2 elements from an n-element set.

W http://en.wikipedia.org/wiki/Binomial_theorem

159/248	SUBMITTED TEXT 57 WORDS	78% MATCHING TEXT 57 WORDS
	a – b) 5 = a 5 – 5a 4 b + 10a 3 b 2 – 10a 2 b 3 + 5ab 4 – b 5 . For, a – b = a + (–b),	a + b (a + b) 2 = a 2 + 2ab + b 2 (a + b) 3 = a 3 + 3a 2 b + 3ab 2 + b 3 (a + b) 4 = a 4 + 4a 3 b + 6

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

160/248	SUBMITTED TEXT 125 WORDS	31% MATCHING TEXT 125 WORDS
	a 8 + 8a 7 b + 28a 6 b 2 + 56a 5 b 3 -- is 28· 6, divided by 3: Note Unit 5: Binomial Theorem 152 28· 6 / 3 = 28· 2 = 56. The next -- (a + b) 8 = a 8 + 8a 7 b + 28a 6 b 2 + 56a 5 b 3 + 70a 4 b 4 -- is 56· 5, divided by 4: 56· 5 / 4 = 14· 5 = 70. We have now come to	a+ b)2 = (a+b)(a+b) = ab + b2 of 3 For an exponent of 3 just multiply a+b)3 = (a2 + 2ab + b2)(a+b) = a3 + 3 a2b + 3 ab2 + b3 We have enough now to

W http://www.mathsisfun.com/algebra/binomial-theorem.html

161/248	SUBMITTED TEXT 84 WORDS	85% MATCHING TEXT 84 WORDS
	a + b) 8 = a 8 + 8a 7 b + 28a 6 b 2 + 56a 5 b 3 + 70a 4 b 4 + 56a 3 b 5 + 28a 2 b 6 + 8ab 7 + b 8 .	a + b) 0 = 1 (a + b) 1 = a + b (a + b) 2 = a 2 + 2ab + b 2 (a + b) 3 = a 3 + 3a 2 ab 2 + b 3 (

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

162/248 **SUBMITTED TEXT** 1059 WORDS **30%** **MATCHING TEXT** 1059 WORDS

n!). x n = [{(2n-1) (2n-3) ----- 3.1.} { 2n (2n-2) ------ 4.2.}] / (n! x n!). x n = [{1. 3. 5. ---- (2n-1)} 2 n {1.2 ---- n}] / (n! x n!) . x n = [{1.3.5----(2n-1)}. 2 n] / (n! x n!). x n = T n+1 = 1. 3. 5 – (2n-1) / (n!). 2 n x n Example Find the term independent of 'x' in the expansion of (i) (1+x+2x 3) [(3 x 2 / 2) – (1/3x)] 9 (ii) [(x 1/3 / 2) + x -1/5] 8 Business Mathematics Note 153 Sol.: (i) (1+x+2x 3) [(3/2)x 2 - (1/3x)] 9 = (1+x+2x 3) { [(3/2)x 2] 9 - 9 C 1 [(3/2)x 2] 8 1/3x + ---------- + + 9 C 6 [(3/2)x 2] 3 (1/3x) 6 - 9 C 7 [(3/2)x 2] 2 (1/3x) 7 ---} = (1+x+2x 3) { [(3/2)x 2] 9 – 9 C 1 (3 7 / 2 8)x 15 + ---- + 9 C 6 (1 x 1 / 2 3 x 3 3) – 9 C 7 1/ (2 2 x 3 5) 1/ x 3 + ----} Term independent of 'x' : 9 C 6 x 1/ (2 3 x 3 3) - 9 C 7 2 / (2 2 x 3 5) = 9! / (6! x 3!) . 1/ (8 x 27) - 9!/(7! x 2!) . 1/ (2 x 243) = (9. 8. 7. 6!) / (6!. 3. 2. 1.) x 1/(8. 27) - (9. 8. 7!) / (7! . 2). 1/ (2.243) = 7 / 18 - 2 / 27 = 17 / 54 (ii) [(1 / 2) x 1/3 + x -1/5] 8 Sol.: General Term T r+1 = n C r [(1/2) x 1/3] n-r . (x -1/5) r n-r -r = n C r [(1/2) n-r] x 3 x 5 Here n = 8 = 8 C r (1/2) 8-r x (8-r)/3 -r/5 40 -8r T r+1 = 8

n C 0 = 1 C 1 = n, n C 2 = n (n – 1), n C 3 = n (n – 1) (n – 2) 2! 3! However, we can also see that (1 + x) n = 1 + n C 1 x + n C 2 x 2 + n C 3 x 3 + ... + n C x r + ... + n n x n C x r Illustration 2: Expand (i) (1 – 2x) 4 (ii) (1 + x 2) 3 in powers x (i) (1 – 2x) 4 = (1 + (-2x)) 4 = 1 + 4 C 1 (-2x) 1 + 4 C 2 (-2x) 2 + 4 C 3 (-2 C 4 (-2x) 4 = 1 + 4 (-2x) + 6(4x 2) + 4(-8x 3) + 16x 4 = 1 – 8x + 24x 2 – 32 x 3 + 16 x 4 (ii) (1 + x + x 2) 3 = [1 + (x + x 2)] 3 = 1 + 3(x + x 2) + 3(x 2) 2 + (x + x 2) 3 = 1 + 3x 2 + 3(x 2 + 2x 3 + x 3 + 3x 4 + 3x 5 + x 6) = 1 + 3x + 6x 2 + 7x 3 + 6x 4 + 3x 5 + x 6 Illustration 3: Find the coefficient of x 8 in (x 2 + 2y / x) 10 Solution: The (r + 1)th term in the expansion of (x 2 + 2x) 10 is 10 C r (x 2) 10-r (2y / x) r 10 C r 2 r y r x 20-2r-r Thus, for the term in x 8 , we have 10 C r 2 r r x 20-3r , 20 – 3r = 8, 12 = 3r ⇒ r = 4 ∴

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163/248 **SUBMITTED TEXT** 124 WORDS **72%** **MATCHING TEXT** 124 WORDS

a + b) n = n C 0 a n + n C 1 a n – 1 b + n C 2 a n – 2 b 2 + n C 3 a n – 3 b 3 = a n + na n – 1 b + a n – 2 b 2 + a n – 3 b 3 .

SA unit 2.docx (D32968061)

164/248 **SUBMITTED TEXT** 150 WORDS **66%** **MATCHING TEXT** 150 WORDS

Find the coefficient of 'x' in the expansion of (1-2x 3 + x 5) [1 + (1/x)] 8 Sol.: (1-2x 3 + 3x 5) [1 + (1/x)] 8 = (1-2x 3 + 3x 5) [1 + 8 C 1 (1/x) + 8 C 2 (1/ x 2) + 8 C 3 (1/ x 3) + 8 C 4 (1/ x 4)+ 8 C 5 (1/ x 5)+ --- + 8 C 8 (1/ x 8) coefficient of x = -2. 8

SA unit 2.docx (D32968061)

165/248 **SUBMITTED TEXT** 384 WORDS **25%** **MATCHING TEXT** 384 WORDS

x m] [n C o – n C 1 x + n C 2 x 2 + ------- + (-1) n n C n x n] Coefficient of x = m C 1 x n C o – m C 0 . n C 1 = m! / (1! x m-1!) x 1 – 1 x n! / (1! x n-1!) = m – n = 3 --------------- < (i) Coefficient of x 2 = - m C 1 x n C 1 + n C 0 x m C 2 + m C 0 x n C 2 = - m!/ (1! x m-1!) x n! / (1! x n-1!)+ 1 x m! / (2! x m-2!) + 1 x n! / (2! x

x) n = 1 + n C 1 n C 2 x 2 + n C 3 x 3 + ... + n C r x r + ... + n C n x n = n C r x r Illustration 2: Expand (i) (1 – 2x) 4 (ii) (1 + x + x 2) 3 in powers of x (i) (1 – 2x)) 4 = 1 + 4 C 1 (-2x) 1 + 4 C 2 (-2x) 2 + 4 C 3 (-2x) 3 + 4 C 4 (-2x) 4 = 1 + 4 (-2 x) + 6(4 x 2) + 4(-8x 3) + 16x 4 = 1 – 8x + 24x 2 – 32

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

166/248	SUBMITTED TEXT	261 WORDS	23%	MATCHING TEXT	261 WORDS

r + 2)th term in the expansion of (1 + x) 43 are equal, find 'r'. Sol.: In (1 + x) 43 : T 2r+1 = 43 C 2r . x 2r Coefficient = 43 C 2r And T r+2 = 43 C r+1 x r+1 Coefficient = 43 C r According to the questions: 43 C 2r = 43 C r+1 2r + r + 1 = 43

r + 1)th term in the expansion of (x 2 + 2 y / x) 10 is 10 C r (x 2) 10- r (2 y / x) r 10 C r 2 r y r x 20-2r- r Thus, for the term in x 8 , we have 10 C r 2 r r x 20-3r = 8, 12 = 3

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

167/248	SUBMITTED TEXT	177 WORDS	52%	MATCHING TEXT	177 WORDS

C r . (x 2) 15-r . 1/ x r T r+1 = 15 C r . x 30-3r ------------- -< (i) Putting : 30 – 3r = 0 ? r = 10 From (i) T 11 = 15 C 10 = 15!/(10! x 5!) = (15 x 14 x 13

C r x 12-r (-½x) r = 12 C r x 12-r (1 / x r) (-½) r Now x 12-r (1 / x r) = x 12-2

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

168/248	SUBMITTED TEXT	360 WORDS	24%	MATCHING TEXT	360 WORDS

x + x logx) 5 is 10,00,000. Find the value of 'x'. Sol.: Putting log 10 x = z in the given expression : We have : (x + x z) 5 T 3 = T 2+1 = 5 C 2 (x) 5-2 (x z) 2 = 5 C 2 x 3 . x 2z = 5! / (2! x 3!) x 2z+3 = (5 x 4) / 2! x 2z+3 = T 3 = 10x 2z+3 ? 10,00,000 = 10. x 2

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

169/248	SUBMITTED TEXT	284 WORDS	67%	MATCHING TEXT	284 WORDS

a) n-4 (-b) 4 T 5 = n C 4 a n-4 b 4 T 6 = T 5+1 = n C 5 (a) n-5 (-b) 5 = - n C 5 a n-5 b 5 T 5 + T 6 = 0 ? n C 4 a n-4 b 4 – n C 5 a n-5 b 5 = 0 or n C 4 a n-4 b 4 = n C 5 a n-5 b 5

SA unit 2.docx (D32968061)

170/248	SUBMITTED TEXT	201 WORDS	50%	MATCHING TEXT	201 WORDS

a n-4 / (n-4)(n-5!) = a n-5 / 5(n-5!) b or a n-4 / a n-5 = b / 5 (n-4) or a (n-4)-(n-5) = (n – 4) / 5 .b or a = (n – 4)/5 . b or a/b = (n – 4) / 5

SA unit 2.docx (D32968061)

171/248	SUBMITTED TEXT	20 WORDS	83%	MATCHING TEXT	20 WORDS

Find the coefficient of x r in the expansion of [x + (1/x)]

SA unit 2.docx (D32968061)

172/248	SUBMITTED TEXT	68 WORDS	100%	MATCHING TEXT	68 WORDS

a 2 – 1) / (a 2 – a) = (a+1) (a-1) / [a (a-1)] = (a + 1) / a = 1 + 1 / a ? (

SA CMP501 CMP 250-Mathematics for computers.pdf (D164861862)

173/248	SUBMITTED TEXT	183 WORDS	82%	MATCHING TEXT	183 WORDS

y – 1) / (y – y 1/2) = 1 + 1 /vy ? (y + 1) / (y 2/3 – y 1/3 +1) - (y – 1) / (y – y 1/2)] 10 = [y 1/3 + 1 – 1 – (1/ y 1/2)] 10 = (y 1/3 – y -1/2) 10 In (y 1/3 – y -1/2) 10 ,

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

174/248	SUBMITTED TEXT	94 WORDS	58%	MATCHING TEXT	94 WORDS

x 4!) = (10 x 9 x 8 x 7) / (4 x 3 x 2 x 1) T 5 = 210 Example x 4-r occurs in the expansion of [x + (1/ x 2)] 4

SA unit 2.docx (D32968061)

175/248	SUBMITTED TEXT	179 WORDS	40%	MATCHING TEXT	179 WORDS

the term independent of 'x' in the expansion of [x + (1/x)] 2n is [1. 3. 5. ---- (2n-1) / (n!)] 2 n Sol.: General Term T r+1 = 2n C r (x) 2n-r (1/x) r = 2n C r . x 2n-2r ---------< (

the term independent of x in the expansion of (x – ½x) 12 Solution: The general term will be 12 C r x 12-r (-½x) r = 12 C r x 12-r (1 / x r) (-½) r Now x 12-r (1 /

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

176/248	SUBMITTED TEXT	310 WORDS	70%	MATCHING TEXT	310 WORDS

n C n = 2n! / (n! x n!) = [2n (2n-1) ------ 3. 2. 1] / (n! x n!) = { 2n (2n-2) ---- 4. 2 } { (2n-1) (2n-3) ----- 3.1.} / (n! x n!) = [2 n {n (n-1) -----2.1.}] { (2n-1) ------- 4.3.1.} / (n! x n!) = 2n. n!{(2n-1) ---- 5. 3. 1. / (n! x n!) = {1. 3. 5. ----- (2n -1)} 2 n / n!

n – 1) + 1 .... (2) (n – 2) 3 3(n – 2) 2 + 3(n – 2) + 1 .... (3) - - - - - - - - - - - 3 3 – 2 3 3.2 3 + 3.2 + 1 .... (n) Adding these n identities (n + 1) 3 – 1 3 3(1 2 + 2 2 + ... + n 2) + 3(1 + 2 + 3 + ... + n) + 2 3S 2 + n(n+1) + n 3S 2 (n + 1) 3 – 1 3 – 3 (n + 1) – n = n 3 + 3n 2 + 3n – 3 (n + 1) – n = {2n 2 + 6n + 6 – 3n – 3 – 2} = {2n + 1} = (n + 1),

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

177/248 **SUBMITTED TEXT** 563 WORDS **27%** **MATCHING TEXT** 563 WORDS

n C 2 x n-2 . a 2 = 84 ------------<(i) T 4 = n C 3 x n-3 a 3 = 280----------<(ii) and T 5 = n C 4 x n-4 a 4 = 560 --------<(iii) egn (i) x egn(iii) : [n C 2 x n-2 a 2] [n C 4 x n-4 a 4] = 84 x 560 = n!/[2! x (n-2)!] x n! / [4! x (n-4)!] . x 2n-6 a 6 = 84 x 560 or n (n-1) / 2 x n(n-1) (n-2) (n-3) / 4! x x 2n-6 a 6 = 84 x 560 -------< (iv) Squaring of eg n (ii), we have : (n C 3 x n-3 a 3) 2 = 280 2 ? n C 3 x n C 3 x x 2n-6 x a 6 = 280 2 = n! / [3! x (n-3)!] x n! / [3! x (n-3)!] x x 2n-6 a 6 = 280 2 or n (n-1)(n-2) / 6 x n(n-1) (n-2) (n-3) / 3! x x 2n-6 a 6 = 280 x 280 -------< (v) egn (v) egn(iv) : ? n 2 (n-1) 2 (n-2) 2 / (6 x 3!) x 2 x 4! / [n 2 (n-1) 2 (n-2)(n-3)] = (280 x 280) / (84 x 560) or 4 (n-2) / 3 (n-3) = 5 / 3

n – 1) x n – 1 + (n n) x n , {\ displaystyle (1+x)^{n}={n \choose 0}x^{0}+{n \choose 1}x^{1}+{n \choose 2}x^{2}+\ cdots +{n \ choose {x^{n-1}+{ n \ choose x^{n},} {\displaystyle (1+x)^{n}={n \choose 0}x^{0}+{n \ choose 1}x^{1}+{n \choose 2}x^{2}+\cdots +{n \choose {n-1}}x^{n-1}+{n \choose n}x^{n},} or equivalently (1 + x) n = ∑ k = 0 n (n x k , {\displaystyle (1+x)^{n}=\sum _{k=0}^{n}{n \choose k}x^{k},} {\displaystyle (1+x)^{n}=\sum _{k=0}^{n}{n \choose x^{k},} or more explicitly[13] x) n = 1 + n x + n (n – 1) 2 ! x 2 + n (n – 1) (n – 2) 3 ! x 3 + ⋯ + n x n – 1 + x n . {\displaystyle (1+n}=1+nx+{\frac {n(n-1)}{2!}}x^{2}+{\frac {n(n-1)(n-2)}{3!}}x^{3}+\cdots +nx^{n-1}+x^{n}.} {\displaystyle (1+x)^{n}=1+nx+{\n(n-1)}{2!}}x^{2}+{\frac {n(n-1)(n-2)}{3!}}x^{3}+\cdots +nx^{n-1}+x^{n}.}

W http://en.wikipedia.org/wiki/Binomial_theorem

178/248 **SUBMITTED TEXT** 225 WORDS **60%** **MATCHING TEXT** 225 WORDS

n!) / [(4/3)n-r]! x [(4n/1) – 4(n-r)/3]! = (4n!) / [(4/3)n-r]! x [(4/3) 2(n+r)]! Example Find the coefficient of x 50 in (1+x) 41 (1-x+x 2) 40 . Sol.: (1+x) 41 (1-x+x 2) 40 = (1+x) (1+x) 40 (1-x+x 2) 40 = (1+x) [(1+x) (1-x + x 2)] 40 = (1+

SA unit 2.docx (D32968061)

179/248 **SUBMITTED TEXT** 18 WORDS **64%** **MATCHING TEXT** 18 WORDS

The binomial theorem describes the algebraic expansion of powers of a binomial, hence it is referred to

the binomial theorem (or binomial expansion) describes the algebraic expansion of powers of a binomial. According to the theorem, it is possible to

W http://en.wikipedia.org/wiki/Binomial_theorem

180/248 **SUBMITTED TEXT** 489 WORDS **22%** **MATCHING TEXT** 489 WORDS

x 5 a 2) = 280 / 84 [7! / (3! x 4!)a] / [7! / (2! x 5!)x] = 10 / 3 or 7! / (3! x 4!) x (2! x 5!) / 7! x a / x = 10 / 3 or 5 / 3 x a / x = 10 / 3 or a = 2x Putting this value in eqn (vi): 7 C 2 . x 5 . 4x 2 = 84 or 7! / (2! x 5!)x 7 = 21 ? (7 x 63) / 2 x 7 = 21 x 7 = 1 ? x = 1 Putting this value in (ix) = a = 2 Example The 6 th term in the expansion of [(1/ x 8/3) + x 2 log 10 x] 8 is 5600. Prove that x =10. Sol.: T 6 = T 5+1 = 8 C 5 (1/ x 8/3) 8-5 (x 2 log 10 x) 5 or 8 C 5 x (1 / x 8) x c 10 x (log 10 x) 5 = 5600 ? 8! / (5! x 3!) x c 2 (log 10 x) 5 = 5600 ? 8. 7. 6. / 6 x c 2 (log 10 x) 5 = 5600 or x 2 (

SA unit 2.docx (D32968061)

181/248	SUBMITTED TEXT	26 WORDS	93%	MATCHING TEXT	26 WORDS

According to the theorem, it is possible to expand the power (x + y) n into a sum involving terms of the form

According to the theorem, it is possible to expand the polynomial (x + y)n into a sum involving terms of the form

W http://en.wikipedia.org/wiki/Binomial_theorem

182/248	SUBMITTED TEXT	35 WORDS	100%	MATCHING TEXT	35 WORDS

where the exponents b and c are nonnegative integers with b + c = n, and the coefficient a of each term is a specific positive integer depending on n and b. ?

where the exponents b and c are nonnegative integers with b + c = n, and the coefficient a of each term is a specific positive integer depending on n and b.

W http://en.wikipedia.org/wiki/Binomial_theorem

183/248	SUBMITTED TEXT	65 WORDS	73%	MATCHING TEXT	65 WORDS

is known as the binomial coefficient n b ? ? ? ? ? ? or n c ? ? ? ? ? ? (the two have the same value). ? The coefficients for varying n and b can be arranged to form Pascal's triangle. ? Numbers also arise in combinatorics, where n b ? ? ? ? ? ? gives the number of different combinations of b elements that can be chosen from an n-element set.

is known as the binomial coefficient (n b) {\displaystyle {\tbinom {n}{b}}} {\displaystyle {\tbinom {n}{b}}} or (n c) {\displaystyle {\tbinom {n}{c}}} {\displaystyle {\tbinom {n}{c}}} (the two have the same value). These coefficients for varying n and b can be arranged to form Pascal's triangle. These numbers also occur in combinatorics, where (n b) {\displaystyle {\tbinom {n}{b}}} {\displaystyle {\tbinom {n}{b}}} gives the number of different combinations of b elements that can be chosen from an n-element set.

W http://en.wikipedia.org/wiki/Binomial_theorem

184/248	SUBMITTED TEXT	19 WORDS	75%	MATCHING TEXT	19 WORDS

Example Find the simple interest on $2,000 for 10 years at the rate of 10% per annum.

SA Objective Mathematics -II (BMGE- 02)- FINAL.pdf (D162012805)

185/248	SUBMITTED TEXT	20 WORDS	85%	MATCHING TEXT	20 WORDS

Example Calculate the simple interest on $ 4,000 for 5 years at the rate of 6% per annum.

SA Objective Mathematics -II (BMGE- 02)- FINAL.pdf (D162012805)

186/248	SUBMITTED TEXT	19 WORDS	75%	MATCHING TEXT	19 WORDS

the simple interest on $7000 for 8 years at the rate of 7% per annum. 9. Find the

SA Objective Mathematics -II (BMGE- 02)- FINAL.pdf (D162012805)

187/248	SUBMITTED TEXT	14 WORDS	100%	MATCHING TEXT	14 WORDS

at the rate of 5% per annum. 10. Find the rate of interest

SA Objective Mathematics -II (BMGE- 02)- FINAL.pdf (D162012805)

188/248	SUBMITTED TEXT	56 WORDS	61%	MATCHING TEXT	56 WORDS

Solution: Here, a = 2 and b =9. a 2 × a = a 3 ; a 2 × 3b = 3a 2 × b; b 2 × 3a = 3a × b 2 ; b 2 × b =

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

189/248	SUBMITTED TEXT	23 WORDS	89%	MATCHING TEXT	23 WORDS

a a d a d a d ? ? ? Here a is the first term and 'd' is the common difference.

a + d, a + 2d, a + 3d, a + 4d, . . . where a = 1 is the first term, and d = 2 is the common difference.

W http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-apgp-2009-1.pdf

190/248	SUBMITTED TEXT	20 WORDS	90%	MATCHING TEXT	20 WORDS

a d a a d a d a d a d ? ? ? ? ? ? ? ?

a − d) 2 + a 2 + (a + d) 2 + (a + 2d) 2 = 165 a 2 − 4ad + 4d 2 + a 2 − 2ad + d 2 +

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

191/248	SUBMITTED TEXT	59 WORDS	61%	MATCHING TEXT	59 WORDS

Solution: Here, a = 7 and b = 1 a 2 × a = a 3 ; a 2 × 3b = 3a 2 × b; b 2 × 3a = 3a × b 2 ; b 2 × b =

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

192/248	SUBMITTED TEXT	21 WORDS	75%	MATCHING TEXT	21 WORDS

n Terms of an A.P. The sum to n terms of A.P. , , 2 , 3 ,......., (1)

n terms of an A.P To find the sum S of n terms of A.P

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

193/248	SUBMITTED TEXT	135 WORDS	31%	MATCHING TEXT	135 WORDS

a a d a d a d a n d ? ? ? ? ? is ( ) (2) .... [(1)] a a d a d a n d ? ? ? ? ? ? ? ? which is given by ? ? 2 (1) 2 n n S a n d ? ? ? ...(2) Since (1) , l a n d ? ? ? we can write the formula for S n as ? ? (1) 2 n n S a a n d ? ? ? ? ? ? 2 n n S a l ? ? ? ? ? ? ? 2 (1) 2 2 n n n S a n d a l ? ? ? ? ? ? where (1) n l T a n d ? ? ? ? .

a + (a + d) + (d) + ⋯ + (a + (n − 2) d) + (a + (n − 1) d) . {\S_{n}=a+(a+d)+(a+2d)+\dots +(a+(n-2)d)+(a+(d).} {\displaystyle S_{n}=a+(a+d)+(a+2d)+\dots +(a+(n-2)d)+(a+(n-1)d).} Rewriting the in reverse order: n = (a + (n − 1) d) + (a + (n − 2) d) + ⋯ + (a + 2 d) + (a + d) + a . {\displaystyle S_{n}=(a+(n-1)d)+(a+(n-2)d)+\dots +(a+2d)+(a+d)+displaystyle S_{n}=(a+(n-1)d)+(

W http://en.wikipedia.org/wiki/Arithmetic_progression

194/248	SUBMITTED TEXT	27 WORDS	60%	MATCHING TEXT	27 WORDS

th Term of an A.P. The n th term of the A.P. , , 2 , 3 ,...... a a d a d

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

195/248	SUBMITTED TEXT	83 WORDS	45%	MATCHING TEXT	83 WORDS

the A.P. 3, 1,1,3,5,...... ? ? Solution: The A.P. is 3, 1,1,3,5,...... ? ? 3and 2, 10 a d n ? ? ? ? ? term (– 1) ? ? th n a n d 10th term 3 (10 1)2 ? ? ? ? 3 18 15 ? ? ? ? Example: If the 5th term of an A.P. is 10 and 8th term is 16, find the first term and the common difference Solution: 5th term

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

196/248	SUBMITTED TEXT	89 WORDS	41%	MATCHING TEXT	89 WORDS

the 4th term of an A.P. is 7 and the 7th term is 13, find the 12th term. Solution: 4 term 5 3 7 th a d ? ? ? ? ...(1) 7 term 13 6 13 th a d ? ? ? ? ...(2) Let us solve these equations. (2) (1) 3 6 d ? ? ? 2 d? ? Substituting 2 d ? in (1), we get 3(2) 7 7 6 1 a a ? ? ? ? ? ? 1, 2

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

197/248	SUBMITTED TEXT	70 WORDS	57%	MATCHING TEXT	70 WORDS

Find the common difference, nth term and 15th term of the A.P. 3, 5, 13,...... ? ? Solution: 3, 5, 13,...... ? ? is an A.P. 3, 5 3 8 a d ? ? ? ? ? ? ? commondifference 8 ? ? ? thterm (1) 3 (1)(8) n a n d n ? ? ? ? ? ? ? 3 8 8 n ? ? ?

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

198/248	SUBMITTED TEXT	42 WORDS	54% MATCHING TEXT	42 WORDS

n 2 . Solution: 2 n S n? 2 1 1 2 2 2 2 (1) (1) 2 1 n n n n S n T S S n n n n n ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 1 2 3 2 1 2(1) 1 1 2(2) 1 3 2(3) 1 5 etc., n T n

n – 2) + 1 .... (3) - - - - - - - - - - - 3 3 – 2 3 3.2 3 + 3.2 + 1 .... (n-1) 2 3 – 1 3 3.1 3 + 3.1 + 1 .... (n) Adding these n identities (n + 1) 3 – 1 3 3(1 2 + 2 2 + ... + n 2) + 3(1 + 2 + 3 + ... + n) + 2 3S 2 + n(n+1) + n 3S 2 (n + 1) 3 – 1 3 – 3 (n + 1) – n = n 3 + 3n 2 + 3n – 3 (n + 1) –

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199/248	SUBMITTED TEXT	40 WORDS	70% MATCHING TEXT	40 WORDS

n ? ? ? ? ? ? ? ? 2 (1) 2 2(1) (1)3 2 2 3 3 2 (3 5) 2 n n S a n d n n n n n n ? ? ? ? ? ? ? ? ? ? ? ? put 10 n ? 10 10(30 5) 2 10 25 5 25 125 2

n(n+1) + n 3S 2 (n + 1) 3 – 1 3 – 3 (n + 1) – n = n 3 + 3n 2 + 3n – 3 (n + 1) – n = {2

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

200/248	SUBMITTED TEXT	84 WORDS	68% MATCHING TEXT	84 WORDS

n S n a n d a d n n n n ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 90 6 150 2 n n ? ? ? 2 2 2 2 2(45 3) 150 2 (45 3) 150 45 3 150 15 50 (dividingby3) 15 50 0 10 5 50 0 (10) 5(10) 0 (10)(5) 0 10, 5 n n n n n n n n n n n n n n n n n n n n n ?

n – 1) 3 3(n – 1) 2 + 3(n – 1) 3 – (n – 2) 3 3(n – 2) 2 + 3(n – 2) + 1 .... (3) - - - - - - - - - - - 3 3 – 2 3 3.2 3 + 3.2 + 1 .... (n-1) 2 3 – 1 3 3.1 3 + 3.1 + 1 .... (n) Adding n identities (n + 1) 3 – 1 3 3(1 2 + 2 2 + ... + n 2) + 3(1 + 2 + 3 + ... + n) + 2 3S 2 + n(n+1) + n 3S 2 (n + 1) 3 – 1 3 – 3 (n + 1) – n = n 3 + 3n 2 + 3n – 3 (n + 1) – n = {2 n 2 + 6n + 6 – 3n – 3 – 2} = {2n + 1} = (

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

201/248	SUBMITTED TEXT	62 WORDS	54% MATCHING TEXT	62 WORDS

n l a n d S a n d ? ? ? ? ? ? ? ? ? 52 5 (1) and 470 2(5) (1) 2 n n d n d ? ? ? ? ? ? ? ? ? ? ? ? 57 (1) and 470 10 (1) 2 n n d n d ? ? ? ? ? ? ? Note

n – 2)d] + [a + (n – 3)d] + ... + a .... 2 Hence, (1) + (2) gives: 2S = [2a + (n – 1)d] + [2a + (n – 1)d] + [2a + (n – 1)d] + ... to nterms = n [2a + (n – 1)d[a + (n – 1)Note:

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202/248	SUBMITTED TEXT	110 WORDS	40% MATCHING TEXT	110 WORDS

The sum of 3 numbers in A.P. is –24 and then product is 288. Find the numbers. Solution: Let , , a d a a d ? ? be the 3 numbers in A.P. Sum 24 3 24 8 a d a a d a a ? ? ? ? ? ? ? ? ? ? ? ? ? ? Product ( ) ( ) 288 a d a a d ? ? ? ? substituting 8 a ? ? , we get 2 2 2 (8)(8)(8) 288 (64) 36 64 36 100 10 d d d d d d ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? the numbers are 8 10, 8, 8 10

The sum of three consecutive terms an A.P is 18, and their products is 120. Find the term Solution: Let the term be: a – d, a, a + d (we take the term a, a+d, a+2d but it is simplier to a – d, a, a + d) a – d+ a + a + d = 18 3a =18 a = 6 The Product = (a – d) a (a +) = 6(6 – d)(6 + d) = 120 6(6 2 – d 2) = 120 6(36 – d 2) = 120 36 – d 2 = 20 d 2 = 36 – 20 d = + 4 If the terms are 2, 6, 10

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

203/248	SUBMITTED TEXT	107 WORDS	57% MATCHING TEXT	107 WORDS

S a n d T a n d ? ? ? ? ? ? ? ? 11 1 82 2 11 165 2 (11 1) 2 2 11(2 10) 165 S a d a d ? ? ? ? ? ? ? ? ? ? 2 10 15 ? ? a d ...(1) 15 12 ? T ? (15 1) 12 ? ? ? a d ? 14 12 ? ? a d ...(2) ? 2 28 24 ? ? a d (multiplying by (2)) 2 10 15 2 28 24 ? ? ? ? a d a

S a + (n − 1) d) + (a + (n − 2) d) + ⋯ + (a + 2 d) + (d) + a . {\displaystyle S_{n}=(a+(n-1)d)+(a+(n-2)d)+\dots + (a+2d)+(a+d)+a.} {\

W http://en.wikipedia.org/wiki/Arithmetic_progression

204/248	SUBMITTED TEXT	89 WORDS	40% MATCHING TEXT	89 WORDS

A.P. is 12:5. Find the ratio of the 13th term to the 4th term. Solution: Let the A.P. be , , 2 ,..... a a d a d ? ? 7thterm 6 12 3rdterm 2 5 ? ? ? ? a d a d 5 30 12 24 a d a d ? ? ? ? 7 6 a d ? ? ...(1) 13thterm 12 4thterm 3 7(12) (multiplybothnumeratoranddenominatorby7) 7(3) 7 84 = 7 21 6 84 [using (1)] 6 21 90 10 27 3 ? ? ? ? ? ? ? ? ? ? ? ? ? ? a d a d a d a d a d a d d d d d d d Therefore the 13

A.P is 25 and the sum of the series be d, then the terms are (a − 2 d), (a − a, a + d, a + 2d. From the a − 2d) + (a − d) + a + (a + d) + (a + 2d) = 25 5a = 25 a = 5 Also, (a − 2d) 2 + (a − d) 2 + a 2 + (a + d) 2 + (a + 2d) 2 = 165 a 2 − 4ad + 4d 2 + a 2 − 2ad + d 2 + d 2)+ (a 2 +4ad+4d 2) = 165 5a 2 + 10d 2 = 165 i.e a 2 + 2d 2 = 33 2d 2 = 8 d 2 = 4 d = + 2 Hence, the

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205/248	SUBMITTED TEXT	129 WORDS	35% MATCHING TEXT	129 WORDS

A.P. is 16 and the sum of their squares is 84. Find the numbers. Solution: Let 3 , , , 3 a d a d a d a d ? ? ? ? be the four terms of an A.P. um 3 3 16 4 16 4 S a d a d a d a d a a ? ? ? ? ? ? ? ? ? ? ? ? ? Business Mathematics Note 213 2 2 2 2 2 2 2 2 2 2 2 2 2 Sum of their squares (3) ( ) ( ) (3) (4 3) (4) (4) (4 3) 16 24 9 16 8 16 8 16 24 9 64 20 a d a d a d a d d d d d d d

A.P is 25 and the sum of the series be d, then the terms are (a − 2d), (a − d), a, a + d, a + 2d. From the question (a − 2 d) + (a − d) + a + (a + d) + (a + 2d) = 25 5a = 25 a = 5 Also, (a − 2 d) 2 + (a − d) 2 + a + d) 2 + (a + 2d) 2 = 165 a 2 − 4ad + 4d 2 + a 2 − 2ad + d 2 + d 2)+ (a 2 +4ad+4d 2) = 165 5a 2 + 10d 2 = 165 i.e a 2 + 2d 2 = 33 2d 2 = 8 d = + 2

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206/248	SUBMITTED TEXT	92 WORDS	27% MATCHING TEXT	92 WORDS

d d d d d ? 2 2 2 64 20 84 20 84 64 20 1 1 d d d d ? ? ? ? ? ? ? ? ? ? ? ? Therefore, the numbers are 4 3,4 1,4 1,4 3 or4 3,4 1,4 1,4 3 i.e.,1,3,5,7or7,5,3,1 ? ? ? ? ? ? ? ? Example: If the 5th term of an A.P. exceeds the 2nd term by 12 and 15th term is 28, find the

d)(6 + d) = 120 6(6 2 − d 2) = 120 6(36 − d 2) = 120 36 − d 2 = 20 d 2 = 36 − 20 d = + 4 If d = 4, the terms are 2, 6, 10 and if d = - 4 they are 10, 6, 2. The numbers are the same but form two different A.Ps. 2. The first term of an arithmetic series is 7, the last term is 70, and the sum is 385. Find the

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207/248	SUBMITTED TEXT	143 WORDS	21%	MATCHING TEXT	143 WORDS

a d a d (4) ( ) 12 3 12 4 a d a d d d ? ? ? ? ? ? ? ? ? 15thterm 14 28 14 4 28 28 56 28 ? ? ? ? ? ? ? ? ? ? ? ? a d a a a ? The A.P. is 28, 24, 16, 12, 8, 4,0,4,8,..... ? ? ? ? ? ? Example If the nth terms of the A.Ps 3, 10, 17, ... and 63, 65, 67,.... are equal, find the value of n. Solution: 3, 10, 17,....... 3, 7 a d ? ? thterm (1) 3 (1)7 7 4 ? ? ? ? ? ? ? ? ? ? n a n d n n 63, 65, 67......... 63, 2 a d ? ? thterm (1) 63 (1)2 2 61 7 4 2 61 5 65 13 n a n d n n n

a+(n-1)d)+(a+(n-2)d)+\dots +(a+2d)+(a+d)+a.} {\displaystyle S_{n}=(a+(n-1)d)+(a+(n-2)d)+\dots + (a+2a+d)+a.} Adding the corresponding terms of both sides of the two equations and halving both sides: S n = n 2 [2 n – 1) d] . {\displaystyle S_{n}={\frac {a+(n-1)d].} {\displaystyle S_{n}={\n}{2}}[2a+(n-1)d].} This formula can be simplified as: S n = n 2 [a + a + (n – 1) d] . = n 2 (a n) . = n 2 (

W http://en.wikipedia.org/wiki/Arithmetic_progression

208/248	SUBMITTED TEXT	95 WORDS	38%	MATCHING TEXT	95 WORDS

th term of the series 3, 6, 9, 12,...... Solution: 3, 3, 17 a d n ? ? ? 17thterm 3 (17 1)3 3 16(3) 51 ? ? ? ? ? ? ? ? Example: Which term of the A.P. 1,3,7,...... ? is 79 ? Solution: 1,3,7,...... ? is the given A.P. 1, 4 thterm (1) 79 1 (1)4 79 1 4 4 79 4 5 79 4 84 21 ? a d n a n d n n n n n ? 21st term of the

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209/248	SUBMITTED TEXT	32 WORDS	58%	MATCHING TEXT	32 WORDS

ar ? ? ...(4) Sum to ‘n’ Terms of a G.P. The sum to ‘n’ terms of the G.P. 2 3 1 , , , ,....

ar 2 Sum of the first n terms of a G.P To find the sum of the first n terms of a G.P,

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

210/248	SUBMITTED TEXT	18 WORDS	76%	MATCHING TEXT	18 WORDS

n a ar ar ar ar ? is 2 3 1 ..... n a ar ar ar ar ? ? ? ? ? ?

n = a + ar + ar 2 + ar 3 + . . . + ar n-1 rSn = ar + ar 2 + ar 3 + . . . + ar

W http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-apgp-2009-1.pdf

211/248	SUBMITTED TEXT	59 WORDS	28%	MATCHING TEXT	59 WORDS

Find the 10th term of the G.P. 1, 2, 4, 8, ... Solution: The G.P. is 1, 2, 4, 8,.... 1 10 1 9 1, 2 thterm 10thterm 1(2) 2 512 ? ? ? ? ? ? ? ? ? n a r n ar Example: If the 5th term of a G.P. is 1 32 and 8th term is 1 256 , find the first term and the common ratio.

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212/248	SUBMITTED TEXT	32 WORDS	71%	MATCHING TEXT	32 WORDS

r ? ? ? 1 2 r ? ? Substituting 1 2 r ? in (1), we get 4 1 1 2 32 1 1 32 1 2 16 a a ? ? ? ? ? ? ? ? ? ? ? first term is 1 2 and the common ratio

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

213/248	SUBMITTED TEXT	31 WORDS	79%	MATCHING TEXT	31 WORDS

n n n r r r r r r r r r r r r r r r r r ?

SA Maths Extended Essay.pdf (D31546002)

214/248	SUBMITTED TEXT	71 WORDS	61%	MATCHING TEXT	71 WORDS

r r ? ? ? Multiply by 4r 2 2 2 2 2 12 12 12 63 12 12 63 12 0 12 51 12 0 4 17 4 0 4 16 4 0 4 (4) 1(4) 0 (4)(4 1) 0 1 4or 4 r r r r r r r r r r r r r r r r r r ?

SA Maths Extended Essay.pdf (D31546002)

215/248	SUBMITTED TEXT	18 WORDS	78%	MATCHING TEXT	18 WORDS

The sum of three numbers in G.P. is 42 5 and their product is –8. Find the numbers.

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

216/248	SUBMITTED TEXT	47 WORDS	77%	MATCHING TEXT	47 WORDS

r r r r r r r r r r r r r r r r r r r ?

SA Maths Extended Essay.pdf (D31546002)

217/248	SUBMITTED TEXT	62 WORDS	69%	MATCHING TEXT	62 WORDS

r r r r r r r r r r ? ? ? ? ? ? ? ? ? ? ? ? ? ? 4 2 2 2 2 2 2 2 2 2 2 i.e.,4 16 4 0 4 (4) 1(4) 0 4 0,4 1 0 1 4, 4 1 2, 2 r r r r r r r r r r r r ?

SA Maths Extended Essay.pdf (D31546002)

218/248	SUBMITTED TEXT	40 WORDS	57%	MATCHING TEXT	40 WORDS

n n n T ? ? ? ? ? ? ? ? ? ? ? ? ...(1) Sum to n terms (1) 1 n a r r ? ? ? 1 1 1 3 2 1 1 2 n ? ? ? ? ? ? ? ? ? ? 1 2 1 2 3 2 n n ? ? ? ? ? ? ? ? ? 1 1 2 1 i.e., 3 2 ? ? ? ? ? ? ? ? ? ? n n n

SA Maths Extended Essay.pdf (D31546002)

219/248	SUBMITTED TEXT	15 WORDS	76%	MATCHING TEXT	15 WORDS

The sum of the first two terms is 15 and the sum of the

the sum of the first 8 terms is twice the sum of the

W http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-apgp-2009-1.pdf

220/248	SUBMITTED TEXT	11 WORDS	96%	MATCHING TEXT	11 WORDS

log log( ) 2log log log log log log log log log log log

Log Log - = - - ⇒ ( ) 2 5 5 8 10 10 10 Log Log x Log Log - = - ⇒ 2 5 8 2 5 5 8 2 5 10 10 10 10 10 10 10 Log Log Log Log Log Log Log

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

221/248	SUBMITTED TEXT	14 WORDS	100%	MATCHING TEXT	14 WORDS

Let S n be the sum of n terms of the

let S n be the sum of n terms of the

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222/248	SUBMITTED TEXT	74 WORDS	35%	MATCHING TEXT	74 WORDS

x if 1, 2, 1 x x x ? ? ? are in G.P. Solution: 1, 2, 1 x x x ? ? ? are in G.P. 2 1 1 2 x x x x ? ? ? ? ? ? ? By cross multiplication, we get 2 (2) (1)(1) x x x ? ? ? ? ? 2 2 i.e., 4 4 1 4 5 4 x x x x x ? ? ? ? ? ? ? ? ?

SA chapter 2-Summation of Series.docx (D33334824)

223/248	SUBMITTED TEXT	25 WORDS	50%	MATCHING TEXT	25 WORDS

n S 2a n 1 d 2 n 2.1 n 1 1 2 n n 1 S 2 n n 1 is n 2 7.6.5 The Sum of The Squares of First N Natural Numbers

n 3S 2 (n + 1) 3 – 1 3 – 3 (n + 1) – n = n 3 + 3n 2 + 3n – 3 (n + 1) – n = {2n 2 + 6n + 6 – 3n – 3 – 2} = {2n 2 + 3n + 1} = (n + 1) (2n + 1), S 2 = (c) Let S 3 be the sum of the cubes of the first n natural numbers

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224/248	SUBMITTED TEXT	12 WORDS	90% MATCHING TEXT	12 WORDS

the sum of the squares of n natural numbers be S.

the sum of the squares of the first n natural numbers be S 2

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225/248	SUBMITTED TEXT	145 WORDS	70% MATCHING TEXT	145 WORDS

n n 1 3n 3n 1 Putting n 1,2,3,..........., n 1 ,n ,we have 1 0 3.1 3.1 1 2 1 3.2 3.2 1 3 2 3.3 3.3 1 .... n 1 n 2 3 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? 2 3 3 2 . n 1 3 n 1 1 n n 1 3n 3n 1 Adding these, we have ? ? ? ? ? ? ? ?? ? ? ? ? ? ? ? 3 2 2 2 2 3 3 1 2 3 .... 3 1 2 3 .... 1 3 3. 2 3 1 or 3 2 3 1 1 1 2 3 1 1 2 n n n n n n S n n n S n n n n n n n n n n ?? ? 2 1 1 2 1 2 1 6 n n n n n n S ? ? ? ? ? ? ?

n + 1) 3 – n 3 3n 2 + 3n + 1 .... (1) Replacing n by (n-2), ..., 2, 1 in succession, n 3 – (n – 1) 3 3(n – 1) 2 + 3(n – 1) + 1 .... (2) (n – 1) 3 – (n – 2) 3 3(n – 2) 2 + 3(n – 2) + 1 .... (3) - - - - - - - - - - - 3 3 – 2 3 3.2 3 + 3.2 + 1 .... (n-1) 2 3 – 1 3 3.1 3 + 3.1 + 1 .... (n) Adding n identities (n + 1) 3 – 1 3 3(1 2 + 2 2 + ... + n 2) + 3(1 + 2 + 3 + ... + n) + 2 3S 2 + n(n+1) + n 3S 2 (n + 1) 3 – 1 3 – 3 (n + 1) – n = n 3 + 3n 2 + 3n – 3 (n + 1) – n = {2n 2 + 6n 2 + 3n + 1} = (n + 1) (2n + 1), S 2 = (

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

226/248	SUBMITTED TEXT	72 WORDS	28% MATCHING TEXT	72 WORDS

n n n n n n n n n n r S a dr dr dr a n d r a dr r r r a n d r dr r a a n d r r dr r a n d r a S r r r ?? ? ? ? n 1 n n Cor : If r 1Then r ,r

SA Maths Extended Essay.pdf (D31546002)

227/248	SUBMITTED TEXT	23 WORDS	83% MATCHING TEXT	23 WORDS

y ? o x ? x y x 1 X 2 1 1 , y x 1 2 , y x x y X Y

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

228/248	SUBMITTED TEXT	73 WORDS	57% MATCHING TEXT	73 WORDS

the n th term of the series. E.g. ? 2 2 2 2 2 2 3 3 3 3 3 2 n 1 2 3 4 .... n n 1 2 3 4 .... n n 2n 1 1.3 2.5 3.7 .... n(2n 1) n 1 2 3 4 .... n

The r th term of the series r – 1) 3 8r 3 - 12r 2 + 6r – 1, sum of n terms = = = = 2n 2 (n+1) 2 – 2n(n+1)(2n+1) + 3n(n+1) – n{(2

W http://unaab.edu.ng/funaab-ocw/opencourseware/Algebra%20For%20Biological%20Science.pdf

229/248	SUBMITTED TEXT	65 WORDS	51%	MATCHING TEXT	65 WORDS

x 1 > x 2 . If f(x 2) < f(x 1) when x 1 > x 2 , then f(x) is increasing. If f(x 2) > f(x 1) when x 1 > x 2 , then f(x)

SA lesson lesson lesson.pdf (D134982954)

230/248	SUBMITTED TEXT	15 WORDS	100%	MATCHING TEXT	15 WORDS

new data points within the range of a discrete set of known data points.

SA lINEAR 2.docx (D125227109)

231/248	SUBMITTED TEXT	17 WORDS	93%	MATCHING TEXT	17 WORDS

a, a+d, a+2d............... a is the first term and d is the common difference. ?

a + 2d, a + 3d, a + 4d, . . . where a = 1 is the first term, and d = 2 is the common difference.

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232/248	SUBMITTED TEXT	16 WORDS	100%	MATCHING TEXT	16 WORDS

new data points within the range of a discrete set of known data points 29.

SA lINEAR 2.docx (D125227109)

233/248	SUBMITTED TEXT	23 WORDS	55%	MATCHING TEXT	23 WORDS

n terms of an A.P. is given by 2 5 4 n n ? , find the nth term and the A.P. 7.

n terms of a series is 2 n 2 − n. Find the nth term and show that the series is an A.P.

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234/248	SUBMITTED TEXT	31 WORDS	39%	MATCHING TEXT	31 WORDS

the sum to n terms of an A.P. is given by 2 2n n? , find the nth term and the A.P. 8. If the sum to n terms is 2 2

The sum of the first n terms of a series is 2 n 2 − n. Find the nth term and show that the series is an A.P. Solution: Using n = 1 in the sum for n terms, it is

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235/248	SUBMITTED TEXT	40 WORDS	50%	MATCHING TEXT	40 WORDS

term of a G.P. T n = ar n-1 ? Sum to n terms of a G.P. S n
= (1) 1 n a r r ? ?

SA Fundamental of Mathematics- Block-I, II.pdf (D144415340)

236/248	SUBMITTED TEXT	15 WORDS	100%	MATCHING TEXT	15 WORDS

Find three numbers in A.P. whose sum is 9 and product is
15. 10.

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237/248	SUBMITTED TEXT	15 WORDS	100%	MATCHING TEXT	15 WORDS

Find three numbers in A.P. whose sum is 15 and product
is 80. 11.

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238/248	SUBMITTED TEXT	11 WORDS	100%	MATCHING TEXT	11 WORDS

Find three numbers in A.P. whose sum is 12 and

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239/248	SUBMITTED TEXT	14 WORDS	100%	MATCHING TEXT	14 WORDS

three numbers in A.P. whose sum is 18 and product is
162. 13.

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240/248	SUBMITTED TEXT	15 WORDS	100%	MATCHING TEXT	15 WORDS

Find three numbers in A.P. whose sum is 15 and product
is 105. 14.

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241/248	SUBMITTED TEXT	11 WORDS	100%	MATCHING TEXT	11 WORDS

Find three numbers in A.P. whose sum is 15 and

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242/248	SUBMITTED TEXT	11 WORDS	100%	MATCHING TEXT	11 WORDS

Find three numbers in A.P. whose sum is 3 and

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243/248	SUBMITTED TEXT	16 WORDS	78%	MATCHING TEXT	16 WORDS

the 10th term of an A.P. is 23, and the 32nd term is 67, find

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244/248	SUBMITTED TEXT	32 WORDS	53%	MATCHING TEXT	32 WORDS

of a G.P. are 1 and 1 8 respectively, find the ninth term. 36. The sum of three numbers in G.P. is 26 and their product is 216. Find the numbers. 37.

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245/248	SUBMITTED TEXT	15 WORDS	87%	MATCHING TEXT	15 WORDS

Find three numbers in G.P. whose sum is 28 and product is 512. 40.

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246/248	SUBMITTED TEXT	15 WORDS	87%	MATCHING TEXT	15 WORDS

Find three numbers in G.P. whose sum is 31 and product is 125. 41.

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247/248	SUBMITTED TEXT	23 WORDS	55%	MATCHING TEXT	23 WORDS

The sum of the first two terms is 15 and the sum of the second and third terms is 30. Find the

the sum of the first 8 terms is twice the sum of the first 5 terms. Find the

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248/248	SUBMITTED TEXT	36 WORDS	85%	MATCHING TEXT	36 WORDS

is 91. 42. The first term and the last term of a G.P. are respectively 3 and 768 and the sum is 1533. Find the common ratio and the number of terms. 43. Find the

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